# Basic DMP model

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## 1 Setup

This note explains the basic DMP model, as in Pissarides (1985) and Shimer (2005). The aggregate number of matches at each period is dictated by the constant-returns-to-scale matching function $M(v_t, u_t)$, where $v_t$ is the aggregate vacancy, and $u_t$ is the number of unemployed workers at time $t$. At the individual level, matching is stochastic, and the probability of a worker finding a job is $p(\theta_t) \equiv M(\theta_t, 1)$, where $\theta_t \equiv v_t/u_t$. The probability of a firm finding a worker is $q(\theta_t) \equiv M(1, 1/\theta_t)$. The separation probability of a matched job-worker pair is $\sigma$. The job-worker match produces $z_t$ unit of consumption goods, and $z_t$ follows a Markov process.

### 1.1 Unemployment dynamics

The total population is 1, and therefore the number of employed workers is $1 - u_t$. The dynamics of the unemployment is dictated by

$$u_{t+1} = \sigma(1 - u_t) + (1 - p(\theta_t))u_t. \tag{1}$$

### 1.2 Value functions

From a firm's perspective, the value of being matched with a worker, $J_t$, is:

$$J_t = z_t - w_t + \beta E[(1 - \sigma)J_{t+1} + \sigma V_t], \tag{2}$$

where $V_t$ is the value of vacancy and $w_t$ is the wage paid to the worker. The expectation $E[\cdot]$ is taken with the information at time $t$. The value of vacancy is

$$V_t = -\kappa + \beta E[q(\theta_t)J_{t+1} + (1 - q(\theta_t))V_{t+1}]. \tag{3}$$

For the worker's side, the value of being employed, $W_t$, is

$$W_t = w_t + \beta E[(1 - \sigma)W_{t+1} + \sigma U_{t+1}], \tag{4}$$

and the value of being unemployed, $U_t$, is

$$U_t = b + \beta E[p(\theta_t)W_{t+1} + (1 - p(\theta_t))U_{t+1}]. \tag{5}$$

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### 1.2.1 Wage determination

Let

$$\tilde{J}_t(w) = z_t - w + \beta E[(1-\sigma)J_{t+1} + \sigma V_{t+1}]$$

and

$$\tilde{W}_t(w) = w + \beta E[(1-\sigma)W_{t+1} + \sigma U_{t+1}].$$

The wage is determined by the generalized Nash bargaining with the worker's bargaining power $\gamma \in (0,1)$. Then $w$ solves

$$(1-\gamma)(\tilde{W}_t(w) - U_t) = \gamma(\tilde{J}_t(w) - V_t). \tag{6}$$

### 1.2.2 Free entry and equilibrium

We assume free entry to vacancy posting, $V_t = 0$. From (3),

$$\kappa = \beta q(\theta_t) E[J_{t+1}] \tag{7}$$

holds. If the jobs are owned by a "large firm" with constant-returns to scale technology, this is the optimal hiring condition.

(2) can be rewritten as

$$J_t = z_t - w_t + \beta(1-\sigma)E[J_{t+1}].$$

Therefore,

$$J_t = z_t - w_t + \frac{(1-\sigma)\kappa}{q(\theta_t)}.$$

Using this to the right-hand side of (7) yields

$$\frac{\kappa}{q(\theta_t)} = \beta E\left[z_{t+1} - w_{t+1} + \frac{(1-\sigma)\kappa}{q(\theta_{t+1})}\right]. \tag{8}$$

From (4) and (5),

$$W_t - U_t = w_t - b + \beta E[(1-\sigma-p(\theta_t))(W_{t+1} - U_{t+1})].$$

This can be rewritten as

$$w_t = W_t - U_t + b - \beta E[(1-\sigma-p(\theta_t))(W_{t+1} - U_{t+1})].$$

From (6),

$$W_t - U_t = \frac{\gamma}{1-\gamma} J_t$$

under equilibrium wage. Thus

$$w_t = \frac{\gamma}{1-\gamma} J_t + b - \beta(1-\sigma-p(\theta_t)) \frac{\gamma}{1-\gamma} E[J_{t+1}].$$

Once again, from (7),

$$w_t = \frac{\gamma}{1-\gamma} J_t + b - \frac{\gamma}{1-\gamma} \frac{(1-\sigma-p(\theta_t))\kappa}{q(\theta_t)}.$$

Forwarding one period, taking expectation, and using (7) once again,

$$E[w_{t+1}] = \frac{\gamma}{1-\gamma} \frac{\kappa}{\beta q(\theta_t)} + b - \frac{\gamma}{1-\gamma} E\left[\frac{(1-\sigma-p(\theta_{t+1}))\kappa}{q(\theta_{t+1})}\right]. \tag{9}$$

Combining (8) and (9) we obtain

$$\frac{\kappa}{q(\theta_t)} = \beta E\left[(1-\gamma)(z_{t+1}-b) + \frac{(1-\sigma-\gamma p(\theta_{t+1}))\kappa}{q(\theta_{t+1})}\right]. \tag{10}$$

Note that the variable $u$ do not appear in (10). This implies that the dynamics of $\theta_t$ (a jump variable) is not influenced by $u$ (only influenced by $z$). Once we know the dynamics of $\theta_t$ from (10), we can determine the dynamics of unemployment by (1) and $u_0$.

Assume that $M(v,u) = \chi v^{1-\eta} u^{\eta}$, so that $p(\theta) = \chi\theta^{1-\eta}$ and $q(\theta) = \chi\theta^{-\eta}$, where $\chi > 0$ and $\eta \in (0,1)$. Then, log-linearizing (10) around the steady-state yields (the "tilde" ($\tilde{}$) denotes the value at the steady state and the "hat" ($\hat{}$) denotes the log deviation from the steady state)

$$\mathcal{A}\hat{\theta}_t = E[\tilde{z}\hat{z}_{t+1} + \mathcal{B}\hat{\theta}_{t+1}],$$

where $\mathcal{A} \equiv \kappa\eta\tilde{\theta}^{\eta}/(1-\gamma)\beta\chi$ and $\mathcal{B} \equiv [(1-\sigma)\kappa\eta\tilde{\theta}^{\eta}/(1-\gamma)\chi] - [\gamma\kappa\tilde{\theta}/(1-\gamma)]$.

Assume that $\hat{z}_{t+1} = \rho\hat{z}_t + \varepsilon_{t+1}$, where $\rho \in (0,1)$ and $\varepsilon_{t+1}$ is a mean zero random variable (thus $\tilde{z} = 1$). Since the equilibrium $\hat{\theta}$ has to take the form

$$\hat{\theta}_t = \mathcal{C}\hat{z}_t,$$

using the method of undetermined coefficients,

$$\mathcal{C} = \frac{\rho}{\mathcal{A} - \rho\mathcal{B}} = \frac{1-\gamma}{\kappa\tilde{\theta}^{\eta}\left(\left[\frac{1}{\rho\beta} - (1-\sigma)\right]\frac{\eta}{\chi} + \gamma\tilde{\theta}^{1-\eta}\right)}. \tag{11}$$

This makes it clear that, for example, for given $\tilde{\theta}$ the amplification ($\mathcal{C}$) is large when $\kappa$ is small. This is the background of Hagedorn and Manovskii's (2008) main result. (In order to keep $\tilde{\theta}$ and other parameters constant, a small $\kappa$ requires a large value of $b$.)

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