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Repeated Moral Hazard with Persistence[*]

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Abstract

This paper considers the optimal contract when the current (hidden) action of an agent has a persistent effect on the future outcome. In this setting, the current outcome is not only a signal of the current action taken by the agent, but also conveys information about his past actions. The optimal contract in a two-effort choice, two-period setting is characterized analytically and numerically. In particular, it is shown that persistence tends to make compensation less responsive to the first-period outcome. At the extreme, there are cases where the agent is perfectly insured against the first-period outcome: the agent obtains the same utility regardless of the first-period outcome. The model is extended to a setting with three effort choices, a three-period setting, and an N-period setting with two-period persistence. Also discussed is an application of our model to the optimal unemployment insurance program. Some empirical evidence is then presented.

Keywords: Repeated Moral Hazard, Persistence, Human Capital, Unemployment Insurance

JEL Classifications: D82, J31, J65

1 Introduction

The existing models of repeated moral hazard typically focus on a time-separable framework where the (unobservable) current effort only affects the current outcome.[1] This separability assumption substantially simplifies the characterization of the optimal contract. Real-life contractual relations however are seldom time-separable. One can find many examples of contractual relations where there are (hidden) intertemporal links across time periods.

For example, consider a situation where an employer wants to train a worker in order to increase his productivity. The effectiveness of the training depends on the effort of the worker, but typically the actual effort level cannot be observed by the employer. The employer wants to design a wage contract conditional on output, since output depends on the productivity of the worker. The worker's productivity, in turn, depends on his past training effort as well as his current effort.

Another example is when there exists a learning-by-doing effect in performing a task. Suppose that the productivity of a worker depends on the current unobservable effort. Suppose also that there is a learning-by-doing effect: the workers who worked harder in the past have higher productivity today. The employer has to design a wage contract recognizing that the current productivity depends on both the current and the past effort.

When the government tries to design an unemployment insurance system, it has to take into account the fact that it cannot observe the worker's search effort. The outcome of search can be influenced not only by the current search effort, but also the past search effort.

We characterize the optimal contract in a two-period principal-agent framework where the second-period outcome is not only a function of the second-period effort but also of the first-period effort. This setting generalizes the environment of Rogerson (1985) in the sense that we allow for the effect of the first-period effort to be persistent while maintaining his basic model structure. The optimal contract is characterized analytically and numerically. It

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[1]See, for example, Lambert (1983), Rogerson (1985), Spear and Srivastava (1987), Phelan and Townsend (1991), and Hopenhayn and Nicolini (1997).

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is shown that an agent with higher first-period output receives a (weakly) larger first-period wage. Given the same history, an agent with higher second-period output receives a strictly larger second-period wage. It is shown that an agent with higher first-period output receives a (weakly) higher second-period wage (controlling for the second-period output). It is also shown that the celebrated martingale result of Rogerson (1985) can be generalized to the current non-separable setting. As a consequence, whether the expected wage is increasing or decreasing depends solely on the curvature of the utility function.

In a series of numerical examples, it is shown that persistence tends to make the compensation less responsive to the first-period outcome. At the extreme, there is a possibility that the agent is *perfectly* insured against the first-period outcome: the agent obtains the same utility regardless of the first-period outcome. This perfect-insurance result is unique to our setting: when there is no persistence, the agent is insured only *partially*.

How can the principal provide incentives without using the first period's compensation? When the agent's effort has a persistent effect, the agent will be concerned about the effect of his first-period effort on the future. If this concern is very strong, he may choose to make an effort in the first period, even if there is no reward for obtaining a good outcome in this period. When effort has a persistent effect, the second-period compensation scheme may be sufficient to provide incentives for the exertion of high effort in both periods. In contrast, in the time-separable setting of Rogerson (1985), it is always optimal to provide a direct reward in the first period. We present a set of necessary and sufficient conditions and show that the perfect-insurance result is likely to occur when the past-period effort has a strong effect on the current outcome.

We extend the model and characterize the optimal contract in a setting where the agent has more than two effort choices. The numerical example shows that perfect insurance can occur in a three-effort choice setting. Then, we extend the model to three periods. It is shown that the perfect-insurance result can occur here as well: the agent can obtain the same utility level regardless of the first- and second-period outcomes. Moreover, when this

occurs, consumption is *perfectly smoothed* in the first two periods: his first-period wage is equal to his second-period wage.

Our model can be applied to examine the optimal unemployment insurance system. The job-search process requires an effort which is, in general, unobservable by the government. In a time-separable setting (e.g. Shavell and Weiss [1979] and Hopenhayn and Nicolini [1997]), the optimal unemployment insurance involves a sequence of benefits which is strictly decreasing over the unemployment spell. The government tries to provide incentives for job search by gradually punishing a worker who stays unemployed for a long period. In our three-period setting, perfect insurance and perfect consumption smoothing imply that the benefit is constant for the first two periods and then drops considerably if the agent cannot find a job in the third period. This scheme is closer to the current U.S. unemployment insurance system (unemployment insurance payments are constant until the termination date) than the previous study.

Applying Fernandes and Phelan's (2000) insight, our model can be extended to a general N-period model when the persistence is limited to two periods. We utilize the dynamic programming technique and numerically characterize a three-period model.

Our analysis has a clear empirical implication: the correlation between pay and performance should be lower in the earlier periods of a contract. Gibbons and Murphy (1992) present evidence of contractual relationships where the structure of the variation in wages follows the predicted pattern. They show that the pay-for-performance elasticity is lower for recently-appointed chief executive officers (CEOs) and that it increases as the CEOs approach retirement. This result is consistent with our predictions: the pay-for-performance relationship is less sensitive to performance in the early periods. Gompers and Lerner (1999) study the pay-for-performance elasticity for venture capital organizations. They consider both cross-sectional and time-series variation in compensation. Their finding is that compensation for smaller and younger venture capitalist organizations is less sensitive to performance than older and larger venture capitalist organizations. Moreover, the pay-for-performance

elasticity increases as the firm's seniority increases.

Some recent studies also consider moral hazard settings in which there are hidden intertemporal links. For example, Fernandes and Phelan (2000) derive the recursive formulation of infinitely-lived optimal contracts in the presence of intertemporal links. They show that it is possible to apply their recursive formulation to a privately observed effort model, where the current output depends both on the past and current effort choice of the agent. As Kocherlakota (2002) points out however, this type of formulation tends to become too complicated to analyze unless one makes drastic simplifying assumptions. Our two- and three-period formulations have the advantage of tractability, and our work can be considered as complementary to the studies which utilize the recursive formulation with an infinite-horizon.

In the next section, the setup of the model is described. In Section 3, some general results are presented. The perfect-insurance result is analyzed in detail in Section 4. In Section 5, we extend the model to three effort choices. In Section 6, the model is extended to three periods. Section 7 contains the application of the model to the optimal unemployment insurance system. In Section 8, we discuss the extension to an N-period model with two-period persistence. In Section 9, some empirical support is presented. Section 10 concludes.

2 Model

2.1 Basic Setup

The relationship between a firm and a worker is analyzed in a principal-agent framework. The firm (principal) hires a worker (agent) and the worker works for the firm for two periods. In each period the agent chooses an effort level that is unobservable to the principal. The effort level, e, can take two values, low effort (L) and high effort (H): $e \in \{L, H\}$ where $L < H$. High effort is costly to the agent. We denote this dependence by the cost function $\alpha(e)$. In particular, high effort is associated with a cost of $\alpha(H) = c$, while low effort entails no cost: $\alpha(L) = 0$. In each period, after the effort level is chosen, output is realized, and

based on the output, the wage payment occurs. Output is stochastic and is affected by the effort level chosen by the agent. Output y can take two values, $y \in \{l, h\}$, where $l < h$.

The principal is risk-neutral and the agent is risk-averse. The utility function of the agent is represented by a strictly increasing and strictly concave function $u(\cdot)$.

Let p_i be the probability that the first-period output is equal to h when the first-period effort is i, where $i \in \{L, H\}$. Then the probability that the first-period output realization (y_1) is high given the first-period effort (e_1) is

$$p_i = \Pr[y_1 = h \,|\, e_1 = i].$$

In the repeated moral hazard model by Rogerson (1985), an agent's effort in each period only affects output in that period. This assumption is relaxed in this paper. Accordingly, the effect of the first-period effort, e_1, persists for two periods. This generalizes Rogerson's setup, and for some purposes (such as the analysis of human capital accumulation under asymmetric information), this specification seems more realistic.

Under this specification, the probability of a high output realization may not be the same in the second period, even if the second-period effort is the same. Since the effect of the first-period effort is persistent, the probabilities depend on the first-period effort, e_1, as well as the second-period effort, e_2. The second-period probabilities are defined as

$$p_{ij} = \Pr[y_2 = h \,|\, e_1 = i, e_2 = j],$$

where $i, j \in \{L, H\}$. Note that when the effect of the effort is not persistent (Rogerson's case), $p_L = p_{LL} = p_{HL}$ and $p_H = p_{LH} = p_{HH}$.

As in Rogerson's model, a high output realization is (strictly) more likely when the agent chooses high effort at any given period. We consider the cases of (weakly) positive persistent effects. (We permit no persistence to include Rogerson's case as a special case.) Namely, high output is (weakly) more likely if the previous effort choice was high. In sum, we assume that

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Assumption 1

1. $p_H > p_L$,

2. $p_{HH} > p_{HL} \geq p_{LL}$,

3. $p_{HH} \geq p_{LH} > p_{LL}$.

Assumption 1 ensures that the monotone likelihood ratio condition is satisfied.

2.2 Contracting Problem

The contract between the principal and the agent is a set of contingent wages for the first and the second period. Wages are contingent on the observable history, namely the realization of output. The contingent wages determined by the contract are $\{w_l,\ w_h,\ w_{ll},\ w_{hl},\ w_{lh},\ w_{hh}\}$ where w_i corresponds to the first-period wage when $y_1 = i$ and w_{ij} corresponds to the second-period wage when $y_1 = i$ and $y_2 = j$ with $i, j \in \{l, h\}$.

Alternatively, the contract can be characterized as a utility contract that provides the agent with a set of contingent utilities $\{u_l, u_h, u_{ll}, u_{hl}, u_{lh}, u_{hh}\}$. The utility level corresponding to w_i $[w_{ij}]$ is denoted as $u(w_i) = u_i$ $[u(w_{ij}) = u_{ij}]$. Let $u^{-1}(\cdot) = v(\cdot)$. Then $v(u_i)$ is the cost of providing the agent with utility u_i. Note that since $u(\cdot)$ is a strictly increasing and strictly concave function, $v(\cdot)$ is a strictly increasing and strictly convex function.

Following the literature, we concentrate on the case where the principal wants to implement high effort in both periods.[2] Let $h = y$ and $l = y - x$, where $y > x > 0$. The principal chooses the utility values, or equivalently the wages, that maximize her profit. Both the principal and the agent discount the future at the rate β.

In order to implement high effort in both periods, the contract must provide the agent with higher expected utility under $(e_1, e_2) = (H, H)$ than under any other effort choices. This requires that the wage contract has to satisfy the incentive-compatibility constraints.

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[2]This is the case when h is sufficiently high and l is sufficiently low.

The agent's strategy consists of the history-contingent effort choices. We assume that the agent employs a pure strategy. In the first period, the agent chooses one of two effort levels. In the second period, the history[3] depends on the first-period outcome, y_1. Given each history, the agent chooses one of two effort levels. When there are i possible first-period effort choices, j possible second-period effort choices, and k possible histories (first-period outcomes), there are $i \times j^k$ possible strategies. Here, $i = 2$, $j = 2$, and $k = 2$, therefore there are 8 possible strategies.

The principal wants to implement $\{e_1 = H; e_2 = H\}$. Thus, there are seven deviation strategies.

- Deviation strategy 1: $\{e_1 = L; e_2 = H \text{ if } y_1 = h \text{ and } e_2 = H \text{ if } y_1 = l\}$
- Deviation strategy 2: $\{e_1 = H; e_2 = L \text{ if } y_1 = h \text{ and } e_2 = L \text{ if } y_1 = l\}$
- Deviation strategy 3: $\{e_1 = L; e_2 = L \text{ if } y_1 = h \text{ and } e_2 = L \text{ if } y_1 = l\}$
- Deviation strategy 4: $\{e_1 = H; e_2 = H \text{ if } y_1 = h \text{ and } e_2 = L \text{ if } y_1 = l\}$
- Deviation strategy 5: $\{e_1 = H; e_2 = L \text{ if } y_1 = h \text{ and } e_2 = H \text{ if } y_1 = l\}$
- Deviation strategy 6: $\{e_1 = L; e_2 = H \text{ if } y_1 = h \text{ and } e_2 = L \text{ if } y_1 = l\}$
- Deviation strategy 7: $\{e_1 = L; e_2 = L \text{ if } y_1 = h \text{ and } e_2 = H \text{ if } y_1 = l\}$

The incentive-compatibility constraints can be written as

$$
\begin{aligned}
&p_H \left(u_h + \beta \left(1 - p_{HH}\right)u_{hl} + \beta \, p_{HH}u_{hh}\right) \\
&+(1 - p_H)\left(u_l + \beta \left(1 - p_{HH}\right) u_{ll} + \beta \, p_{HH} \, u_{lh}\right) - (1 + \beta)\alpha(H) \\
&\qquad \geq p_i \left(u_h + \beta \left(1 - p_{ij}\right) u_{hl} + \beta \, p_{ij} \, u_{hh}\right) \\
&\qquad\quad +(1 - p_i)\left(u_l + \beta \left(1 - p_{ik}\right) u_{ll} + \beta \, p_{ik} \, u_{lh}\right) - \alpha(i) - \beta(1 - p_i)\alpha(k) - \beta p_i \alpha(j).
\end{aligned}
\tag{1}
$$

where $i, j, k \in \{H, L\}$ except for $i = j = k = H$. We will refer to the incentive-compatibility constraints corresponding to the deviation strategies 1 to 7 as $(IC1)$ to $(IC7)$.

The contract must also provide the agent with higher utility than autarky to ensure his participation. We assume that both sides commit to the long-term contract, so that there is no participation issue in the second period. The participation constraint[4] (PC) dictates

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[3]Since the agent only uses pure strategies, we do not need to include the past action in the history.

[4]Alternatively, we can interpret $\bar{U}$ as the promised utility level from the principal to the agent, and call (PC) the "promise-keeping" constraint.

that the expected utility of the agent is higher than the utility of his outside option. Thus the following has to be satisfied:

$$p_H \left(u_h + \beta \left(1 - p_{HH} \right) u_{hl} + \beta \, p_{HH} \, u_{hh} \right)$$
$$+ (1 - p_H) \left(u_l + \beta \left(1 - p_{HH} \right) u_{ll} + \beta \, p_{HH} \, u_{lh} \right) - (1 + \beta)c \geq \bar{U}. \qquad (PC)$$

The principal's problem is

$$\max_{u_l, u_h, u_{ll}, u_{hl}, u_{lh}, u_{hh}} \quad [y - (1 - p_H) \, x] + \beta \, [y - (1 - p_{HH}) \, x]$$
$$-p_H [v(u_h) + \beta \left(1 - p_{HH} \right) v(u_{hl}) + \beta \, p_{HH} v(u_{hh})] \qquad (P1)$$
$$-(1 - p_H)[v(u_l) + \beta \left(1 - p_{HH} \right) v(u_{ll}) + \beta \, p_{HH} v(u_{lh})]$$

subject to the incentive-compatibility and participation constraints.

The following lemma helps to simplify the problem.

Lemma 1 *(IC2) is implied by (IC4) and (IC5).*

Proof: Adding up both terms in $(IC4)$ and $(IC5)$, then arranging terms yields $(IC2)$. $\square$

Thus, we can ignore $(IC2)$. Let λ_1, λ_3, λ_4, λ_5, λ_6, λ_7, and μ be the nonnegative Lagrange multipliers associated with $(IC1)$, $(IC3)$, $(IC4)$, $(IC5)$, $(IC6)$, $(IC7)$, and (PC), respectively. The following are the first order conditions:

u_l:

$$v'(u_l) = \mu + \left[1 - \frac{1 - p_L}{1 - p_H} \right] (\lambda_1 + \lambda_3 + \lambda_6 + \lambda_7). \qquad (2)$$

u_h:

$$v'(u_h) = \mu + \left[1 - \frac{p_L}{p_H} \right] (\lambda_1 + \lambda_3 + \lambda_6 + \lambda_7). \qquad (3)$$

u_{ll}:

$$v'(u_{ll}) = \mu + \left[1 - \frac{(1 - p_L)(1 - p_{LH})}{(1 - p_H)(1 - p_{HH})} \right] (\lambda_1 + \lambda_7) + \left[1 - \frac{1 - p_{HL}}{1 - p_{HH}} \right] \lambda_4$$
$$+ \left[1 - \frac{(1 - p_L)(1 - p_{LL})}{(1 - p_H)(1 - p_{HH})} \right] (\lambda_3 + \lambda_6). \qquad (4)$$

u_{hl}:

$$v'(u_{hl}) = \mu + \left[1 - \frac{p_L(1 - p_{LH})}{p_H(1 - p_{HH})} \right] (\lambda_1 + \lambda_6) + \left[1 - \frac{1 - p_{HL}}{1 - p_{HH}} \right] \lambda_5$$
$$+ \left[1 - \frac{p_L(1 - p_{LL})}{p_H(1 - p_{HH})} \right] (\lambda_3 + \lambda_7). \qquad (5)$$

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u_{lh}:

$$v'(u_{lh}) = \mu + \left[1 - \frac{(1 - p_L)p_{LH}}{(1 - p_H)p_{HH}}\right](\lambda_1 + \lambda_7) + \left[1 - \frac{p_{HL}}{p_{HH}}\right]\lambda_4$$
$$+ \left[1 - \frac{(1 - p_L)p_{LL}}{(1 - p_H)p_{HH}}\right](\lambda_3 + \lambda_6). \tag{6}$$

u_{hh}:

$$v'(u_{hh}) = \mu + \left[1 - \frac{p_L\, p_{LH}}{p_H\, p_{HH}}\right](\lambda_1 + \lambda_6) + \left[1 - \frac{p_{HL}}{p_{HH}}\right]\lambda_5$$
$$+ \left[1 - \frac{p_L\, p_{LL}}{p_H\, p_{HH}}\right](\lambda_3 + \lambda_7). \tag{7}$$

3   General Results

By the first order conditions, we characterize the optimal contract.

Proposition 1 *The following properties hold.*

1. $u_h \geq u_l$,

2. $u_{lh} > u_{ll}$ *and* $u_{hh} > u_{hl}$,

3. $u_{hl} \geq u_{ll}$ *and* $u_{hh} \geq u_{lh}$.

Proof: See Appendix. □

The first two properties in Proposition 1 imply that with a given history, high output will give the agent a better wage. The third property implies that if the current output is the same, an agent with a better history receives a (weakly) better wage. Notice that part 1 and part 3 of Proposition 1 are weak inequalities. As it will be illustrated below, there are cases where they hold with equality. The "weakness" of these inequalities is unique to the current setting. In a model where there is no second-period action (but the first-period action has a persistent effect), it can easily be shown that all the inequalities must be strict. If there is a second-period action but no persistence (as in Rogerson [1985]), as is shown later, all the inequalities have to be strict.

The martingale result by Rogerson (1985, Proposition 1) can be generalized to our setting.

Proposition 2 $1/u'(w)$ *follows a martingale: that is, the optimal contract satisfies*

$$\frac{1}{u'(w_i)} = (1 - p_{HH})\frac{1}{u'(w_{il})} + p_{HH}\frac{1}{u'(w_{ih})}, \tag{8}$$

for $i = l, h$.

Proof: For $i = l$, the equation is obtained by combining (2), (4), and (6). For $i = h$, (3), (5), and (7) are combined. $\square$

From this proposition, it is easy to characterize the wage dynamics as Rogerson has done. In particular, whether the expected wage is increasing or decreasing depends solely on the curvature of the utility function (see Rogerson's Proposition 3).

4   The Perfect-Insurance Result

In the standard time-separable repeated moral hazard models, the agent is always only *partially* insured against the first-period outcome: $u_h > u_l$ always holds (see the following Corollary 1). When persistence is introduced, the agent may be insured *perfectly* against the first-period outcome. That is, it is possible that $u_l = u_h$. Even though the first-period outcome contains some information about the first-period effort, it is intentionally disregarded. Moreover, in this case, the first-period outcome does not have any effect on wages both in the first and second period. The following proposition shows that $u_l = u_h$ if and only if $u_{ll} = u_{hl}$ and $u_{lh} = u_{hh}$.

Proposition 3 $u_l = u_h$ *if and only if* $u_{ll} = u_{hl}$ *and* $u_{lh} = u_{hh}$.

Proof: See Appendix. $\square$

When does perfect insurance occur? The following proposition establishes a set of necessary and sufficient conditions for perfect insurance.[5]

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[5]In the earlier version of this paper, we only had the necessity part. After completing the earlier version, we became aware of the independent work by Ogawa (2003), whose result implies that the conditions (9) and (10) are also sufficient for $\beta = 1$. Our sufficiency result generalizes Ogawa's result for $\beta \leq 1$.

Proposition 4 *The following is a set of necessary and sufficient conditions for $u_l = u_h$ to hold under the optimal contract.*

$$\beta(p_{HH} - p_{LH}) \geq p_{HH} - p_{HL}, \tag{9}$$

$$\beta(p_{HL} - p_{LL}) \geq p_{HH} - p_{HL}. \tag{10}$$

Proof: See Appendix.□

It is straightforward to show that perfect insurance never occurs in the environment of Rogerson (1985).

Corollary 1 $u_h > u_l$ *always holds if there is no persistence.*

Proof: When there is no persistence, $p_{HH} - p_{LH} = p_H - p_H = 0$ and $p_{HH} - p_{HL} = p_H - p_L > 0$. Thus (9) cannot be satisfied. □

Notice that the left hand side of (9), $\beta(p_{HH} - p_{LH}) = \beta\{(1 - p_{LH}) - (1 - p_{HH})\}$, measures the (discounted) relative probability of a bad second-period output when the agent chooses low effort in the *first* period. The right hand side, $(p_{HH} - p_{HL}) = \{(1 - p_{HL}) - (1 - p_{HH})\}$ measures the relative probability of a bad second-period output when the agent chooses low effort in the *second* period. Inequality (9) ensures that the former (the persistent effect of the first-period effort) is larger than the latter (the instantaneous effect of the second-period effort). Inequality (10) ensures that the same relationship holds for the case where, for the left hand side, the second-period effort is low. When perfect insurance occurs, an incentive scheme for the first period is not necessary because the "second-period prevention" of second-period shirking automatically ensures that the agent will not shirk in the first period. In other words, the second-period output serves as a signal not only for the second-period effort but also for the first-period effort. When effort has a large persistent effect, the first-period signal is intentionally ignored and the agent is perfectly insured in the first period. He will

choose a high effort level in the first period because he is concerned about the occurrence of a bad output (and the consequent punishment) in the second period.

In what kind of economic environment is perfect insurance likely to occur? From (9), one can see that it is necessary that $p_{HL} \geq p_{LH}$. Thus, yesterday's low effort hurts today's output more than today's low effort. Below, three such examples are presented.

Example 1: (Human Capital − 1)

Consider a model of human capital accumulation. The probability (p) of obtaining a high output depends on the worker's human capital level k

$$p = F(k),$$

where $F(\cdot)$ is an increasing function.

The human capital accumulation function is

$$k_t = (1 - \delta)k_{t-1} + I(e_t), \tag{11}$$

where the t subscript denotes period t, the depreciation rate is δ, and $I(e_t)$ is an increasing function of the period-t effort level, e_t.

Let $F(k) = \psi k$ and $I(e) = \phi + \omega e$. Also specify that $u(\cdot) = \log(\cdot)$, $y = 2$, $x = 1$, $c = 0.5$, $\bar{U} = 2$, $\beta = 1$, $\psi = 0.3$, $\phi = 0.4$, $\omega = 3$, $(H, L) = (0.2, 0)$, and $k_0 = 0$.

Figures 1 and 2 show the values of utility under the optimal contract for different values of δ. Here, δ represents the degree of persistence, where persistence is higher when δ is smaller. $\delta = 1$ corresponds to the case of no persistence (Rogerson [1985]).

Figures 1 and 2 show that the effect of the first-period outcome, $u_h - u_l$, $u_{hh} - u_{lh}$, and $u_{hl} - u_{ll}$, declines as δ becomes smaller. That is, wages become less responsive to the first-period outcome as persistence increases. At the extreme, perfect insurance against the first-period outcome occurs when $\delta < 0$.

The implication of (9) is that perfect insurance occurs only if $\delta \leq 0$, since $p_{LH} > p_{HL}$ is satisfied whenever $\delta > 0$. In this example, negative depreciation seems unrealistic. Does it
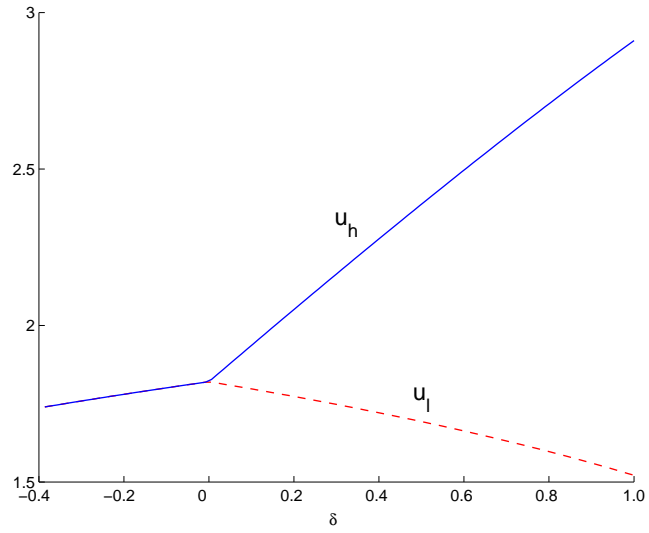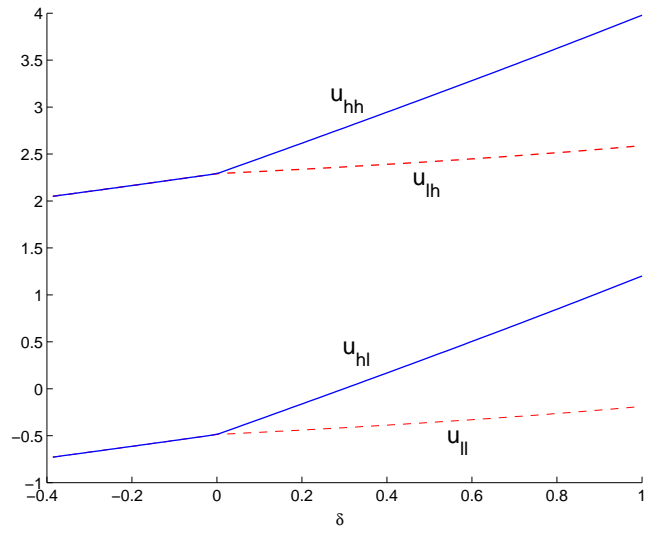
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Figure 1: u_l and u_h for different values of δ.

Figure 2: u_{lh}, u_{hl}, u_{ll}, and u_{hh} for different values of δ.

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mean that perfect insurance never occurs in a model of human capital? In the next example, it is shown that a slight modification allows perfect insurance to occur even when $\delta > 0$.

Example 2: (Human Capital − 2)

Assume that both training and production require effort. The "effort endowment" for a worker is fixed in each period (normalized to 1), and the "effort budget constraint" for each period is

$$e_p + e_k = 1, \tag{12}$$

where e_p is the effort choice for production, and e_k is the effort choice for training. The probability (p) of obtaining a high output depends on the worker's production effort and his human capital k. Assume that this "probability production function" is

$$p = G(k, e_p),$$

where $G(\cdot, \cdot)$ is increasing in both terms. Let us specify that $G(k, e_p) = \varphi \cdot e_p \cdot k$, where φ is a constant. Then, considering (12),

$$p = \varphi(1 - e_k)k.$$

Human capital evolves according to (11). Specify the function and parameters in the same way as in the previous example, except now $\varphi = 0.5$.

Again, Figures 3 and 4 illustrate that persistence reduces the responsiveness of the wages to the first-period outcome. Notice that perfect insurance occurs for some range of positive δ's. Figure 5 shows that $p_{HL} > p_{LH}$ holds when perfect insurance occurs.

Example 3: (Learning by Doing)

As an alternative environment, consider the case where there is learning-by-doing in production. Production depends on the worker's (hidden) production effort, e. In the first period, the probability of high output is a function of the first-period effort,
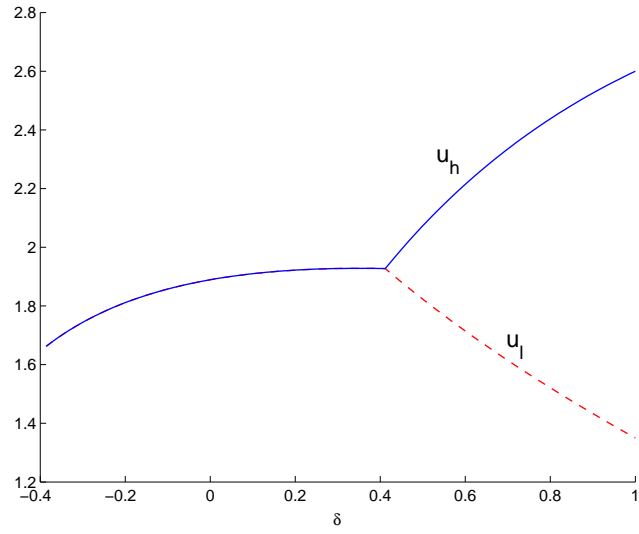
$$p = F^1(e_1),$$
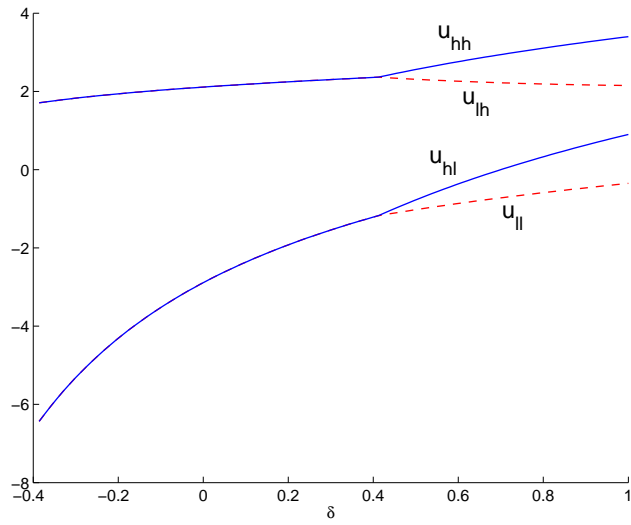
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Figure 3: u_l and u_h for different values of δ.

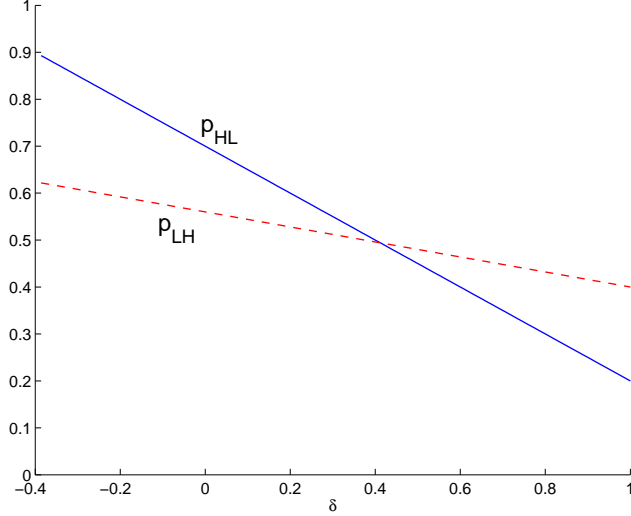Figure 4: $u_{lh}, u_{hl}, u_{ll},$ and u_{hh} for different values of δ.

Figure 5: p_{HL} and p_{LH} for different values of δ.

where $F^1(\cdot)$ is an increasing function. In the second period, the probability depends not only on e_2, but also on e_1: there is a learning-by-doing effect. Let

$$p = F^2(e_1, e_2),$$

where $F^2(\cdot, \cdot)$ is increasing in both terms.

Specify $F^1(e_1) = \eta e_1$ and $F^2(e_1, e_2) = \nu(e_2 + \zeta e_1)$. Here ζ measures the degree of persistence: persistence increases as ζ increases. Note that $\zeta = 0$ corresponds to the no-persistence case.

As a numerical example, specify that $u(\cdot) = \log(\cdot)$, $y = 2$, $x = 1$, $c = 0.5$, $\bar{U} = 2$, $\beta = 1$, $\eta = 2$, $\nu = 1$, and $(H, L) = (0.3, 0.1)$. Figures 6 and 7 illustrate the results. Again, as persistence increases, wages become less responsive to the first-period output. When ζ is very large (when the learning-by-doing effect is strong), perfect insurance occurs.

5 Extension to Three Effort Levels

In this section we extend the two-effort-choice specification to the case where there are more than two effort choices. Recall that when there are i possible first-period effort choices,
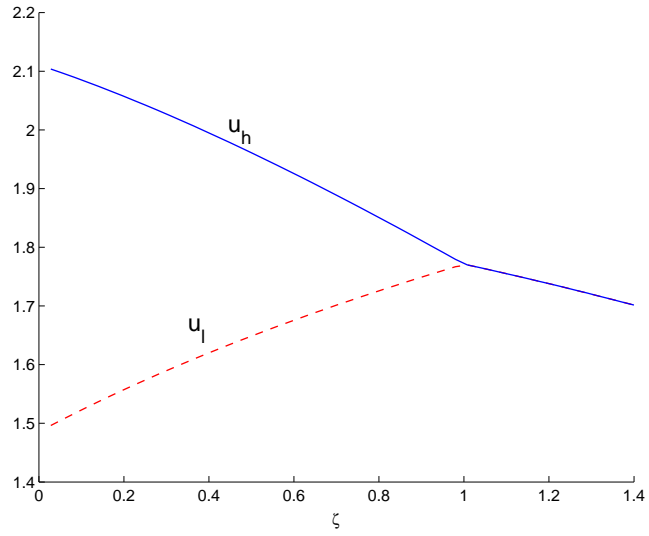
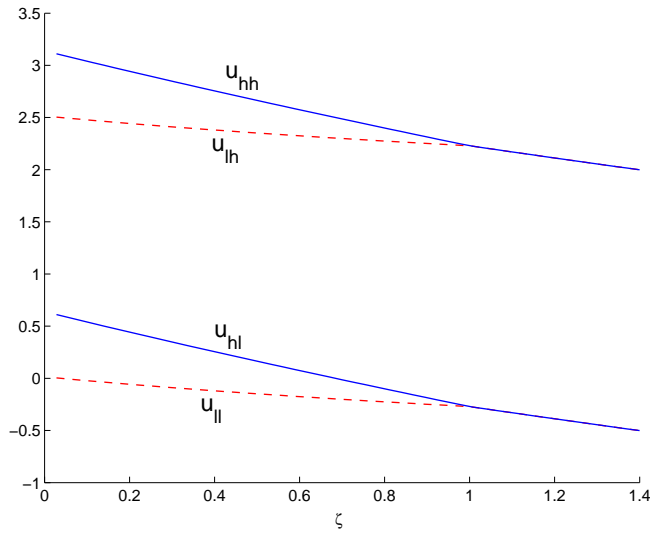Figure 6: u_l and u_h for different values of ζ.

Figure 7: u_{lh}, u_{hl}, u_{ll}, and u_{hh} for different values of ζ.

j possible second-period effort choices, and k possible histories, there are $i \times j^k$ possible strategies. For example when $i = 3$, $j = 3$, and $k = 2$, there are 27 possible strategies.

We extend the basic formulation to three-effort choices by assuming that the principal wants to implement high effort in all periods. Then, the principal's problem becomes

$$
\max_{u_l, u_h, u_{ll}, u_{hl}, u_{lh}, u_{hh}} \quad [y - (1 - p_H)\, x] + \beta\, [y - (1 - p_{HH})\, x]
$$
$$
- p_H [v(u_h) + \beta\,(1 - p_{HH})v(u_{hl}) + \beta\, p_{HH} v(u_{hh})]
$$
$$
- (1 - p_H)[v(u_l) + \beta\,(1 - p_{HH})v(u_{ll}) + \beta\, p_{HH} v(u_{lh})]
$$

subject to the incentive-compatibility and participation constraints. The incentive-compatibility constraints can be written as

$$
p_H \left(u_h + \beta\,(1 - p_{HH})u_{hl} + \beta\, p_{HH}u_{hh} \right)
$$
$$
+ (1 - p_H)\left(u_l + \beta\,(1 - p_{HH})\, u_{ll} + \beta\, p_{HH}\, u_{lh} \right) - (1 + \beta)\alpha(H)
$$
$$
\geq p_i \left(u_h + \beta\,(1 - p_{ij})\, u_{hl} + \beta\, p_{ij}\, u_{hh} \right)
$$
$$
+ (1 - p_i)\left(u_l + \beta\,(1 - p_{ik})\, u_{ll} + \beta\, p_{ik}\, u_{lh} \right) - \alpha(i) - \beta(1 - p_i)\alpha(k) - \beta p_i \alpha(j).
$$

where $i, j, k \in \{H, M, L\}$ except for $i = j = k = H$. Note that this formulation can easily be generalized to settings where the number of effort choices is greater than three.

Example 4: (Human Capital)

Consider the human capital model in Example 1. The three-effort choice model is numerically computed and the values of the utilities are shown in Figures 8 and 9. We use the same functional specifications and parameters. The set of effort choices also includes an intermediate value: $(H, M, L) = (0.2, 0.1, 0)$ and the cost associated with choosing (H, M, L) is given by $(0.5, 0.2, 0)$. Figures 8 and 9 illustrate the same result as in the two-effort model: wages become less responsive to the early periods' outcomes as the effect of the agent's effort becomes more persistent.

Perfect insurance occurs for the three-effort case as well. When δ is small enough, $u_l = u_h$, and $u_{lh} = u_{hl} = u_{ll} = u_{hh}$ hold. The first- and the second-period outcomes are ignored in all periods.

We also compute the agent's utilities from the optimal contract for the case where the principal wants to implement M in both periods. The values of the utilities are shown in Figures 10 and 11. As persistence increases, the utility values become less responsive to
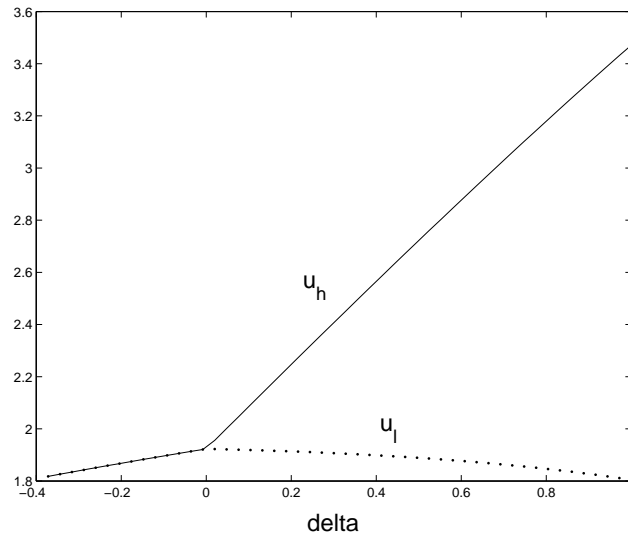
Figure 8: u_l and u_h for different values of δ.
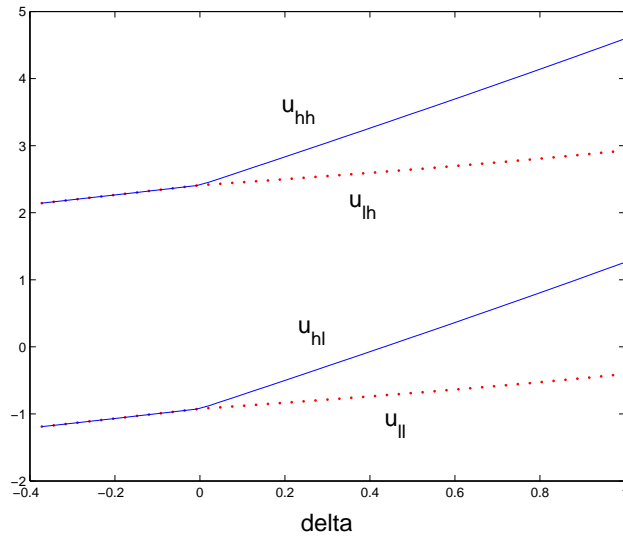

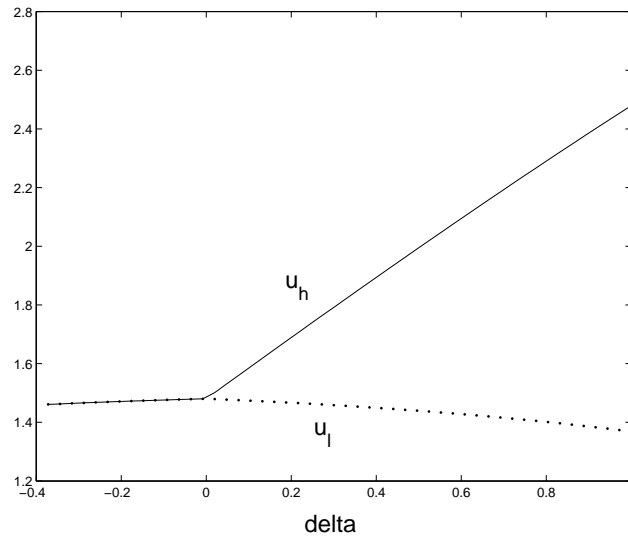Figure 9: u_{lh}, u_{hl}, u_{ll}, and u_{hh} for different values of δ.

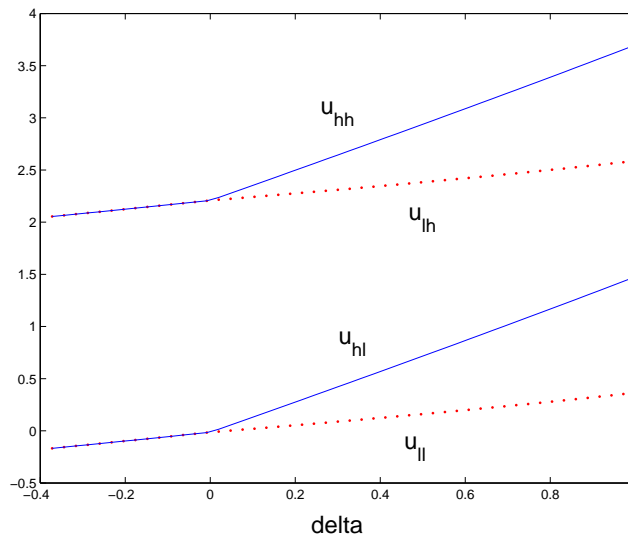Figure 10: u_l and u_h for different values of δ.

Figure 11: u_{lh}, u_{hl}, u_{ll}, and u_{hh} for different values of δ.

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output. This is very similar to the case where the principal wants to implement H.[6]

6   Extension to Three Periods

In this section, the basic framework is extended to a three-period model. Here, the first-period effort, e_1, affects y_1, y_2, and y_3. The second-period effort, e_2, affects y_2 and y_3. When there are i possible first-period effort levels, j possible second-period effort choices, k possible third-period effort levels, m possible first-period outcomes, and n possible second-period outcomes, there are $i \times j^m \times k^{m \times n}$ possible strategies for the agent. Here, $i = 2$, $j = 2$, $k = 2$, $m = 2$, and $n = 2$, therefore there are 128 possible strategies. Again, assume that the principal wants to implement high effort in all periods. Then, the principal's problem becomes

$$\max_{\substack{u_i, u_{jk}, u_{mno} \\ i,j,k,m,n,o = \{h,l\}}} R - C, \tag{P2}$$

subject to the incentive-compatibility and participation constraints, where

$$R \equiv [y - (1 - p_H)\, x] + \beta\, [y - (1 - p_{HH})\, x] + \beta^2\, [y - (1 - p_{HHH})\, x],$$

and

$$
\begin{aligned}
C \equiv \quad & p_H[v(u_h) && +\beta\,(1 - p_{HH})\{v(u_{hl}) + \beta\, p_{HHH} v(u_{hlh}) + \beta\,(1 - p_{HHH})v(u_{hll})\} \\
& && +\beta\, p_{HH}\{v(u_{hh}) + \beta\, p_{HHH} v(u_{hhh}) + \beta\,(1 - p_{HHH})v(u_{hhl})\}] \\
& +(1 - p_H)[v(u_l) && +\beta\,(1 - p_{HH})\{v(u_{ll}) + \beta\, p_{HHH} v(u_{llh}) + \beta\,(1 - p_{HHH})v(u_{lll})\} \\
& && +\beta\, p_{HH}\{v(u_{lh}) + \beta\, p_{HHH} v(u_{lhh}) + \beta\,(1 - p_{HHH})v(u_{lhl})\}].
\end{aligned}
$$

R is the expected return to the principal, and C is the expected wage payment. We will characterize the solution to this problem by extending the human capital model from Example 1 to three periods.

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[6]Quantitatively, when the principal wants to implement H, the difference between u_h and u_l is higher compared to the case where she wants to implement M, since more incentive has to be provided to induce a high effort.
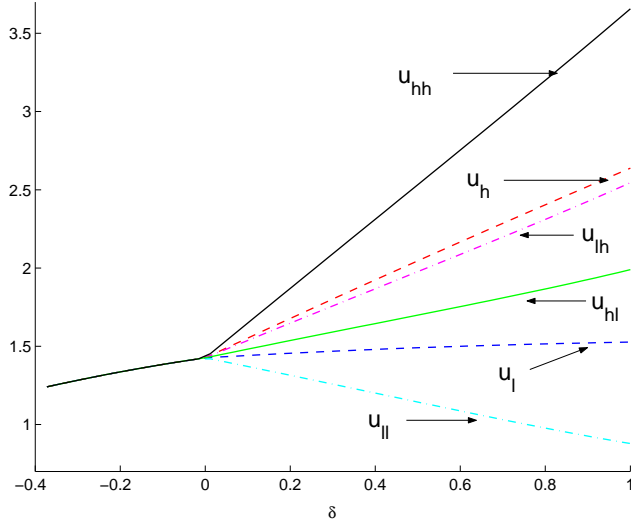
Figure 12: u_l, u_h, u_{lh}, u_{hl}, u_{ll}, and u_{hh} for different values of δ.

Example 5: (Human Capital)

Consider the human capital model from Example 1. The three-period version is numerically computed[7] and the values of the utilities are shown in Figures 12 and 13. The functional specifications and the parameters are the same, except that $\psi = 0.25$ and $\phi = 0.25$. Figures 12 and 13 illustrate the same results as in the two-period model: wages become less responsive to the early periods' outcomes as the effect of the agent's effort becomes more persistent.

Again, perfect insurance occurs. When δ is small enough, $u_l = u_h = u_{lh} = u_{hl} = u_{ll} = u_{hh}$, $u_{lll} = u_{lhl} = u_{hll} = u_{hhl}$, and $u_{llh} = u_{lhh} = u_{hlh} = u_{hhh}$ hold. The first- and the second-period outcomes are ignored in all periods. Moreover, *perfect consumption smoothing*

---

[7]This task involves solving a constrained maximization problem with 128 constraints (127 incentive-compatibility constraints and one participation constraint). One can write the constraints recursively by using the general formulation of incentive-compatibility constraints similar to (1). The fact that the constraints are linear in utilities made the computation somewhat easier. We have reduced the computational complexity by eliminating the redundant constraints. For the two-period model, we have shown that $(IC2)$ is redundant, which reduces the number of inequality constraints to a total of 7. It turns out that 14 constraints are redundant for the three-period case. The redundant constraints are the ones which involve deviations in the third period contingent on more than one set of past outcome (y_1, y_2). These constraints are implied by the constraints involving deviations in the third period conditional on only one set of past outcomes. The numerical examples also provide intuition about the nature of the constraints. For the two-period case, when perfect insurance occurs, only $(IC4)$ and $(IC5)$ bind. These are the constraints that involve deviations in the final period. Our three-period example exhibits a similar pattern. When perfect insurance occurs, only four constraints bind, and these are the constraints that involve deviations only in the final period.
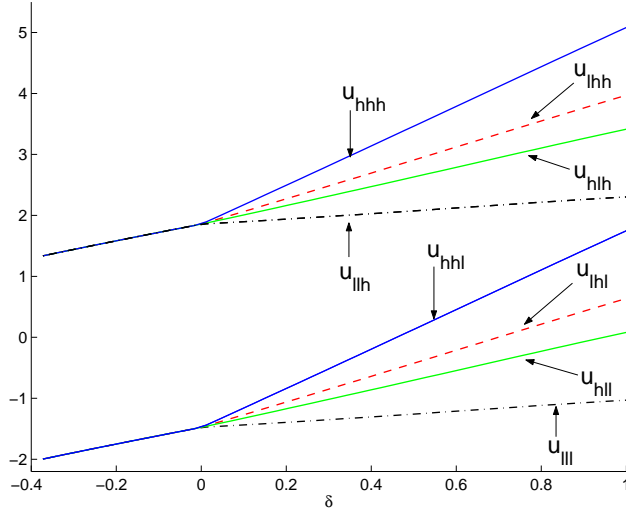
Figure 13: u_{lll}, u_{lhl}, u_{llh}, u_{lhh}, u_{hll}, u_{hhl}, u_{hlh}, and u_{hhh} for different values of δ.

is obtained: wages in the first period and in the second period become equal. The intuition is the same as in the two-period model: as the effect of effort becomes more persistent, the last-period incentive becomes sufficient to induce the agent to choose high effort in all periods.

7   Application to Unemployment Insurance

Our three-period model can be interpreted as a model of unemployment insurance. Assume that the agent is unemployed in the first period. The agent has to make a job-search effort, which is unobservable by the government (the principal). When the agent is unemployed, translate the state h to "finding a job", and the state l to "not finding a job". The agent is more likely to find a job when he chooses a high job-search effort. While he is on the job, he has to make a job-retention effort (as in Wang and Williamson [2002]). When employed, translate outcome h to "keeping the job", and outcome l to "losing the job". The agent is more likely to keep the job if he chooses a high job-retention effort. The government can affect the consumption of the unemployed by providing unemployment insurance, and the consumption of the employed by levying tax on wages (as in Hopenhayn and Nicolini [1997]).

The government conditions the unemployment insurance benefit and the wage tax on the agent's unemployment history.

The effect of effort is persistent. The first-period search effort affects not only the probability of job finding in the first period, but also the probability of job finding in the second and the third period (if he is unemployed in those periods). We assume that even if he finds a job in the first or the second period, the first-period job search effort can affect the subsequent job-retention probability, by influencing the (unobservable) quality of the job-worker match. The second-period search (or job-retention) effort has an analogous persistent effect on the subsequent outcomes.

Using this interpretation, we can extend the implications of our model to the optimal unemployment insurance program. In the literature (e.g. Shavell and Weiss [1979] and Hopenhayn and Nicolini [1997]), the optimal unemployment insurance schedule is characterized by a declining sequence of insurance payments.[8] In our three-period model, the sequence of unemployment insurance is represented by $\{u_l, u_{ll}, u_{lll}\}$. Figures 12 and 13 show that $u_l \geq u_{ll} > u_{lll}$. This implies that the optimal unemployment insurance schedule is a weakly declining sequence. The amount of decline in each period, $u_l - u_{ll}$ and $u_{ll} - u_{lll}$, behaves differently as persistence increases. In particular, $u_l - u_{ll}$ declines as the persistence increases. At the extreme, $u_l = u_{ll} > u_{lll}$ holds. This scheme resembles the current U.S. unemployment insurance system: payments are constant until the termination date (at which time the payment drops to zero). The intuition is simple: by having an incentive scheme in the third period, the government can induce a high effort in the first and the second period as well as in the third period. Thus, it is optimal to have a smooth consumption scheme in the first and the second period. The current U.S. system may not be too far away from the optimal system, if persistence is strong.

---

[8]Several recent studies have shown that the optimal unemployment insurance scheme is not necessarily strictly declining if the agent is allowed to make hidden savings. See Wang and Williamson (1999); Abdulkadiroğlu, Kuruşçu, and Şahin (2001); and Kocherlakota (2002). We follow Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) and do not allow the agent to save (or, alternatively, we impose that all savings are observable to the principal).

It may seem counterfactual that the effect of search effort carries over to the probability of job retention when employed. To see if our result is robust, we construct a three-period model where the agent does not need to exert any effort once he is hired (there is no possibility of losing a job). The timing is as follows. In the first period, the agent is unemployed. He chooses the level of his job-search effort. It is assumed that the effort increases the probability of getting hired: $p_H > p_L$. If he is hired, he obtains u_h in every subsequent period (he never gets fired). If he is not hired, he derives utility u_l from unemployment insurance and remains unemployed until the next period. In the second period, an unemployed agent again decides the level of his job-search effort. If he finds a job, then he receives u_{lh} in the second and the third period. Otherwise, he obtains the utility u_{ll} from unemployment insurance and enters the third period as unemployed. The unemployed agent then decides the level of the third-period job search effort, and if he finds a job, he receives u_{llh}. Otherwise he obtains u_{lll} from unemployment insurance.

Thus, the agent's expected utility is

$$
\begin{aligned}
U(i,j,k) \equiv \quad & p_i(u_h && +\beta u_h && +\beta^2 u_h) \\
& (1-p_i)\{u_l && +\beta p_{ij}(u_{lh} && +\beta u_{lh}) \\
& && +\beta(1-p_{ij})[u_{ll} && +\beta p_{ijk}u_{llh} \\
& && && +\beta(1-p_{ijk})u_{lll}]\} \\
& && && -c_1 - \beta c_2 - \beta^2 c_3,
\end{aligned}
$$

where $i,j,k \in \{H,L\}$ are the first-, second-, and third-period effort levels. c_1, c_2, and c_3 are the costs of effort in each period respectively, where they are zero if the effort level is L and c if the effort level is H.

The optimal unemployment insurance solves (up to a constant)

$$
\begin{aligned}
\min_{u_h,u_l,u_{lh},u_{ll},u_{llh},u_{lll}} \quad & p_H(v(u_h) && +\beta v(u_h) && +\beta^2 v(u_h)) \\
& (1-p_H)\{v(u_l) && +\beta p_{HH}(v(u_{lh}) && +\beta v(u_{lh})) \\
& && +\beta(1-p_{HH})[v(u_{ll}) && +\beta p_{HHH}v(u_{llh}) \\
& && && +\beta(1-p_{HHH})v(u_{lll})]\},
\end{aligned}
\qquad (P3)
$$

subject to the seven incentive-compatibility constraints

$$
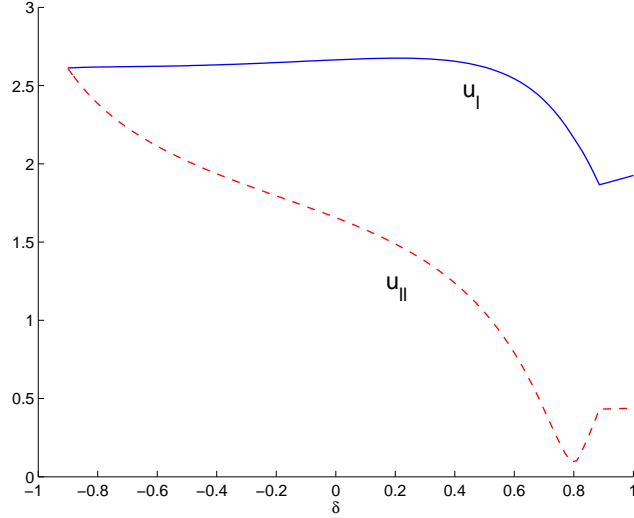U(H,H,H) \geq U(i,j,k) \qquad i,j,k \in \{H,L\} \text{ except for } i=j=k=H,
$$

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Figure 14: u_l and u_{ll} for different values of δ.

and the participation constraint:

$$U(H, H, H) \geq \bar{U}.$$

Example 6 (Unemployment Insurance)

Assume that the probabilities evolve according to the following process. For the first period the probability of finding a job is

$$p_1 = \eta_1 e_1 + (1 - \delta)\eta_2 p_0,$$

where η_1, δ, η_2, and p_0 are given constants. For the second and third period

$$p_t = \eta_1 e_t + (1 - \delta)\eta_2 p_{t-1},$$

where $t = 2, 3$. We interpret δ as the persistence parameter.

Figure 14[9] depicts the same tendency as in Example 5: u_l and u_{ll} become closer as persistence increases (δ decreases), implying less varying unemployment insurance sequences. Figure 15 illustrates the other utility values. (Note that it shows only the part where $\delta \geq 0.4$.) The punishment in the third period becomes more drastic as the persistence increases: u_{lll}

---

[9]The parameter values are: $(H, L) = (0.2, 0)$, $p_0 = 0.5$, $\beta = 0.8$, $c = 0.5$, $\bar{U} = 2$, $\eta_1 = 1$, and $\eta_2 = 0.5$.
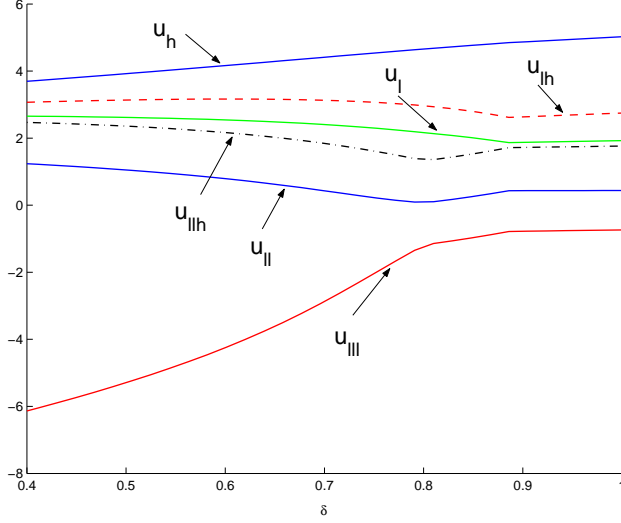
Figure 15: $u_h, u_l, u_{lh}, u_{ll}, u_{llh}$, and u_{lll} for different values of δ.

declines rapidly as δ decreases. The optimal unemployment insurance induces the agent to exert high effort in all periods based almost solely on the third-period incentive scheme.

8 N-period Model with Two-period Persistence

In general, it is difficult to extend our basic model to many periods, since the number of the incentive-compatibility constraints increases very rapidly as the number of periods expand. In this section, we limit our attention to the case where there is only two-period persistence (that is, the effort at the n-th period affects only the n-th and the $n + 1$-st period output). Fernandes and Phelan (2000) showed that an infinite-horizon version of this model can be formulated recursively. Following their insight, here we formulate an N-period version of our model utilizing the dynamic programming technique. As in Fernandes and Phelan (2000), the key is to keep track of the promised utility (W) of an agent whose action is H in the current period, and the promised utility $(\hat{W})$ of an agent who takes the action L in the current period. The following shows the first, $2 \leq n \leq N$, and the final (N) period problem. The proof that the solution to these problems corresponds to the original problem's solution is similar to the proof in Fernandes and Phelan (2000), and is therefore omitted.

- First period:

$$\min_{u_h^1, u_l^1, W_h^2, W_l^2, \hat{W}_h^2, \hat{W}_l^2} \quad p_H[v(u_h^1) + \beta V_2(W_h^2, \hat{W}_h^2)] + (1 - p_H)[v(u_l^1) + \beta V_2(W_l^2, \hat{W}_l^2)]$$

subject to

$$U \leq p_H(u_h^1 + \beta W_h^2) + (1 - p_H)(u_l^1 + \beta W_l^2) - c, \tag{13}$$

$$p_H(u_h^1 + \beta W_h^2) + (1 - p_H)(u_l^1 + \beta W_l^2) - c \geq p_L(u_h^1 + \beta \hat{W}_h^2) + (1 - p_L)(u_l^1 + \beta \hat{W}_l^2), \tag{14}$$

$$(W_h^2, \hat{W}_h^2) \in \mathcal{W}^2, (W_l^2, \hat{W}_l^2) \in \mathcal{W}^2. \tag{15}$$

- n-th period $(2 \leq n \leq N - 1)$:

$$V_n(W, \hat{W}) = \min_{u_h^n, u_l^n, W_h^{n+1}, W_l^{n+1}, \hat{W}_h^{n+1}, \hat{W}_l^{n+1}}$$
$$\left\{ p_{HH}[v(u_h^n) + \beta V_{n+1}(W_h^{n+1}, \hat{W}_h^{n+1})] + (1 - p_{HH})[v(u_l^n) + \beta V_{n+1}(W_l^{n+1}, \hat{W}_l^{n+1})] \right\}$$

subject to

$$W = p_{HH}(u_h^n + \beta W_h^{n+1}) + (1 - p_{HH})(u_l^n + \beta W_l^{n+1}) - c, \tag{16}$$

$$\hat{W} = \max \left\langle p_{LH}(u_h^n + \beta W_h^{n+1}) + (1 - p_{LH})(u_l^n + \beta W_l^{n+1}) - c, \right. \\ \left. p_{LL}(u_h^n + \beta \hat{W}_h^{n+1}) + (1 - p_{LL})(u_l^n + \beta \hat{W}_l^{n+1}) \right\rangle, \tag{17}$$

$$W \geq p_{HL}(u_h^n + \beta \hat{W}_h^{n+1}) + (1 - p_{HL})(u_l^n + \beta \hat{W}_l^{n+1}), \tag{18}$$

$$(W_h^{n+1}, \hat{W}_h^{n+1}) \in \mathcal{W}^{n+1}, (W_l^{n+1}, \hat{W}_l^{n+1}) \in \mathcal{W}^{n+1}. \tag{19}$$

- N-th period:

$$V_N(W, \hat{W}) = \min_{u_h^N, u_l^N} \left\{ p_{HH} v(u_h^N) + (1 - p_{HH}) v(u_l^N) \right\}$$

subject to

$$W = p_{HH} u_h^N + (1 - p_{HH}) u_l^N - c, \tag{20}$$

$$\hat{W} = \max \left\langle p_{LH} u_h^N + (1 - p_{LH}) u_l^N - c, p_{LL} u_h^N + (1 - p_{LL}) u_l^N \right\rangle, \tag{21}$$

$$W \geq p_{HL} u_h^N + (1 - p_{HL}) u_l^N. \tag{22}$$

Here, (13) is the participation (or promise-keeping) constraint in the first period; (16) and (20) are the promise-keeping constraints in the n-th period and the N-th period; (17) and (21) are the threat-keeping constraints; (14), (18), and (22) are the incentive-compatibility constraints; and (15) and (19) are feasibility constraints on $(W, \hat{W})$. u_i^n denotes the utility of an agent at period n with output i. W_i^n is the promised utility of an agent from the period n on when the output is i and the agent takes the effort level H. $\hat{W}_i^n$ is the promised utility of an agent from the period n on when the output is i and the agent takes the effort level L. The feasibility set, $\mathcal{W}^n$, is defined as the set of $(W^n, \hat{W}^n)$ such that there exists an allocation satisfying the promise-keeping, the threat-keeping, and the incentive-compatibility constraints in all the following periods.

Example 7: (Human Capital)

Utilizing this formulation, we characterize an example with $N = 3$.[10] The setting and the parameter values are the same as Example 1, except that human capital is formed by

$$k_t = (1 - \delta)I(e_{t-1}) + I(e_t), \tag{23}$$

when $t = 2, 3$.

Figures 16, 17, and 18 plot the utility values for each δ. Here, as in the previous examples, we see the tendency that u_h and u_l become closer as δ becomes smaller. Perfect insurance in the first period does not occur in this example. To see why, let us separate this three-period model into two parts: "first and second period" part and "second and third period" part. From the perspective of "second and third period" part (as a two-period model), as δ decreases it becomes optimal to concentrate the incentive scheme to the third period and make the second period utility constant. However, from the perspective of the "first and second period" part, as δ decreases it becomes optimal to concentrate the incentive scheme

---

[10]We chose $N = 3$ due to computational limitations. In principle, the same method can be used to characterize any N-period contract. Fernandes and Phelan (2000) compute an example of a hidden-endowment model with an infinite horizon. Their method cannot be applied straightforwardly to a repeated moral hazard model, since we cannot guarantee the convexity of the set $\mathcal{W}$. We thank Chris Phelan for providing us with the program used in Fernandes and Phelan (2000).
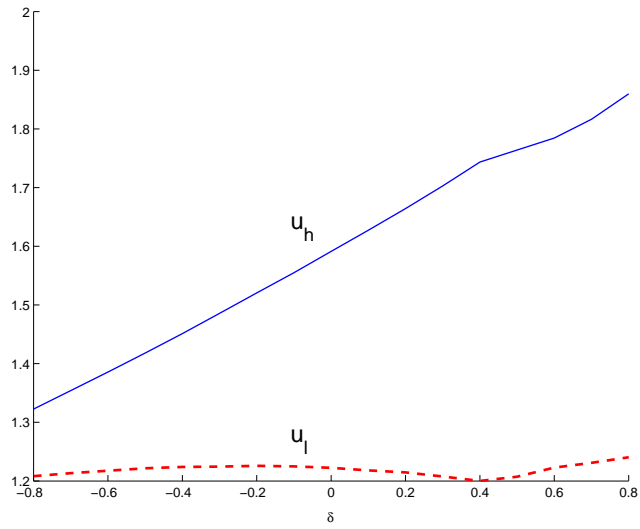
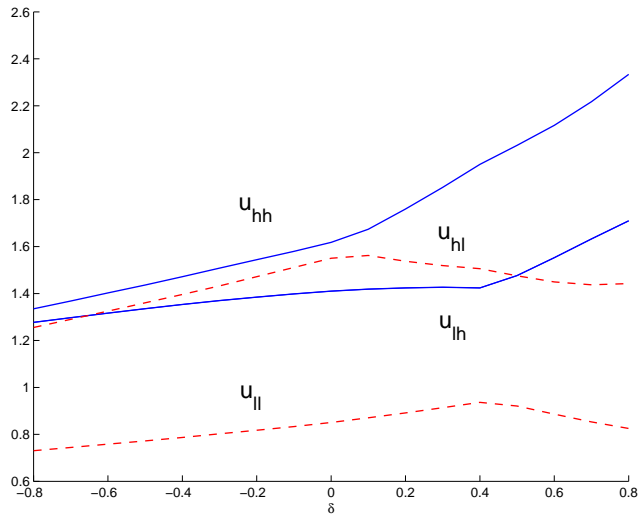Figure 16: u_l and u_h for different values of δ.

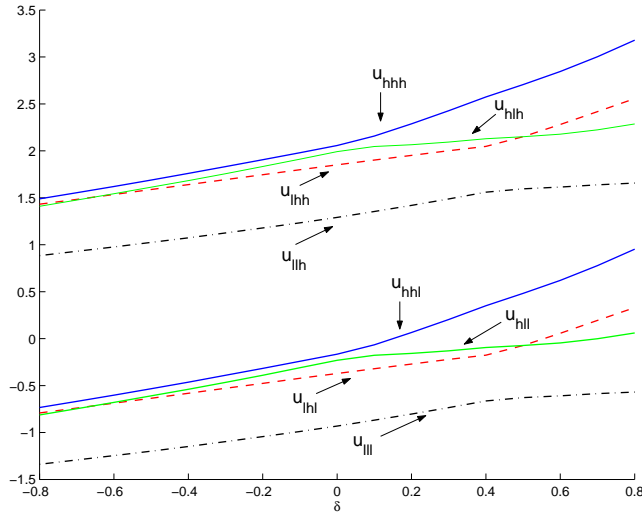Figure 17: u_{lh},u_{hl},u_{ll}, and u_{hh} for different values of δ.

Figure 18: u_{lll}, u_{lhl}, u_{llh}, u_{lhh}, u_{hll}, u_{hhl}, u_{hlh}, and u_{hhh} for different values of δ.

in the second period. Therefore, the increase in persistence has two opposite effects on the second period incentive scheme.[11] The first effect prevents the principal from levying a strong enough incentive scheme in the second period to make the first-period utility constant. Due to these (opposing) two effects, the comparison between the first-period and the second-period incentive schemes becomes somewhat unclear. However, it is clear that the third-period incentive scheme (response of the reward to the output) tends to be much larger than the first-period and the second-period incentive schemes when δ is small (persistence is large). This tendency is consistent with the basic model.

9 Empirical Implications

In above examples, we have shown that when the effect of effort is persistent, the correlation between the output and the wage of an agent tends to be lower in the earlier periods compared to the later periods of the principal-agent relationship. Moreover, for certain parameter values, the optimal contract provides perfect insurance for the agent in the earlier periods,

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[11]This conflict arises since the effect of the first-period effort persists only for two periods. In Example 2, perfect insurance occurred for small δ since the third-period incentive scheme also affected the first-period effort choice by the agent.

implying a zero-correlation between pay and performance.

This result has a clear empirical implication: the correlation between pay and performance should be lower in the earlier periods of a contract. A natural way of examining the empirical plausibility of this prediction is to look at the correlation between pay and performance at different points of a contract. We expect to see that pay is more sensitive to performance in the later periods. Here, we present two empirical studies which support this prediction.

Gibbons and Murphy (1992)

Gibbons and Murphy (1992) study the cross-sectional variation in the pay-for-performance relationship for CEOs. They use the firms' stock price as a measure of the CEO performance. They find that the pay-for-performance elasticity is lower for recently-appointed CEOs and it increases as the CEOs approach retirement. This result is consistent with our predictions: the pay-for-performance relationship is less sensitive to performance in the early periods.

The theoretical analysis in Gibbons and Murphy (1992) is different from our analysis,[12] yet in their framework (as in ours) there is a non-trivial link between the earlier periods and the later periods of the principal-agent relationship.

Gompers and Lerner (1999)

Gompers and Lerner (1999) study the pay-for-performance elasticity for venture capital organizations. They consider both cross-sectional and time-series variation in compensation. They find that compensation for smaller and younger venture capitalist organizations is less sensitive to performance than for older and larger venture capitalist organizations. Moreover, the pay-for-performance elasticity increases as the firm's seniority increases.[13]

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[12]Gibbons and Murphy (1992) explain their finding in terms of career concerns. Even without an explicit incentive scheme, recently-appointed CEOs work harder to obtain a good outcome, because a good outcome sends a good signal about their ability and has a positive influence on their future compensation.

[13]They explain the difference in the pay-for-performance elasticity with a learning model, where the venture capitalist is concerned about reputational effects. Younger firms try to establish good reputation which will in turn bring them higher compensation in the later periods. As the firms get older, these reputational concerns

The findings of Gompers and Lerner (1999) are consistent with our results. The presence of intertemporal links makes it possible to provide incentives by using a pay-for-performance compensation scheme only in the second period. The correlation between pay and performance is weak in the first period since the second-period pay-for-performance scheme is sufficient to provide incentives for high effort in both periods.

Both Gibbons and Murphy (1992) and Gompers and Lerner (1999) provide empirical support for our compensation scheme. The key to all of these findings is the presence of a link across periods. When the agent's effort is persistent, the agent will be concerned about the effect of his first period's effort on the future. Thus the agent will choose to work hard in the first period even if he is not fully compensated for high output. The persistence – intertemporal links across periods – can be attributed to human capital accumulation/learing-by-doing (our case), career concerns (Gibbons and Murphy [1992]), or to reputational concerns (Gompers and Lerner [1999]).

10 Conclusion

This paper analyzed a repeated moral hazard model where the current (hidden) action of an agent has a persistent effect on the future outcome. First, the optimal contract in a two-period setting was characterized analytically and numerically. Unlike the existing models of repeated moral hazard, there is a possibility that the agent is perfectly insured against the first-period outcome: the agent obtains the same utility regardless of the first-period output. In the numerical examples, it was shown that persistence tends to make compensation less responsive to the outcome in the first period.

Next, two extensions of the model were presented. In the first extension, the model was generalized to include more than two effort choices. In the second extension, the model was extended to three periods. The numerical examples showed that the main result in the

are less important and pay-for-performance incentives are required to make the firms work harder.

two-effort, two-period model holds for the extended models as well: when persistence exists, wages tend to be less responsive to the outcomes in the earlier periods. Moreover, in the three-period setting, the consumption of the agent is completely smooth between the first and the second periods.

Applied to the problem of the optimal unemployment insurance, our result implies that unemployment insurance payments tend to be constant in the initial periods of unemployment. This feature remains in the model where an employed worker does not need to make a job-retention effort.

We also considered a computational method to calculate an N-period model with two-period dependence. As an example, we numerically characterized a three-period model with two-period dependence. An important future topic is to extend the analysis to an infinite-horizon contract.

Empirical evidence supports the main implication of our analysis: pay-for-performance elasticity is lower for younger CEOs and younger venture capitalist organizations. One interesting application would be to look at the pay-for-performance relationship for a more diverse worker pool and to test our prediction. It would also be interesting to extend our model to permit the separation between the firm (the principal) and the worker (the agent). In such an environment, the punishment can take the form of a separation which involves a wage loss. The extended model should predict that a worker with higher tenure should suffer a larger wage cut when separation occurs.[14]

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[14]Topel (1990) argues that the wage loss of a displaced worker tends to be larger when the worker's previous job duration is longer.

Appendix

A Proofs

Proof of Proposition 1:

Part 1:

From Assumption 1, the coefficients of λ_1, λ_3, λ_6, and λ_7 are strictly negative in (2), and strictly positive in (3). Since $\lambda_1, \lambda_3, \lambda_6, \lambda_7 \geq 0$ and $v(\cdot)$ is strictly convex, part 1 follows.

Part 2:

$(IC4)$ can be rewritten as

$$(p_{HH} - p_{HL})(u_{lh} - u_{ll}) \geq c. \tag{24}$$

From Assumption 1, $(p_{HH} - p_{HL}) > 0$. Thus, $(u_{lh} - u_{ll}) > 0$ and the first inequality of part 2 follows. The second inequality of part 2 can be proved by applying the same logic to $(IC5)$.

Here, we note that the following corollary holds.

Corollary 2

1. *At least one of $(IC1)$, $(IC3)$, $(IC4)$, $(IC6)$, $(IC7)$ is binding.*
2. *At least one of $(IC1)$, $(IC3)$, $(IC5)$, $(IC6)$, $(IC7)$ is binding.*

Proof: The first part follows from the fact that $u_{lh} > u_{ll}$ and (4) and (6). The second part follows from $u_{hh} > u_{hl}$ and (5) and (7). □

Part 3:

Proving part 3 requires several steps. The goal is to prove that $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$. In the end, we will show that those inequalities hold for all the combinations of positive and zero Lagrange multipliers $\{\lambda_1, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7\}$. Before that, we will establish several properties which will be used later.

Lemma 2 *At least one of $(u_{hh} - u_{lh})$ and $(u_{hl} - u_{ll})$ has to be (weakly) positive.*

Proof: Suppose, by contradiction, $u_{hh} < u_{lh}$ and $u_{hl} < u_{ll}$.

From (4) and (5), the following holds.

$$
\begin{aligned}
v'(u_{hl}) - v'(u_{ll}) = \; & \frac{1-p_{LH}}{1-p_{HH}}\left[\frac{1-p_L}{1-p_H} - \frac{p_L}{p_H}\right]\lambda_1 + \frac{1-p_{LL}}{1-p_{HH}}\left[\frac{1-p_L}{1-p_H} - \frac{p_L}{p_H}\right]\lambda_3 \\
& + \left[1 - \frac{1-p_{HL}}{1-p_{HH}}\right](\lambda_5 - \lambda_4) \\
& + \left[\frac{(1-p_L)(1-p_{LL})}{(1-p_H)(1-p_{HH})} - \frac{p_L(1-p_{LH})}{p_H(1-p_{HH})}\right]\lambda_6 \\
& + \left[\frac{(1-p_L)(1-p_{LH})}{(1-p_H)(1-p_{HH})} - \frac{p_L(1-p_{LL})}{p_H(1-p_{HH})}\right]\lambda_7.
\end{aligned}
\tag{25}
$$

Note that the coefficients of λ_1, λ_3, and λ_6 are always strictly positive. The coefficient of $(\lambda_5 - \lambda_4)$ is strictly negative.

From (6) and (7), the following holds.

$$
\begin{aligned}
v'(u_{hh}) - v'(u_{lh}) = \; & \frac{p_{LH}}{p_{HH}}\left[\frac{1-p_L}{1-p_H} - \frac{p_L}{p_H}\right]\lambda_1 + \frac{p_{LL}}{p_{HH}}\left[\frac{1-p_L}{1-p_H} - \frac{p_L}{p_H}\right]\lambda_3 \\
& + \left[1 - \frac{p_{HL}}{p_{HH}}\right](\lambda_5 - \lambda_4) \\
& + \left[\frac{(1-p_L)p_{LL}}{(1-p_H)p_{HH}} - \frac{p_L p_{LH}}{p_H p_{HH}}\right]\lambda_6 \\
& + \left[\frac{(1-p_L)p_{LH}}{(1-p_H)p_{HH}} - \frac{p_L p_{LL}}{p_H p_{HH}}\right]\lambda_7.
\end{aligned}
\tag{26}
$$

Note that the coefficients of λ_1, λ_3, and λ_7 are always strictly positive. The coefficient of $(\lambda_5 - \lambda_4)$ is also strictly positive.

Utilizing (25) and (26), we can calculate

$$
J \equiv (1 - p_{HH})[v'(u_{hl}) - v'(u_{ll})] + p_{HH}[v'(u_{hh}) - v'(u_{lh})].
\tag{27}
$$

Under the initial supposition, J has to be strictly negative. However, since

$$
J = \left[\frac{1-p_L}{1-p_H} - \frac{p_L}{p_H}\right](\lambda_1 + \lambda_3 + \lambda_6 + \lambda_7) \geq 0,
\tag{28}
$$

this is a contradiction. $\square$

Lemma 3

1. When $\lambda_6 > 0$, $u_{hh} - u_{lh} \geq u_{hl} - u_{ll}$.

2. When $\lambda_7 > 0$, $u_{hl} - u_{ll} \geq u_{hh} - u_{lh}$.

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Proof: We will prove only the first part. The second part can be proved in a similar manner. When $\lambda_6 > 0$, $(IC6)$ holds with equality. This implies that [RHS of $(IC6)$] $\geq$ [RHS of $(IC3)$]. Rearranging terms, we obtain

$$(p_{LH} - p_{LL})(u_{hh} - u_{hl}) \geq c. \tag{29}$$

Similarly, rearranging [RHS of $(IC6)$] $\geq$ [RHS of $(IC1)$] yields

$$(p_{LH} - p_{LL})(u_{lh} - u_{ll}) \leq c. \tag{30}$$

Equations (29) and (30) imply that $u_{hh} - u_{hl} \geq u_{lh} - u_{ll}$, which can be rewritten as $u_{hh} - u_{lh} \geq u_{hl} - u_{ll}$. $\square$

Lemma 4

1. When $\lambda_4 > 0$, $u_{hh} - u_{lh} \geq u_{hl} - u_{ll}$.
2. When $\lambda_5 > 0$, $u_{hl} - u_{ll} \geq u_{hh} - u_{lh}$.

Proof: We will prove only the first part. The second part can be proved in an analogous manner. When $\lambda_4 > 0$, $(IC4)$ binds, and therefore

$$(p_{HH} - p_{HL})(u_{lh} - u_{ll}) = c$$

holds. $(IC5)$ can be rewritten as:

$$(p_{HH} - p_{HL})(u_{hh} - u_{hl}) \geq c. \tag{31}$$

Thus, $u_{hh} - u_{hl} \geq u_{lh} - u_{ll}$, which can be rewritten as $u_{hh} - u_{lh} \geq u_{hl} - u_{ll}$. $\square$

Now, we will show that the desired inequalities hold for all the possible combinations of the Lagrange multipliers. We will start from the following cases:

Lemma 5

1. When $\lambda_4 > 0$ and $\lambda_5 > 0$, $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$.
2. When $\lambda_4 > 0$ and $\lambda_7 > 0$, $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$.

3. When $\lambda_5 > 0$ and $\lambda_6 > 0$, $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$.

4. When $\lambda_6 > 0$ and $\lambda_7 > 0$, $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$.

Proof: We will prove only the first part. The other parts can be proved in a similar manner, utilizing Lemma 3 and Lemma 4. When $\lambda_4 > 0$ and $\lambda_5 > 0$, Lemma 4 implies that $u_{hh} - u_{lh} = u_{hl} - u_{ll}$. From Lemma 2, $u_{hh} - u_{lh}$ and $u_{hl} - u_{ll}$ cannot be both negative. Therefore, $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$ follow. $\square$

Lemma 5 completes the proof for all the cases which involve strictly positive values of $\{\lambda_4, \lambda_5\}$, $\{\lambda_4, \lambda_7\}$, $\{\lambda_5, \lambda_6\}$, and $\{\lambda_6, \lambda_7\}$. There still are some cases left. First note that since the coefficients of λ_1 and λ_3 in (25) and (26) are both strictly positive, having λ_1 or λ_3 strictly positive only strengthens the positivity of $u_{hh} - u_{lh}$ and $u_{hl} - u_{ll}$, so adding strictly positive λ_1 or λ_3 does not alter the result. (The cases of when only λ_1 and/or λ_3 are positive are straightforward from (25) and (26).) Also note that the cases where only $\lambda_4 > 0$ or where only $\lambda_5 > 0$ never occur by Corollary 2. Corollary 2 also implies that at least one of the λs always have to be strictly positive. Therefore, the other cases are categorized into the following five:

1. $\lambda_6 > 0$ and $\lambda_4 = \lambda_5 = \lambda_7 = 0$,

2. $\lambda_7 > 0$ and $\lambda_4 = \lambda_5 = \lambda_6 = 0$,

3. $\lambda_6 > 0$, $\lambda_4 > 0$ and $\lambda_5 = \lambda_7 = 0$,

4. $\lambda_7 > 0$, $\lambda_5 > 0$ and $\lambda_4 = \lambda_6 = 0$,

5. $[\lambda_1 > 0$ and/or $\lambda_3 > 0]$, $[\lambda_4 > 0$ or $\lambda_5 > 0]$, and [the other λs $= 0$].

The following Lemma establishes the first case and the third case. The second case and the fourth case can be proved in a similar manner.

Lemma 6 When $\lambda_6 > 0$ and $\lambda_5 = \lambda_7 = 0$, $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$.

Proof: From Lemma 3, $u_{hh} - u_{lh} \geq u_{hl} - u_{ll}$. Therefore, we only need to prove that $u_{hl} \geq u_{ll}$. This inequality holds from (25), $\lambda_4 \geq 0$, and $\lambda_5 = \lambda_7 = 0$. $\square$

The following Lemma establishes the fifth case for $[\lambda_1 > 0, \lambda_4 > 0,$ and $\lambda_5 = \lambda_6 = \lambda_7 = 0]$. The other combinations of positive λs can be proved in a similar manner.

Lemma 7 *When $[\lambda_1 > 0, \lambda_4 > 0,$ and $\lambda_5 = \lambda_6 = \lambda_7 = 0]$, $u_{hh} - u_{lh} \geq 0$ and $u_{hl} - u_{ll} \geq 0$.*

Proof: From Lemma 4, $u_{hh} - u_{lh} \geq u_{hl} - u_{ll}$. Therefore, we only need to prove that $u_{hl} \geq u_{ll}$. This inequality holds from (25) and $\lambda_5 = \lambda_7 = 0$. $\square$

This completes the proof of part 3 of Proposition 1. $\square$

Proof of Proposition 3:

$[u_l = u_h] \Rightarrow [u_{ll} = u_{hl}$ and $u_{lh} = u_{hh}]$:

From (2) and (3), $u_l = u_h$ implies that $\lambda_1 = \lambda_3 = \lambda_6 = \lambda_7 = 0$. This, together with Corollary 2, implies that $(IC4)$ and $(IC5)$ are both binding. Rewriting $(IC4)$ and $(IC5)$,

$$(p_{HH} - p_{HL})(u_{lh} - u_{ll}) = c \tag{32}$$

and

$$(p_{HH} - p_{HL})(u_{hh} - u_{hl}) = c \tag{33}$$

hold. Thus,

$$u_{lh} - u_{ll} = u_{hh} - u_{hl}. \tag{34}$$

Suppose, by contradiction, $u_{ll} < u_{hl}$. From (4) and (5), this implies $\lambda_4 > \lambda_5$. Then, from (6) and (7), $u_{hh} < u_{lh}$ follows. In sum,

$$u_{ll} < u_{hl} < u_{hh} < u_{lh},$$

which contradicts (34). In the same manner, $u_{ll} > u_{hl}$ leads to a contradiction. Thus $u_{ll} = u_{hl}$ follows. $u_{lh} = u_{hh}$ can be proved by the same logic.

$[u_{ll} = u_{hl}$ and $u_{lh} = u_{hh}] \Rightarrow [u_l = u_h]$:

From (2) and (3), it is sufficient to show that $\lambda_1 = \lambda_3 = \lambda_6 = \lambda_7 = 0$. Let's calculate the value J from (27). Since $u_{ll} = u_{hl}$ and $u_{lh} = u_{hh}$, $J = 0$. From (28), this implies $\lambda_1 = \lambda_3 = \lambda_6 = \lambda_7 = 0$. $\square$

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Proof of Proposition 4:

Necessity:

From Proposition 3, $u_{ll} = u_{hl}$ and $u_{lh} = u_{hh}$ hold whenever $u_l = u_h$. Using this to rewrite $(IC1)$ yields

$$\beta(p_{HH} - p_{LH})(u_{lh} - u_{ll}) \geq c. \tag{35}$$

Substituting for $(u_{lh} - u_{ll})$ using (32) yields (9). Applying the same procedure to $(IC3)$ provides (10). In the same way, $(IC6)$ and $(IC7)$ yield

$$\beta\{p_{HH} - p_L p_{LH} - (1 - p_L)p_{LL}\} \geq \{1 + \beta(1 - p_L)\}(p_{HH} - p_{HL}),$$

and

$$\beta\{p_{HH} - p_L p_{LL} - (1 - p_L)p_{LH}\} \geq (1 + \beta p_L)(p_{HH} - p_{HL}).$$

It is easy to show that these inequalities are implied by (9) and (10).

Sufficiency:

Consider a modified version of $(P1)$, where only $(IC4)$, $(IC5)$, and (PC) are the constraints. We will solve the model for this modified problem, whose solution exhibits perfect insurance. Then, we will show that the other IC constraints are satisfied under this solution, given (9) and (10).

Clearly, the first-order conditions for this modified problem are (2) to (7), with $\lambda_1 = \lambda_3 = \lambda_6 = \lambda_7 = 0$. From (2) and (3), $u_l = u_h$. Proposition 3 holds for the modified problem, and $u_{ll} = u_{hl}$ and $u_{hl} = u_{hh}$ follow. Since $(IC4)$ and $(IC5)$ both bind (see the proof of Proposition 3), (32) and (33) hold.

Under this solution, $(IC1)$ can be rewritten as (35). Using (32), it is clear that this is satisfied given (9). In a similar manner, it can be easily shown that $(IC3)$, $(IC6)$, and $(IC7)$ are also satisfied.□

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