Education as Self-Discovery*

Toshihiko Mukoyama†
Department of Economics
Concordia University
and
CIREQ

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Abstract

In this paper, we analyze the interaction between labour market conditions and education demand in a job matching model. Education is modeled as a process of self-discovery: schooling raises a worker's likelihood of a good match to the job by helping him to find his comparative advantage. It is shown that there is a close relationship between education demand and job mobility.

Keywords: Job Matching; Schooling; Education Demand; Mobility.

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†1455 de Maisonneuve Blvd. West, Montreal, Quebec, H3G 1M8 Canada. Tel: +1-514-848-3927, Fax: +1-514-848-4536, E-mail: mukoyama@alcor.concordia.ca

1 Introduction

The importance of education in economic life has been recognized for a long time, and there has been substantial studies about the economic role of education. The economics of education can be mainly categorized into two literature. One is the *human capital* literature (e.g. Becker [1993], Mincer [1974]), which analyzes the aspect that education provides workers with skills (general or specific) by means of training. Another is the *signaling* literature (e.g. Spence [1973]), which investigates the role of education as a device to resolve the problem of asymmetric information about workers' ability.

Here, we propose another view. Schooling period can serve as a process of self-discovery: education has a role in finding one's comparative advantage. In real life, people do not always know their ability or their preferences for various kinds of existing jobs until they have already had some experience with them. Especially, it is difficult for one to know what he is good at and bad at. One of the greatest roles of education is to help people discovering their advantages and disadvantages, and give them orientations for their occupational choices in the future. In other words, we consider schooling as a device to reduce the friction in job assignment.[1] For modeling our view, we employ a variant of the Jovanovic (1979) matching model. In our model, schooling enhances workers' ability to match. Our analysis sheds light on the relationship between education demand and labour market conditions.

There are several important antecedents. Johnson (1978) analyzes the effect of education on labour market when education reveals a worker's *general* ability[2]. In our model, it is assumed that the difference in ability is always (*industry-, occupational-, or job-*) *specific*, and education reveals a worker's comparative advantages. We believe that our formulation captures an important real-life characteristics of education. In many empirical works such as Willis and Rosen (1979) and Heckman and Schienkman (1987), it is shown that comparative advantage is important in occupational choice and wage determination. The nature of people's ability is intrinsically complex, and to assess people's comparative advantage, time and experiences are necessary. Through the

[1] The effectiveness of job search assistance in public training suggests that assignment problem may be more serious than skill acquisition. See Heckman, Lochner, Smith, and Taber (1997).

[2] Neal and Rosen (2000) call the matching models with general ability as "sorting model" and the models with industry (or firm) specific ability as "matching model". Johnson analyzes only the "sorting" feature of education. Pioneering contributions to the sorting models of education include Arrow (1973) and Burdett (1978). Arrow also analyzes the matching role of education in his Sections 3 and 4.

process of communication with teachers and friends, trials and errors, people become more aware of the relationship between themselves and the world. Moreover, this paper's formulation has an advantage that the ability differences in the model can also be interpreted as the differences in job preferences.

MacDonald (1980) explicitly assumes that schooling reveals the worker's comparative advantage and analyzed the education demand. His spirit is the closest to ours. However, his analysis is limited for one-period matching.[3] The matching occurs only once in a lifetime, and the interaction between schooling-based information acquisition and on-the-job information acquisition is not considered. Moreover, many of his results rely on the structure of risk-aversion of workers. In contrast, we allow the interaction between schooling-based information acquisition and on-the-job information acquisition, and we assume risk neutrality of workers. Recently, economists are aware that a matching- (or search-) theoretic environment gives a risk-loving bias to people's preferences,[4] by providing a hedging device for the downside risk. Our model captures this feature. In this sense, people's attitude toward risk is given by the economic environment. This type of endogenous risk-attitude is absent in MacDonald's model.

Utilizing the following model, we show that mobility in the labour market has an impact on schooling demand. In Section 2.1, we set up the general structure of the model. In Section 2.2, we consider a simple one period model and illustrate some direct consequences of our assumptions. In Section 2.3, a model with job mobility is analyzed. We construct a simpler version of Jovanovic (1979) matching model, and the relationship between job mobility and education is examined. We show that the demand for education rises with mobility when education level is low and falls when education level is high. Then in Section 2.4, we endogenize the economy's job mobility by introducing a job switching cost. We derive the condition under which education demand rises with the worker's job switching cost. Section 3 concludes.

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[3] In fact, it is possible to reproduce some of his results utilizing our one-period model in the next section.

[4] For example, Gomes, Greenwood and Rebelo (2001) obtains the result that in a search-environment, people may prefer to have business cycles.

2 Model

2.1 Basic Setup

The economy consists of workers and firms. There are a continuum of industries[5], indexed between $[0, 1]$. Each industry has measure zero. Workers are endowed with one unit of labour for each period. Production requires only labour, and the output is perfectly observable for workers and firms. We assume that each industry is perfectly competitive. The wage contract is formed in the beginning of each period, so that the wage is based on the expected output conditional on the information available at the beginning of the period. Information is assumed to be symmetric between workers and firms. Hence, perfect competition ensures that the income of a worker i in an industry j at time t, $y_{ij,t}$, equals the expected output conditional on the information available at the beginning of the period t, $E[Y_{ij,t}|\Omega_t]$.

Assumption 1 *Workers are risk neutral and the credit market is perfect, so that they maximize the discounted sum of their income,* $E[\sum_{t=1}^{n} \beta^{t-1} y_{ij,t}]$.

Workers differ in their ability and the ability is unknown to both workers and firms.

Assumption 2 *For each industry* $j \in [0, 1]$*, a worker* i *is in either of two matches,* $\mu_{ij} \in \{fit = F, unfit = U\}$*. Match is industry-specific, and the realization of the match is unknown to both workers and firms.*

Assumption 3 *The output is uncertain. There are two states,* $s \in \{good = G, bad = B\}$*. The probability of occurrence of each state is conditional on the match. The probability is independent across industries. The probability of each state conditional on the match is summarized as follows.*

$$P[G|F] = \pi_{G|F} \qquad P[B|F] = \pi_{B|F}$$

$$P[G|U] = \pi_{G|U} \qquad P[B|U] = \pi_{B|U},$$

where $\pi_{i|j} \in [0, 1]$ *for* $i \in \{G, B\}, j \in \{F, U\}$*. Note that* $\pi_{G|F} + \pi_{B|F} = 1$ *and* $\pi_{G|U} + \pi_{B|U} = 1$*. We assume that* $\pi_{G|F} > \pi_{G|U}$*, which means that it is likely to obtain a good result if the worker is fitted to the industry. We assume that all* π*'s are common across the industries.*

[5] The model's implication is unchanged by using "job" or "occupation" as a unit, instead of "industry".

The output of worker i in industry j, Y_{ij}, is conditional on the state,

$$Y_{ij} = \begin{cases} g & \text{if } s = G \\ b & \text{if } s = B, \end{cases}$$

where $g > b$.

Schooling increases the likelihood of a good match.

Assumption 4 *The unconditional probability of the fit in a randomly chosen industry, p, depends on the amount of the worker's education level, e. The dependence is summarized by a function $p = p(e)$, $\Re_+ \to [0,1]$. We assume $p'(e) > 0$ and $p''(e) < 0$. $p(0) = \underline{p}$, which means that when workers obtain no education and choose a job randomly, $\underline{p}$ is the probability for them to choose a right job. The cost of education is summarized by a function $c = c(e)$, $\Re_+ \to \Re_+$. We assume $c'(e) > 0$ and $c''(e) > 0$. Further, we impose the Inada-conditions to both functions to ensure an interior solution.*

$p(\cdot)$ function is increasing in e since schooling helps students to find their ability and therefore narrow down the industries they fit. The more they invest in education, the more information they can get about themselves and the more likely they can choose the right job.

From the next section, we will examine a worker's decision. Since all the workers and industries are symmetric, we will denote $Y_t \equiv Y_{ij,t}$ and $y_t \equiv y_{ij,t}$.

2.2 One-Period Model

First, we construct a model with one-period lived workers. We assume that workers decide how much to be educated before working, and they work for one period. There are no decision-making for the choice of the job, since the expected income is the same in all industries.

The optimization problem for a worker is

$$\max_e \quad W(e) = y(e) - c(e),$$

where $y(e)$ is the income of the worker, where the dependence on e is made explicit. $W(e)$ can be rewritten as

$$\begin{aligned} y(e) - c(e) &= E[Y(e)] - c(e) \\ &= P[G]g + P[B]b - c(e) \\ &= (P[G,F] + P[G,U])g + (P[B,F] + P[B,U])b - c(e) \\ &= \{\pi_{G|F}p(e) + \pi_{G|U}(1 - p(e))\}g + \{\pi_{B|F}p(e) + \pi_{B|U}(1 - p(e))\}b - c(e). \end{aligned}$$

Then the first order condition reads

$$\{(\pi_{G|F} - \pi_{G|U})g + (\pi_{B|F} - \pi_{B|U})b\}p'(e) = c'(e). \tag{1}$$

Next comparative statics follow.

Proposition 1 *The optimal amount of education, e, rises as $(g - b)$ and $\Delta\pi \equiv \pi_{G|F} - \pi_{G|U}$ rise.*

Proof. Using $\pi_{G|F} + \pi_{B|F} = 1$ and $\pi_{G|U} + \pi_{B|U} = 1$, the first order condition (1) can be rewritten as

$$\Delta\pi \cdot (g - b) = c'(e)/p'(e).$$

(Note that $\Delta\pi$ is also equal to $\pi_{B|U} - \pi_{B|F}$.) From the assumption that $c''(e) > 0$ and $p''(e) < 0$, the right hand side is increasing in e. Then e is increasing in $\Delta\pi$ and $(g - b)$ to satisfy this condition. ∎

Intuitively, the rise of $\Delta\pi$ and $(g-b)$ increases the reward from fitting to a chosen job. $\Delta\pi$ denotes how much the relative probability of obtaining a good outcome rises when a worker can choose a right job instead of a wrong job. $(g - b)$ is the relative reward of the good outcome, which the worker can obtain with higher probability when he chooses a right job. Since education increases the probability of the worker's correct choice of a job, education becomes more beneficial to the worker when $\Delta\pi$ is large.

2.3 Two-Period Model

Although the one-period model can be used to analyze some interesting issues, extending the model to multi-periods is very important since the possibility of job (industry) switching is an essential feature of the matching model. In this section and the following section, we assume that the workers decide how much to be educated before working and then work for two periods. Between the first and the second period, they have an option of switching the job (industry).

The problem for a worker is

$$\max_{e} \quad W(e) = y_1(e) + \beta E[y_2(e)] - c(e),$$

where

$$y_2(e) = \max\{y_2^c(e), y_2^s(e)\}.$$

$y_t(e)$ is the income of t-th period, β is the discount factor, $y_2^c(e)$ is the earning of the second period if the worker changes the industry, and $y_2^s(e)$ is the earning of the second period if he stays in the same industry. The earnings of the second period are conditional on the worker's history. His *history*, $h \in \{G, B\}$, is defined as his outcome at the first period. The worker can control the amount of education, e, and also if he stays in the same industry or change the industry at the second period.

We can rewrite the first-period income as follows.

$$\begin{aligned} y_1(e) &= E[Y_1(e)] \\ &= P[G]g + P[B]b \\ &= (P[G,F] + P[G,U])g + (P[B,F] + P[B,U])b \\ &= \{\pi_{G|F} \cdot p(e) + \pi_{G|U} \cdot [1 - p(e)]\}g + \{\pi_{B|F} \cdot p(e) + \pi_{B|U} \cdot [1 - p(e)]\}b \\ &= (\pi_{G|F} \cdot g + \pi_{B|F} \cdot b)p(e) + (\pi_{G|U} \cdot g + \pi_{B|U} \cdot b)[1 - p(e)] \\ &= \bar{Y}_F \cdot p(e) + \bar{Y}_U \cdot [1 - p(e)]. \end{aligned} \tag{2}$$

where $\bar{Y}_F \equiv E[Y|F] = \pi_{G|F} \cdot g + \pi_{B|F} \cdot b$ and $\bar{Y}_U \equiv E[Y|U] = \pi_{G|U} \cdot g + \pi_{B|U} \cdot b$ are the expected output conditional on whether the worker is fit ($\bar{Y}_F$) or unfit ($\bar{Y}_U$).

Henceforth, we will apply the next normalization for simplicity.

Normalization 1 *We normalize $\bar{Y}_F = 1$ and $\bar{Y}_U = 0$.*

Then (2) can be rewritten as $y_1(e) = p(e)$. This normalization is innocuous unless we change the values of π's, g, or b. When we conduct comparative statics on these parameters (as in Proposition 4), we have to be aware that we implicitly change other parameters to satisfy this normalization. Particularly, $\Delta\pi(g - b) = 1$ is implied by this normalization. This fact implies that when we apply Normalization 1, the effects which were working in Proposition 1 will be completely neutralized. We maintain this normalization in the following analysis since the direct effects that appeared in Proposition 1 are rather obvious and the results would not be so interesting if they heavily depend upon these effects.

We will solve this problem working backwards. We begin by analyzing the period two decision.

The Second Period The worker will decide whether he changes the job (industry) or not based on the history. When he changes the industry, for his next industry he is indifferent among all the industries except the one he already had an experience. Since each industry has measure zero, the expected income when he changes the industry is the same amount as in the first period [i.e. $p(e)$]. Essentially, if a worker changes the industry, he is starting over again from the same situation as in the first period.

The decision rule of the worker is summarized in the following lemma.

Lemma 2 *The worker stays in the same industry if the outcome of the first period was good, $h = G$, and changes the industry if the outcome is bad, $h = B$.*

Proof. $h = G$: From Normalization 1, we obtain

$$y_2(e) = E[Y_2(e)|G] = \max\{p(e), P[F|G]\},$$

where the first argument is the expected income when the worker changes the industry, the second argument is the expected income when the worker remains in the same industry.

By Bayes' rule,

$$P[F|G] = \frac{P[F,G]}{P[G]} = \frac{\pi_{G|F} \cdot p(e)}{\pi_{G|F} \cdot p(e) + \pi_{G|U} \cdot (1 - p(e))}.$$

It is straightforward to see that $P[F|G] > p(e)$ (from the assumption $\pi_{G|F} > \pi_{G|U}$), and the worker always stays in the same industry.

$h = B$: In the similar manner,

$$y_2(e) = E[Y_2(e)|B] = \max\{p(e), P[F|B]\},$$

where

$$P[F|B] = \frac{\pi_{B|F} \cdot p(e)}{\pi_{B|F} \cdot p(e) + \pi_{B|U} \cdot (1 - p(e))}.$$

It is straightforward to see that $p(e) > P[F|B]$ (from the assumption $\pi_{B|F} < \pi_{B|U}$), and the worker always changes the industry. ■

From Lemma 1, we can calculate the (unconditional) expected income of the worker in the second period:

$$\begin{aligned} E[y_2(e)] &= E[E[y_2(e)|h]] \\ &= P[G] \cdot E[y_2(e)|G] + P[B] \cdot E[y_2(e)|B] \\ &= \{\pi_{G|F} \cdot p(e) + \pi_{G|U} \cdot [1 - p(e)]\} \cdot \frac{\pi_{G|F} \cdot p(e)}{\pi_{G|F} \cdot p(e) + \pi_{G|U} \cdot [1 - p(e)]} \\ &\quad + \{\pi_{B|F} \cdot p(e) + \pi_{B|U} \cdot [1 - p(e)]\} \cdot p(e) \\ &= p(e) + p(e) \cdot [1 - p(e)] \cdot \Delta\pi. \end{aligned} \tag{3}$$

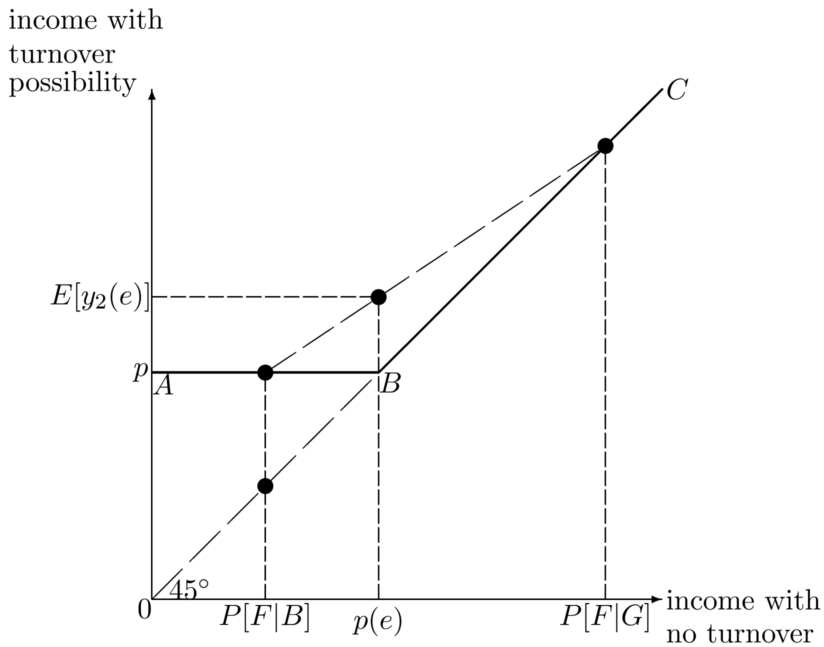

Figure 1:

Lemma 1 is used in the third equality. Note that if there is no turnover, $E[y_2(e)] = p(e)$. The second term in (3) exhibits the value of the option that he can change the job at the second period. Notice that $p(e) \cdot [1 - p(e)]$ is the unconditional variance of y. Variance enters into the expected income since the payoff structure is "convex" (in the sense that the downside risk is avoided) because of the existence of the job turnover possibility. In Figure 1, $E[y_2(e)]$ is larger than $p(e)$ since the payoff function ABC is convex. If the worker observes a bad outcome, he can change his job to earn $p(e)$ instead of $P[F|B]$.

The First Period The worker determines the optimal amount of education. The problem is

$$\max_e \quad W(e) = p(e) + \beta\{p(e) + p(e) \cdot [1 - p(e)] \cdot \Delta\pi\} - c(e).$$

However, this problem may not be concave programming, by the existence of the term $p(e) \cdot [1 - p(e)] \cdot \Delta\pi$. To ensure the concavity, we further assume:

Assumption 5 $V(e) \equiv p(e) \cdot [1 - p(e)]$ *is concave in* e.

Under this assumption, we can describe the optimal decision utilizing the first order condition.

The first order condition is

$$(1+\beta)p'(e) + \beta V'(e)\Delta\pi = c'(e). \tag{4}$$

The second order condition is satisfied since the sum of concave functions is also concave. From (4), we can easily see that a high β yields high education demand. This effect of the interest rate (in this model, $1/\beta - 1$) is familiar in human capital literature, e.g. Mincer (1974). This comes from the fact that education is an "investment" in this model. The effect of $\Delta\pi$ is ambiguous, depending on the sign of $V'(e)$.

First we show two results which are related to the existing literature.

Proposition 3 *Income rises with the industry tenure, and average income rises with experience.*

Proof. From the discussion in the proof of Lemma 1, it is straightforward to see that $E[y_1(e)] = E[y_2(e)|B] = p(e)$ (workers with tenure 1) and $E[y_2(e)|G] > p(e)$ (workers with tenure 2). Average income of the first-period workers is $E[y_1(e)]$, and the second period workers is $E[y_2(e)]$, which is larger than $E[y_1(e)]$ from (3). ■

Proposition 1 replicates the result of Jovanovic (1979). Income rises with tenure since only the people who are likely to fit their job remain in the same job. Average income rises with experience since with longer experience, a larger amount of people find the job which are likely to fit them.

Proposition 4 *Under Normalization 1 [holding $\Delta\pi \cdot (g-b)$ constant], if there are two kinds of jobs which have different $\Delta\pi$, workers prefer the ones which have larger $\Delta\pi$.*

Proof. $\partial W(e^*)/\partial\Delta\pi = \beta V(e^*) > 0$ [see equation (3)]. Note that the indirect effect from the change in e^* is second-order by the Envelope Theorem. ■

This result departs from Johnson's (1978) main result (his Proposition 1). Johnson assumes normality in the distribution of ability and outcome, and derives the result that workers prefer jobs which yield a large *variance* of outcome. Here, we can choose the level of $\pi_{G|F}$ and $\pi_{G|U}$ in each industry so that we obtain the same variance of outcome across industries even if they have different $\Delta\pi$. (When changing $\Delta\pi$, g and b should be changed so that Normalization 1 is met.) In our model, the difference of variance across industries does not matter for the worker's choice. Instead, $\Delta\pi$ plays an important role. When $\Delta\pi$ is large, the outcome of the first period gives more accurate information for his fitness. The important factor is not the variance of the outcome itself, but the information which is contained in the first-period outcome.

From the next proposition, we examine the implications about schooling in this model.

Proposition 5 *The income in each period is an increasing function of the amount of education.*

Proof. For the first period, $\partial E[y_1(e)]/\partial e = p'(e) > 0$. For the second period, from (3), $\partial E[y_2(e)]/\partial e = p'(e) + [p'(e) - 2p'(e) \cdot p(e)] \cdot \Delta\pi = (1 + \Delta\pi - 2p(e) \cdot \Delta\pi) \cdot p'(e) \geq 0$ since $p(e) \leq 1$ and $\Delta\pi \leq 1$. ■

In the first period, only the direct effect is working, that is, the probability of fit is raised by schooling and the probability of getting a good outcome is raised. In the second period, two effects are working. First, the probability of obtaining a good outcome in the first period rises ($P[G]$ rises), so that the worker remains in the same job and receives a better wage.[6] Second, the wage in each state (either change or not change the job) rises by the direct effect of schooling ($E[y_2(e)|G] = P[F|G]$ and $E[y_2(e)|B] = p(e)$ rise).

In the next proposition, we analyze the effect of labour market mobility on education demand.

Proposition 6 *The prohibition of job switching yields a higher education demand of the workers if and only if $p(e) > 1/2$ evaluated at the optimal e.*

Proof. Denote e^* as the education demand when job change is allowed, and e^{**} as the education demand when job change is prohibited. Then the first order conditions for two economies are

$$(1+\beta)p'(e^*) + \beta V'(e^*)\Delta\pi = c'(e^*), \tag{5}$$

$$(1+\beta)p'(e^{**}) = c'(e^{**}). \tag{6}$$

From Assumption 4, $e^{**} > e^*$ if and only if the second term in (5) is negative. This occurs if and only if $p(e^*) > 1/2$. ■

The existence of the critical value, 1/2, is due to the binomial structure of the model.[7] To interpret the result, consider four different economies. The first two, A and B, have lower marginal costs of

[6] Notice that the turnover rate is $(1 - P[G])$, which is decreasing in e.

[7] The critical value does not depend on the normalization. Without Normalization 1, the first order conditions become

$$\{(1+\beta)p'(e^*) + \beta V'(e^*)\Delta\pi\}(\bar{Y}_F - \bar{Y}_U) = c'(e^*)$$

and

$$(1+\beta)p'(e^{**})(\bar{Y}_F - \bar{Y}_U) = c'(e^{**}),$$

from which the critical value is also 1/2.

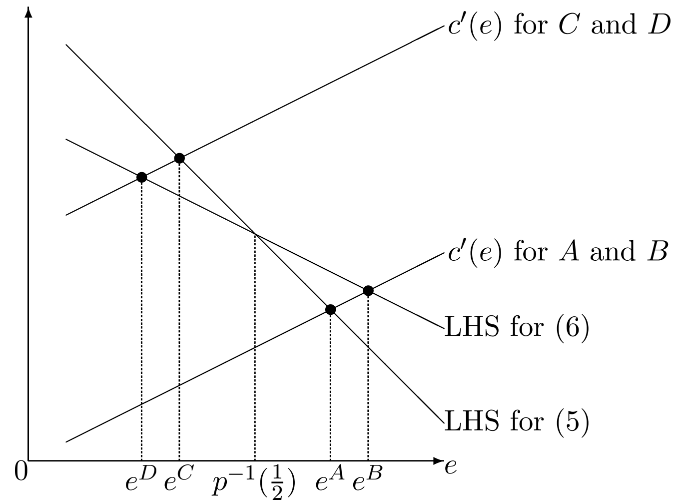

Figure 2:

education and therefore have high education levels. The others, C and D, have higher marginal cost and therefore low education levels. In the economies B and D, people cannot change jobs.

By the convex structure of the second-period payoff, the "sureness" about the fitness for the first job has less value (the preference has a risk-loving bias) in the economy where people can change their job easily (A and C). In these economies, instead of being very assured of their future before starting the first job, they can take advantage of having a second chance. In economy B, people demand more education compared to A, to become more confident for their future. In economy D, people demand less education compared to C, to be more "assured" of their future. When $p(e)$ is low, people almost always choose a wrong job for them. By having *lower* $p(e)$, the people in economy D become more "assured" that they will be in an unfitted job.

Alternatively, we can interpret this result as following. In economy D, workers have fewer chances than in C to make use of the advantage of having better $p(e)$, since there is no opportunity to change the job in the second period. This effect presents also in economy B, but this has a small influence in the worker's decision, because the probability that he needs to use the second chance, $[1 - p(e)]$, is small. Figure 2 exhibits the education demand in each economy, e^A, e^B, e^C, and e^D, utilizing (5) and (6).

The lower mobility yields lower education in low-educated economies and higher education in high-educated economies. This "magnification" of education demand caused by low mobility is larger if $\Delta\pi$ is larger. This proposition suggests that the high demand for high education in some highly-developed economies (notorious phenomena in Japan and Korea, for example) might be the consequence of low labour market mobility caused by the high job switching cost.

Proposition 6 has another empirical implication. Suppose there exists heterogeneity in the education cost among workers. Compare two economies with high and low job mobility, which have the same distribution for the heterogeneous education cost $c(e)$ within each economy. Then, it has to be the case that the variance of education demand is larger in the low-mobility economy. People's educational attainment is more homogeneous when the mobility is high.

2.4 Two-Period Model with Switching Cost

In this section, we extend the result of Proposition 6. Proposition 6 was the comparison between two extremes: perfect job mobility and perfect job immobility. The constraint was given exogenously. Here, a job-switching cost is incorporated, and the job-switching decision is endogenized. It will be shown that there exists a threshold value of switching cost, above which a worker never switches jobs. Moreover, the optimal amount of education depends on the switching cost. Our analysis shows that at the threshold, the education demand changes discontinuously.

Assumption 6 *A worker incurs the switching cost,* $0 \leq m \leq 1$*, if he decides to change his job.*

The range of m is restricted to be less than 1. This is without loss of generality since the benefit from the switch is always less than 1, so that workers with m larger than 1 will never switch jobs.

For simplicity, we impose one more assumption.

Assumption 7 $\pi_{G|U} = 0$.

This implies that if a worker is not fitted to the industry, he always obtains bad outcomes. Therefore, he can infer with certainty that he is in a right industry if he observes a good outcome. This assumption is innocuous, since we already know that a worker will remain in the same industry every time he observes the good signal.[8] Note that this assumption implies that $\Delta\pi = \pi_{G|F} = 1 - \pi_{B|F}$.

The new problem is

$$\max_{e} \quad W(e;m) = y_1(e) + \beta E[y_2(e)] - c(e),$$

[8]This follows from Lemma 1 and the fact that the introduction of m only discourages job switching.

where

$$y_2(e) = \max\{y_2^c(e) - m, y_2^s(e)\}.$$

In the same way as the previous section, we will solve the problem backwards.

The Second Period Given the value of $\{e, h\}$, a worker decides whether to switch jobs. This decision depends on the value of the parameter m. The problem is to choose $\max\{p(e) - m, P[F|h]\}$. From Lemma 1, we know that $P[F|G] > p(e)$ so that $\max\{p(e) - m, P[F|G]\} = P[F|G]$ and the worker always stays in the job when he obtains a good outcome. When he obtains a bad outcome, his decision is not always the same as in the previous section. If $P[F|B] > p(e) - m$, he stays in the same job, and otherwise, he changes his job. Note that if $m = 1$, he always stays in the same job regardless of the first period output, and if $m = 0$, he changes the job when he gets a bad outcome. Proposition 6 can be seen as the comparison of these two extremes.

The First Period Anticipating the next period's decision, workers choose the optimal amount of education at the beginning of the first period. After some calculation, the maximization problem can be rewritten as

$$\max_e \quad W(e; m) = (1 + \beta)p(e) - c(e) + \max\{V(e)\Delta\pi - (1 - p(e)\Delta\pi)m, 0\}.$$

The first argument in the second max corresponds to the case where the worker will change the job if he gets a bad signal, and the second argument corresponds to the case where the worker will never change his job. This problem can be rewritten as

$$\max_e \quad W(e; m) = \max\{(1 + \beta)p(e) - c(e) + V(e)\Delta\pi - (1 - p(e)\Delta\pi)m, (1 + \beta)p(e) - c(e)\}.$$

From Assumptions 4-6, both arguments are concave in e. Then we can rewrite the problem as

$$\max\{W^*(m), W^{**}\},$$

where $W^*(m) \equiv \max_e\{(1 + \beta)p(e) - c(e) + V(e)\Delta\pi - (1 - p(e)\Delta\pi)m\}$ and $W^{**} \equiv \max_e\{(1 + \beta)p(e) - c(e)\}$. First order conditions are

$$(1 + \beta)p'(e^*) + \{V'(e^*) + p'(e^*)m\}\Delta\pi = c'(e^*), \tag{7}$$

$$(1 + \beta)p'(e^{**}) = c'(e^{**}). \tag{8}$$

Equation (7) defines e^* as a function of m and (8) defines e^{**}.

Now we characterize education demand. First, some preliminary results are presented.

Lemma 7 *There exists a threshold value for m, $\bar{m}$, above which the worker never changes his job, and below which the worker changes the job if he obtains a bad outcome.*

Proof. First, note that $W^*(m)$ is monotonically decreasing in m, since $\partial W^*(m)/\partial m = -(1 - \Delta\pi p(e^*)) < 0$ from the Envelope Theorem. It is straightforward to see that $W^*(0) > W^{**}$ and $W^*(1) < W^{**}$. Since $W^*(m)$ is continuous in m (from the Maximum Theorem), there exists a unique $\bar{m}$ which satisfies $W^*(m) = W^{**}$ (from the Intermediate Value Theorem). Above $\bar{m}$, $W^*(m) > W^{**}$ and the worker never changes his job. Below $\bar{m}$, $W^*(m) < W^{**}$ and the worker changes the job if he obtains a bad outcome. ■

Corollary 8 *Above $\bar{m}$, workers demand the same education regardless of m. Below $\bar{m}$, the education demand depends on m. Generically, at $\bar{m}$, education demand changes discontinuously with respect to the change in m*[9] *.*

Corollary 9 *If all workers are the same except for their value of m, the distribution of net lifetime income W is truncated at point W^{**} from below.*

Now we proceed to analyze job mobility and education demand in the whole economy. We will assume, as in Corollary 9, that workers differ only in the value of m. m is distributed across the workers with cumulative distribution $\Phi(m)$ and density $\phi(m)$.

As defined in Lemma 2, the threshold value $\bar{m}$ characterizes the aggregate mobility of the economy. With higher $\bar{m}$, less people are trapped in unfit jobs. In fact, the fraction of workers who change jobs in period 2 is

$$f(\bar{m}) = \int_0^{\bar{m}} (1 - p(e^*(m))\Delta\pi)\phi(m)dm, \tag{9}$$

which is increasing in $\bar{m}$.

Given m, we can draw a figure analogous to Figure 2 for each worker. The left-hand-sides of (7) and (8) are depicted in Figure 3. (7) is drawn for different m's, m_1 and m_2. m_2 is assumed to be less than m_1. The figure shows that the marginal benefit of education [LHS for (7)] is larger when m is larger, since higher education saves the spending of the switching cost and this benefit is larger when m is large.

[9] This statement applies unless it happens to be $\bar{m} = p(e)$ at e which satisfies (7) and (8) at the same time.

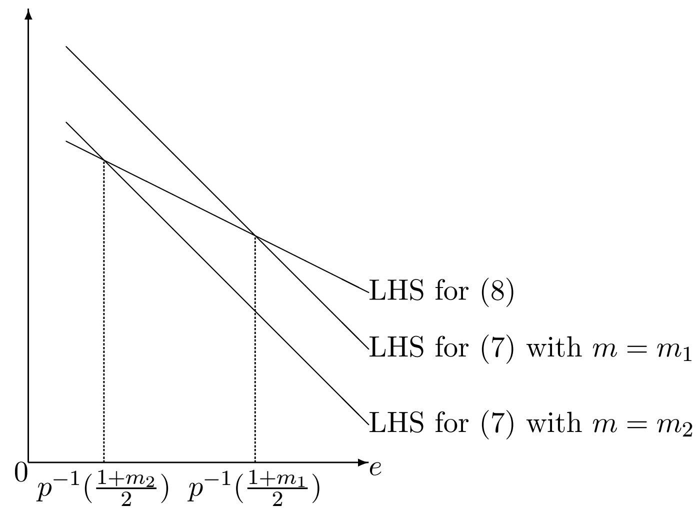

Figure 3:

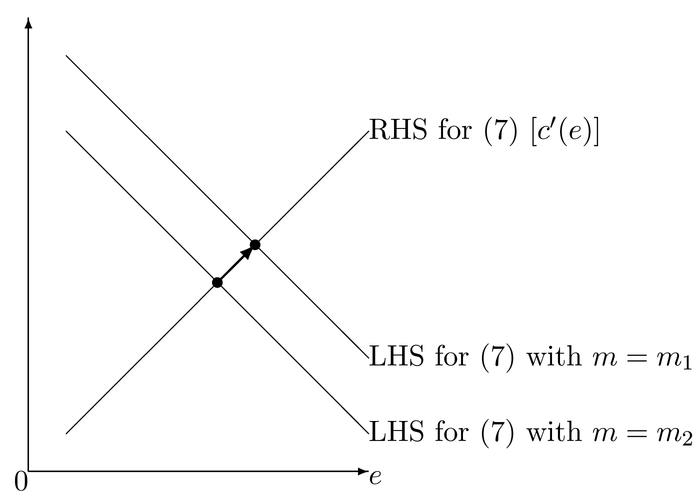

Figure 4: The Movement When m Is Small

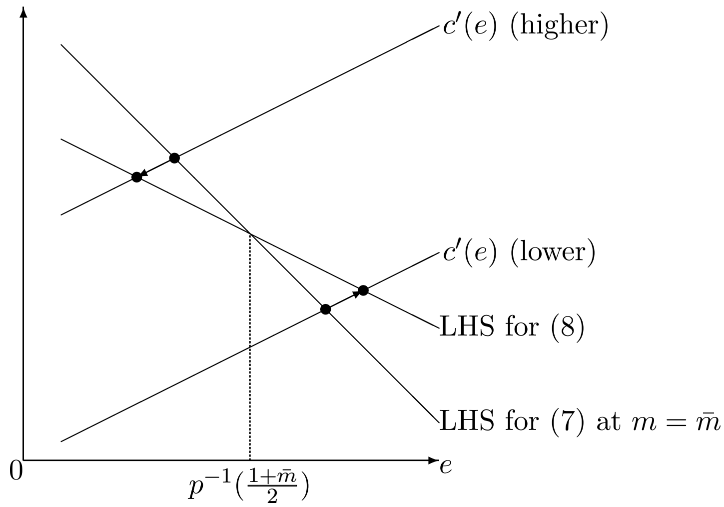

Figure 5: The Direction of Jump

Let us start the comparative statics from a small m. As m becomes larger, education demand rises monotonically [(7) applies]. This is because the larger the switching cost m becomes, the more the worker wants to avoid spending it. This movement is shown in Figure 4. When m becomes larger than the threshold value $\bar{m}$, the education demand changes discontinuously [(8) applies]. This part is shown in Figure 5. The direction of the jump depends on if it is more or less than $p^{-1}(\frac{1+\bar{m}}{2})$. This direction of jump is dictated by the marginal cost of education, in a similar manner as in Proposition 6.

In sum, the result is more complex than Proposition 6 in two directions. First, below m, the education demand changes monotonically. Second, the critical value is this time endogenous, $(1+\bar{m})/2$. This critical value is derived in the same way as in the previous section. That is, in (7), $V'(e) + p'(e)m = 0$ if and only if $p = (1+m)/2$.

In Figures 4 and 5, when the marginal cost of education is high, the direction of the monotonic movement and the jump are the opposite. Thus the increase in m may or may not increase the education demand. When the marginal cost of education is low, the demand moves monotonically as m changes.

Proposition 10 *If $p(e^{**}) > (1+\bar{m})/2$, education demand is increasing in m.*

Proof. When $m < \bar{m}$, e monotonically increases. At $m = \bar{m}$, education demand jumps, but always increases if e^{**} is larger than critical value $p^{-1}(\frac{1+\bar{m}}{2})$, by the same argument as Proposition 6. When $m > \bar{m}$, the education demand is e^{**} and does not depend on m. ■
Therefore, the implication of Proposition 6 is preserved in highly-educated economies.

3 Conclusion

In this paper, we constructed a job-matching model in which through schooling a worker can obtain information about the workers' ability. The education demand under several circumstances is analyzed.

When matching is only once in lifetime, the education demand depends on the direct reward from the education investment. It depends on the relative probability of the good outcome when the right job is chosen, and the amount of the difference between the good and bad outcome.

Next we examined the case when the job turnover is permitted in the middle of life. The experience in the first job reveals some information about the fitness of the worker to the current industry. This possibility provides a convex structure in the workers' payoff, as in the standard search-matching literature. It was shown that income in each period is an increasing function of schooling. Then we examined the effect of the prohibition of job switching to the education demand. When the equilibrium education demand is high, the prohibition of job switching raises education demand. When the equilibrium education demand is low, the prohibition lowers education demand.

We extended the model to endogenize both the job switching decision and the education demand. The reaction in the education demand when the switching cost is changed is examined. In general, the education demand reacts in a non-monotonic way to a change in the switching cost. When the equilibrium education demand is sufficiently high, the rise in the switching cost always increases the education demand.

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