A Simple Model of Productivity Slowdown

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Abstract

This paper demonstrates a simple mechanism of productivity slowdown. There are two types of technological change: exogenous and endogenous. After news arrives that there will be an acceleration of exogenous technological progress in the future, endogenous technological progress may slow down.

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1 Introduction

Many OECD countries experienced a slowdown in productivity growth since the 1970s. In the United States, the labor productivity growth rate during 1973-95 was 1.47% on average, which is substantially lower than the 1959-73 period (3.03%).[1] There has also been a resurgence of productivity in recent years: during 1995-2001, the labor productivity growth was 2.61% on average.

This paper demonstrates that this productivity slowdown and resurgence can be explained by a modified version of the standard Ramsey model. There are two types of technological change: exogenous and endogenous. After news arrives that there will be an acceleration of exogenous technological progress in the future, the rate of endogenous technological progress may slow down.

In the standard Solow model and Ramsey model, the technological change is assumed to be *completely exogenous*. In the models of endogenous technological change such as Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), the technological change is assumed to be *completely endogenous*. In reality, some new technologies arrive exogenously (e.g. scientific discoveries in universities), and some new technologies are the outcomes of intentional (endogenous) research and development by firms. We emphasize the interaction of these two.

Other authors also pointed out that the acceleration of exogenous technological progress can initially reduce the measured productivity. Hornstein and Krusell (1996) focus on learning and quality mismeasurement. Greenwood and Yorukoglu (1997) emphasize the increase in (unmeasured) investment for setting up and learning new technologies. Mukoyama (2005) considers the mismeasurement of the aggregate capital stock. The mechanism in the current paper is complementary to these alternative theories.[2]

2 Model

Time is continuous. There is an aggregate production technology $Y(t) = F(A(t), Z(t), L(t))$. Here, $Y(t)$ is the total output at time t, $A(t)$ is the exogenous technology level, $Z(t)$ is the endogenous technology level, and $L(t)$ is the labor input. For simplicity, we abstract from capital accumulation in this section. There is no population growth, and $A(t)$ evolves exogenously with the growth rate γ_A. $Z(t)$ can be enhanced by the R&D investment[3] and

[1] All the numbers in this section are taken from Nordhaus (2005, Table 3). Nordhaus shows that the same pattern can be found using other measures of productivity.

[2] Another related literature is the one that focuses on the role of the general purpose technologies (GPTs). Helpman and Trajtenberg's (1998) model exhibits a slowdown in *output* after the arrival of a new GPT due to the increase in the resources devoted to R&D investment. Aghion and Howitt (1998a) emphasize the role of social learning in creating a delayed slump in aggregate output after the arrival of a new GPT.

[3] This includes not only the measured R&D but also any productivity-enhancing expenditures, such as expenditures for learning and reorganization. The data shows that the measured R&D/GDP ratio declined

evolves according to

$$\dot{Z}(t) = I(t), \tag{1}$$

where $I(t)$ is the amount of R&D investment.[4] Denoting the aggregate consumption level by $C(t)$, it follows that $I(t) = Y(t) - C(t)$. For the ease of exposition, we assume a Cobb-Douglas production function of the form $F(A(t), Z(t), L(t)) = Z(t)^\alpha (A(t)L(t))^{1-\alpha}$, where $\alpha \in (0, 1)$. With this assumption, the current analysis becomes analogous to the analysis of the standard Ramsey model, with $Z(t)$ instead of capital stock. With this production function, the growth rate of total factor productivity (TFP) can be measured as[5]

$$\gamma_{TFP} = \alpha \frac{\dot{Z}(t)}{Z(t)} + (1-\alpha)\frac{\dot{A}(t)}{A(t)}. \tag{2}$$

The representative agent lives for an infinite horizon. He maximizes the utility

$$U = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

where $\rho > 0$ and $\theta \geq 0$. The standard optimization procedure provides the Euler equation

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}\left(\alpha \left(\frac{Z(t)}{A(t)L(t)}\right)^{\alpha-1} - \rho\right).$$

We denote the variables in per efficiency unit of labor with small letters: $y(t) \equiv Y(t)/(A(t)L(t))$, $z(t) \equiv Z(t)/(A(t)L(t))$, and $c(t) \equiv C(t)/(A(t)L(t))$. Then, we obtain the set of differential equations that characterize the path of $(z(t), c(t))$:

$$\dot{z}(t) = z(t)^\alpha - c(t) - \gamma_A z(t),$$

and

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(\alpha z(t)^{\alpha-1} - \rho - \theta\gamma_A).$$

Assume that the economy is in the steady-state ($Y(t)$ and $Z(t)$ are growing at the rate γ_A) initially. Suppose that at time t_0, suddenly and unexpectedly, there arrives news that from time $t_1 (> t_0)$, the exogenous productivity growth rate, γ_A, will increase to a new level γ'_A. This resembles the situation in the U.S. in the 1970s, when people became aware of upcoming new technologies, such as the computer, but which were not yet available to the wide population until more than a decade later.[6] Figures 1 and 2 depict the transition path.

starting in the late 1960s until the end of the 1970s, and then somewhat recovered (Aghion and Howitt 1998b, Figure 12.3).

[4] Alternatively, one can model the endogenous technological change as in the recent endogenous growth literature—e.g. Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

[5] Since we do not have capital stock and there is no population growth, this also corresponds to the growth rate of labor productivity.

[6] In general, the diffusion of a new technology is not immediate. See Mukoyama (2004) and reference therein for studies of diffusion of new technologies.

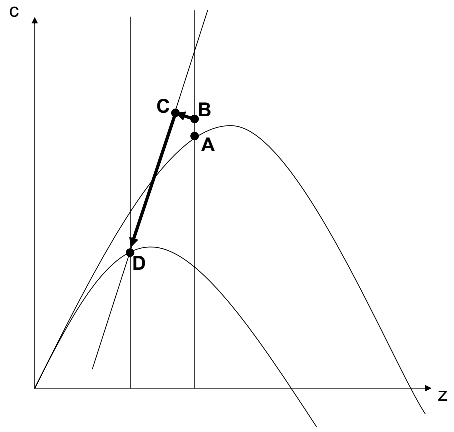

Figure 1: Case I

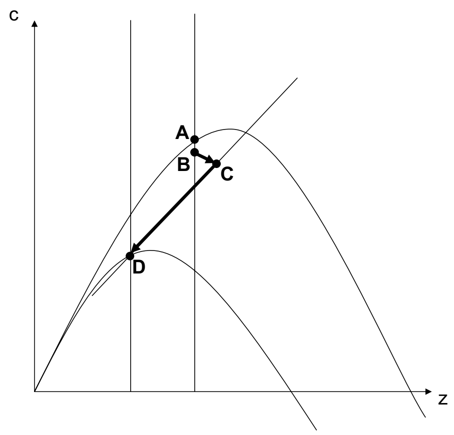

Figure 2: Case II

A is the old steady-state and **D** is the new steady-state. There are two possible cases, Case I (Figure 1) and Case II (Figure 2). In Case I, the saddle-path of the new system of differential equations is relatively steep. For $(z(t_1), c(t_1))$ to be on the new saddle-path at time t_1 (point **C**), the consumption must jump up at time t_0 (point **B**). In Case II, the new saddle-path is relatively flat. For $(z(t_1), c(t_1))$ to be on the new saddle-path at time t_1 (point **C**), the consumption must jump down at time t_0 (point **B**). Case I produces the initial decline in $z(t)$ and therefore the decline in productivity growth. Case II does not exhibit such a decline.[7]

In Case I, $z(t)$ declines even before the new regime arrives. This is due to intertemporal consumption smoothing. Since the consumer is now aware that consumption $C(t)$ will grow at the rate γ'_A in future, with the concave utility it is optimal to adjust consumption gradually. To achieve this, the consumer increases consumption from time t_0 and decreases $z(t)$. There is also an incentive to increase $z(t)$—since the exogenous productivity is higher in future, there is an incentive to increase endogenous productivity from today, so that one can produce more once the new regime arrives. If this effect is strong enough, Case II will take place.

Figure 3 plots the time series[8] of γ_{TFP}, calculated by (2), when $\alpha = 0.3$, $\rho = 0.05$, $\gamma_A = 0.03$, $\gamma'_A = 0.05$, $t_0 = 0$, and $t_1 = 20$. The solid line corresponds to $\theta = 1$ (log utility) and the dotted line corresponds to $\theta = 5$. It turns out that with both values of θ, we obtain Case I. This shows that Case I is not an unreasonable outcome for standard parameter values. Between t_0 and t_1, γ_{TFP} keeps falling. This can be seen as the productivity slowdown. The productivity slowdown is more severe when θ is larger. At time t_1, γ_{TFP} jumps up, and starts increasing. This corresponds to the productivity resurgence. Figures 4 and 5 plot the time series of the growth rate of C and the growth rate of Z.

3 Discussion

Incorporating capital does not alter the qualitative prediction of the model. The simplest way to incorporate capital stock $K(t)$ is to assume that the production technology is $Y(t) = Z(t)^{\alpha/2}K(t)^{\alpha/2}(A(t)L(t))^{1-\alpha}$, and to assume that the capital stock accumulates in the same manner as in (1). Then, in the steady state, the value of Z is equal to the value of K, and starting from the steady-state, it is optimal to always set $Z(t) = K(t)$ for all t. Then, defining $X(t) \equiv Z(t) = K(t)$, the production technology becomes $Y(t) = X(t)^{\alpha}(A(t)L(t))^{1-\alpha}$ and $X(t)$ follows the same transition equation as (1). Therefore, the dynamics of the economy are exactly the same as in the previous section. The growth rate of TFP is now calculated

[7] Case II is less plausible than Case I, since it implies that if the new regime arrives at the same time as the news ($t_1 = t_0$), the consumption level jumps down. Later we will show that with reasonable parameter values, Case I realizes.

[8] The numerical solutions are computed using forward- and backward-shooting methods. See Judd (1998, Chapter 10).

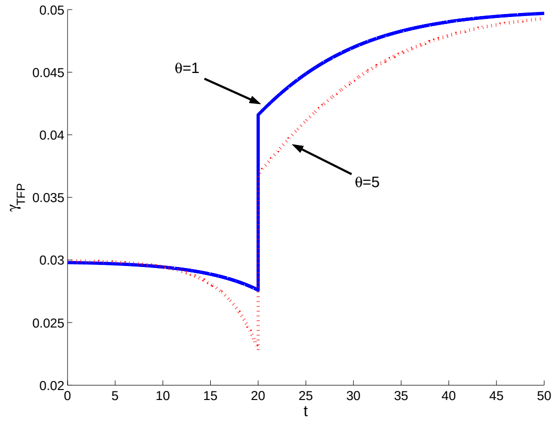

Figure 3: Time Series of γ_{TFP}

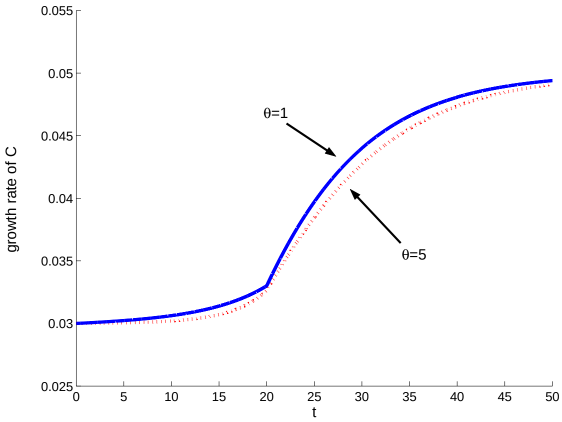

Figure 4: Time series of $\dot{C}/C$

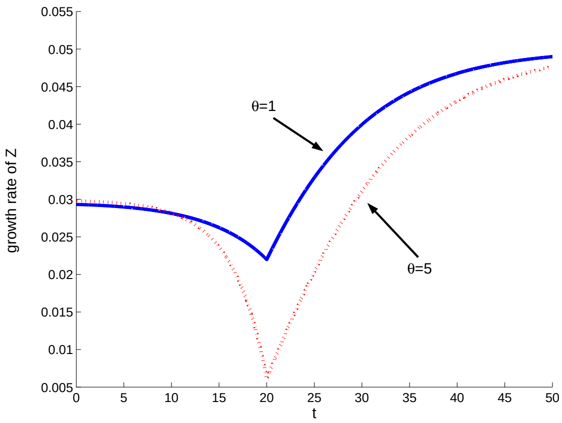

Figure 5: Time series of $\dot{Z}/Z$

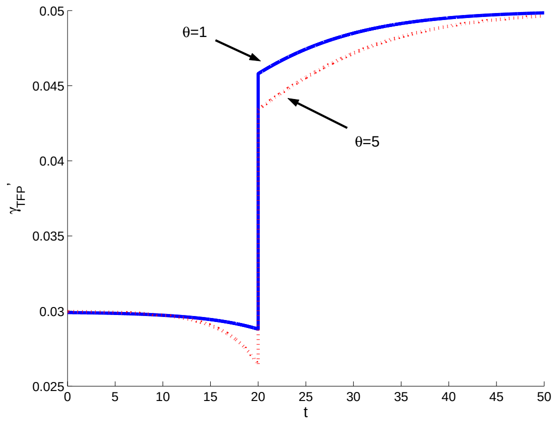

Figure 6: Time Series of γ'_{TFP}

as

$$\gamma'_{TFP} = \frac{\alpha}{2}\frac{\dot{Z}(t)}{Z(t)} + (1-\alpha)\frac{\dot{A}(t)}{A(t)}.$$

Figure 6 plots γ'_{TFP}, which exhibits the same pattern as Figure 3.

In our model, it is assumed that the new regime arrives to the entire economy at time t_1. In reality, the diffusion of technologies is rather gradual. In such a case, the time series of the TFP growth rate will look more smooth. Also in this case, the model will exhibit a productivity slowdown as long as the initial speed of technology diffusion is sufficiently slow.

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