Heterogeneous Jobs and the Aggregate Labor Market

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August 2018

Abstract

This paper analyzes a simple search-and-matching model with heterogeneous jobs. First, I derive an explicit formula that ensures the social efficiency of the equilibrium outcome. This formula generalizes the well-known Hosios condition and clarifies the role of externalities across labor markets for different types of jobs. Second, business cycle fluctuations with heterogeneous jobs are analyzed. Heterogeneity in productivity and job stability plays an important role in generating strong labor-market responses to the aggregate labor market to productivity shocks.

Keywords: Search and matching; Unemployment; Heterogeneous jobs; Efficiency; Business cycles
JEL Classifications: D61, E24, E32, J63, J64

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1 Introduction

The most distinctive property of the labor market, compared to other markets of goods and services, is that the objects traded in the market are highly heterogeneous. The nature of labor services can be markedly different depending on worker, job, and match characteristics. A large literature in modern macroeconomics treats unemployment as an outcome of market frictions stemming from this heterogeneity—it is difficult to find a right person for a particular job. A popular search-and-matching model of unemployment, often called the Diamond-Mortensen-Pissarides (DMP) model, typically treats this heterogeneity in a reduced-form manner, assuming that these frictions can be represented by a black box referred to as the aggregate matching function. In the commonly-used version of the model, such as Pissarides (1985) and Shimer (2005), jobs and workers are homogeneous outside the matching function.

Extensions of the standard DMP model, such as the models that analyze endogenous job destruction and the models that analyze job-to-job transitions, often explicitly incorporate heterogeneity. However, they mostly consider only *ex post* heterogeneity; that is, they typically assume that a random match-specific productivity is realized *after* the match is formed.[1] In this paper, I instead focus on *ex ante* heterogeneity. In particular, I analyze a setting where jobs are heterogeneous, while abstracting from heterogeneity on the worker side. Jobs have different characteristics at the time a vacancy is created, that is, *before* the match is formed. I assume that the influences across different types of vacancies may be limited. Thus, different vacancies can operate in different labor markets, although each vacancy can potentially match with any of the unemployed workers.

In the model, jobs can be different across many dimensions: productivity, recruiting costs, worker's bargaining power, and job stability. The main goal of this paper is to develop theoretical intuitions; therefore, the model is fairly stylized. The different 'types' of jobs in the model are open to many possible interpretations, and thus the model analysis can be applied to many different contexts. For example, these different job types can represent permanent versus temporary contracts, full-time versus part-time jobs, jobs in different sectors, different occupations, or jobs in large firms versus small firms. What is important here is that similar workers can work in different types of jobs; thus the characteristics of *all* jobs that can potentially match with a worker affect her outside options.

The present paper asks two questions. First, how does the existence of heterogeneous jobs affect the efficiency property of the market equilibrium? Second, how does the economy respond to aggregate and type-specific productivity shocks?

For the first question, I provide a generalized version of the Hosios (1990)-type condition that guarantees the social efficiency of the equilibrium outcome. It calls for the bargaining power of the workers for different types of jobs to be related to various factors. One important component in the condition is how each type of vacancy imposes externalities to other (and its own) types of vacancies in the matching process.

For the second question, focusing on the situation where the markets are perfectly seg-

[1] See Pissarides (2000) for a textbook treatment.

mented, I show that positive productivity shocks to one type of jobs have negative effects on the openings of all other types of jobs. The intuition is simple: a positive productivity shock to one type of jobs increases the outside options of all workers and thus pushes up wages. Higher wages reduce the firms' incentive to create new jobs. The quantitative exercise suggests that this effect can be sizable.

This paper is related to several strands of literature. The most closely related is the literature that analyzes the efficiency of the equilibrium in a DMP-style model with heterogeneous jobs. The existing work includes Bertola and Caballero (1994), Acemoglu (2001), Davis (2001), and Ljungqvist and Sargent (2012, Section 28.4). Compared to this literature, the result in Section 2 (Proposition 2) is more general than any existing results that I am aware of. This strand of work does not consider the propagation of shocks.

Another related literature includes the recent papers on heterogeneous firms with matching frictions. Elsby and Michaels (2013) build a model of firm dynamics with DMP-style labor market frictions. They also analyze business cycles. Compared to their model, the model of this paper is closer to the original DMP model and is substantially more tractable; thus, this paper has an advantage that the mechanism is more transparent. They also emphasize the volatility of labor market reacting to aggregate shocks, and the mechanism that I highlight is likely to be at work in their model as well. Kaas and Kircher (2015) and Lise and Robin (2017) also analyze frictional labor market models with heterogeneous firms, while the settings of these papers are substantially different from the setting of this paper; the former looks at a model of directed search and the latter features a model of sorting where workers have no bargaining power when they move from unemployment.

The third strand of related literature is the papers that emphasize heterogeneous workers. Recent examples are Bils et al. (2012) and Mueller (2017). The current paper complements this literature by focusing on the other side of the labor market. For this paper's model, it is important that the same worker can potentially be matched to different types of jobs. Most significantly, it means that different jobs can affect the workers' outside options and consequently what happens to other jobs can affect wages even when the labor market is segmented on the vacancy side.

This paper is organized as follows. Section 2 sets up the continuous-time version of the model and characterizes the steady-state equilibrium analytically. Two results are established: comparative steady state and efficiency result. Section 3 builds the discrete-time version of the model and solves it quantitatively. Here, the focus is on the response of the labor market to productivity shocks. Section 4 concludes.

2 The continuous-time model

I consider a simple extension of the basic continuous-time DMP framework. Each consumer is infinitely-lived and supplies one unit of labor inelastically. A consumer is employed at a firm or unemployed. I normalize the total population of the consumers to one. A consumer maximizes the utility

$$\boldsymbol{U} = E_0 \left[\int_0^\infty e^{-rt} c(t) dt \right],$$

where $E_0[\cdot]$ denotes the expectations taken at time 0, $r > 0$ is the discount rate, and $c(t)$ is the consumption at time t. A consumer is either employed or unemployed at each point in time. Given that the consumers are indifferent about the timing of consumption when the interest rate is also r (which is the case in equilibrium), I assume that $c(t)$ is equal to the wage income when the consumer works and it is equal to the home production value when the consumer is unemployed.[2] Denote the flow value of the home production by h.

There are N different types of jobs. Types are indexed by i. The number of job vacancies for type i jobs is denoted by $v^i(t)$. Let $u(t)$ be the mass of unemployed workers (which is the same as the unemployment rate). Denote the mass of employment in type-i jobs as $n^i(t)$. Then

$$\sum_i n^i(t) + u(t) = 1 \tag{1}$$

always holds. The labor market tightness for type-i jobs is denoted as $\theta^i(t) \equiv v^i(t)/u(t)$. Note that the denominator is $u(t)$: this expression reflects an underlying assumption that a worker can apply for all types of jobs simultaneously. The vector of all $\theta^i(t)$ is represented by $\boldsymbol{\theta}(t)$. I assume that the Poisson rate that a type-i vacancy finds a worker is a function of $\boldsymbol{\theta}(t)$, and express the dependence by the function $q^i(\boldsymbol{\theta}(t))$.

The total number of type-i matches is $v^i(t)q^i(\boldsymbol{\theta}(t))$, and the economy-wide total number of matches is $\sum_i v^i(t)q^i(\boldsymbol{\theta}(t))$. The matching process is entirely random. Thus, the Poisson rate with which a worker finding a type-i job is $\theta^i(t)q^i(\boldsymbol{\theta}(t))$. I assume that a match is separated at a Poisson rate σ^i.

This formulation nests several important special cases. As an example, suppose that all jobs are posted in different markets, while the workers can simultaneously visit any market. In particular, let $M^i(u, v^i)$ be the matching function in the market for type i and the matching function exhibits constant-returns to scale. Then it is straightforward to show that

$$q^i(\boldsymbol{\theta}) = M^i\left(\frac{1}{\theta^i}, 1\right) \tag{2}$$

and

$$\theta^i q^i(\boldsymbol{\theta}) = \theta^i M^i\left(\frac{1}{\theta^i}, 1\right).$$

Below, I call this case *perfect segmentation* case. Here, posting a type-i vacancy imposes no

[2] Since the firms are owned by the consumers and the firms generate profit (there is a positive aggregate profit in the steady state), $c(t)$ should in principle include asset income, even when there are no productive capital stocks. I abstract from asset income here, as in the standard textbook treatment, because (as will be clear later on) it does not affect the positive analysis of the equilibrium outcome in the current context. This is because what matters for the equilibrium is the *difference* of values between being employed and being unemployed. This will no longer be the case if the consumer's utility is not linear and the asset market is not complete. See Krusell et al. (2010) for such an analysis.

direct externalities on the markets of other types of jobs. For example, an increase in the number of vacancies by a construction firm may reduce the probability of another construction firm finding a worker, but it would not affect the worker-finding probability of a retail firm (although a worker can be matched with a construction firm or a retail firm with a random probability).

For another example, suppose that there is only one labor market in the economy with a matching function: $M\left(u, \sum_k v^k\right)$. In this case, all vacancies are pooled in one market and interact with each other. In this case,

$$q^i(\boldsymbol{\theta}) = M\left(\frac{1}{\sum_k \theta^k}, 1\right) \tag{3}$$

and

$$\theta^i q^i(\boldsymbol{\theta}) = \theta^i M\left(\frac{1}{\sum_k \theta^k}, 1\right).$$

With this formulation, $\partial q^i(\boldsymbol{\theta})/\partial v^j < 0$ for $j \neq i$ and therefore a type j vacancy imposes a negative externality on the matching probability of the firms with different types of jobs. I call this case *perfect pooling*. The above two extreme examples correspond to formulations that have been considered in the existing literature.[3]

2.1 Worker flows and the steady-state stocks

Under the above assumptions, $n^i(t)$ follows the differential equation

$$\dot{n}^i(t) = \theta^i(t) q^i(\boldsymbol{\theta}(t)) u(t) - \sigma^i n^i(t), \tag{4}$$

for $i = 1, .., N$ (where $\dot{n}^i(t)$ denotes $dn^i(t)/dt$) and $u(t)$ follows

$$\dot{u}(t) = \sum_i \sigma^i n^i(t) - \left(\sum_i \theta^i(t) q^i(\boldsymbol{\theta}(t))\right) u(t).$$

In the steady state (where I omit the time notation), $\dot{n}^i(t) = \dot{u}(t) = 0$ for all i. Thus, from (4), $n^i = \theta^i q^i(\boldsymbol{\theta}) u/\sigma^i$. From this and (1), the steady-state unemployment rate is

$$u = \frac{1}{1 + \sum_i (\theta^i q^i(\boldsymbol{\theta})/\sigma^i)}. \tag{5}$$

2.2 The steady-state equilibrium

This section focuses on the steady-state equilibrium. A type-i match (that is, a match of a consumer and a type-i job) produces a flow value of p^i. I assume that $p^i > h$, and p^i is

[3]See Bertola and Caballero (1994), Acemoglu (2001), Davis (2001), and Ljungqvist and Sargent (2012, Section 28.4).

sufficiently large for all i so that no match has an incentive to voluntarily separate.[4]

Denoting the value of a type-i job by J^i, the Hamilton-Jacobi-Bellman (HJB) equation for a type-i job that is matched with a worker is

$$rJ^i = p^i - w^i - \sigma^i(J^i - V^i),$$

where w^i is the wage of type-i job and V^i is the value of a type-i vacancy. A type-i vacancy's value satisfies

$$rV^i = -\kappa^i + q^i(\boldsymbol{\theta})(J^i - V^i).$$

where κ^i is the cost of posting a type-i vacancy. On the consumer side, the value of a consumer working at a type-i job, W^i, satisfies

$$rW^i = w^i - \sigma^i(W^i - U),$$

where U is the value of unemployment. An unemployed consumer's HJB equation is

$$rU = h + \sum_i \theta^i q^i(\boldsymbol{\theta})(W^i - U).$$

I assume free entry into vacancy posting. That is, firms post vacancies until the present value of a vacancy is driven down to zero (here, I only consider the situation where $v^i > 0$ for all i in equilibrium):

$$V^i = 0. \tag{6}$$

Wages are determined by the generalized Nash bargaining solution, with the bargaining power to the worker denoted by $\gamma^i \in (0, 1)$. As in the standard textbook model,

$$\gamma^i(J^i - V^i) = (1 - \gamma^i)(W^i - U)$$

holds as a result. Let the total surplus of a matched type-i job be

$$S^i \equiv J^i - V^i + W^i - U = \frac{J^i - V^i}{1 - \gamma^i} = \frac{W^i - U}{\gamma^i}.$$

The equilibrium can be summarized as the job-creation (JC) condition:

$$(r + \sigma^i)S^i = p^i - h - \sum_j \theta^j q^j(\boldsymbol{\theta})\gamma^j S^j. \tag{7}$$

The interpretation is simple. The left-hand side is the flow return from a matched type-i job. The right-hand side shows that the return has two components: the first component is

[4] When there is only one type of job, $p^i > h$ is sufficient to ensure that there is no endogenous separation. It is no longer the case when jobs are heterogeneous. Because the value of unemployment includes the option value of finding a good job, the value of a type-i match can become lower than the value of unemployment (and vacancy) even when $p^i > h$.

the flow (net) benefit from the match in terms of production. The second component is the opportunity cost of moving a worker from unemployment to employment; by being matched, the workers loses an ability to search for a new job. From the free-entry condition (6), S^i satisfies

$$S^i = \frac{\kappa^i}{(1-\gamma^i)q^i(\boldsymbol{\theta})}. \tag{8}$$

Putting (7) and (8) together,

$$p^i - h - \frac{(r+\sigma^i)\kappa^i}{(1-\gamma^i)q^i(\boldsymbol{\theta})} - \sum_j \frac{\theta^j\gamma^j\kappa^j}{1-\gamma^j} = 0 \tag{9}$$

has to hold for all i. Thus, (9) defines a system of N equations with N unknowns that pins down the equilibrium values of θ^i for all i.

In the case of perfect segmentation (that is, no externalities across markets), it is fairly straightforward to show the following comparative static result.

Proposition 1 *Suppose that there are no externalities across the markets, that is, $q^i(\boldsymbol{\theta})$ depends only on θ^i. Let $\hat{p}^i$ and $\hat{\theta}^i$ be the log deviation of the variables p^i and θ^i. Suppose that $\hat{p}^i > 0$ and $\hat{p}^j = 0$ for all $j \neq i$. Then $\hat{\theta}^i > 0$ and $\hat{\theta}^j < 0$ for a small value of $\hat{p}^i$.*

Proof. See Appendix A. ■

2.3 Efficiency

In this section, I compare the equilibrium outcome to the socially efficient outcome. Here, the concept of social efficiency is a "constrained efficiency"; the social planner must face the same frictions as the private sector does. In this paper's context, I consider a social planner who faces the same labor-market frictions as the private sector. The social planner can specify the number of vacancies that the firms post (and therefore indirectly specify the employment of each type). Given the linear utility of consumers, the objective of a benevolent social planner is to maximize the discounted sum of the total value added. The maximization problem of the social planner is

$$\max_{\{n^i(t),\theta^i(t)\}} \int_0^\infty e^{-rt}\left(\sum_i p^i n^i(t) + h\left(1-\sum_i n^i(t)\right) - \sum_i \kappa^i\theta^i(t)\left(1-\sum_j n^j(t)\right)\right)$$

subject to

$$\dot{n}^i(t) = \theta^i(t)q^i(\boldsymbol{\theta}(t))\left(1-\sum_j n^j(t)\right) - \sigma^i n^i(t). \tag{10}$$

Let the costate variable that is associated with the constraint (10) be $\lambda^i(t)$. The present-value Hamiltonian for this optimization problem is then

$$H(t) = e^{-rt}\left(\sum_i p^i n^i(t) + h\left(1 - \sum_i n^i(t)\right) - \sum_i \kappa^i \theta^i(t)\left(1 - \sum_j n^j(t)\right)\right)$$
$$+ \sum_i \lambda^i(t)\left(\theta^i(t) q^i(\boldsymbol{\theta}(t))\left(1 - \sum_j n^j(t)\right) - \sigma^i n^i(t)\right).$$

The first-order condition for $\theta^i(t)$ is (after canceling out the common terms)

$$-e^{-rt}\kappa^i + \lambda^i(t)\left[1 - \eta^i(\boldsymbol{\theta}(t))\right] q^i(\boldsymbol{\theta}(t)) + \sum_{j \neq i} \lambda^j(t)\theta^j(t)\frac{\partial q^j(\boldsymbol{\theta}(t))}{\partial \theta^i(t)} = 0,$$

where $\eta^i(\boldsymbol{\theta}) \equiv -(\partial q^i(\boldsymbol{\theta})/\partial\theta^i)\theta^i/q^i(\boldsymbol{\theta}) > 0$ is the elasticity of $q^i(\boldsymbol{\theta})$ with respect to θ^i. Defining $\mu^i(t) \equiv e^{rt}\lambda(t)$ as the current value of the costate variable ($\mu^i(t)$ is constant in a steady state), this can be rewritten as

$$-\kappa^i + \mu^i(t)\left[1 - \eta^i(\boldsymbol{\theta}(t))\right] q^i(\boldsymbol{\theta}(t)) + \sum_{j \neq i} \mu^j(t)\theta^j(t)\frac{\partial q^j(\boldsymbol{\theta}(t))}{\partial \theta^i(t)} = 0. \tag{11}$$

The first-order condition for $n^i(t)$ is

$$e^{-rt}\left(p^i - h + \sum_j \kappa^j \theta^j(t)\right) - \sum_j \lambda^j(t)\theta^j(t) q^j(\boldsymbol{\theta}(t)) - \lambda^i(t)\sigma^i + \dot{\lambda}^i(t) = 0,$$

which can be rewritten as

$$p^i - h + \sum_j \kappa^j \theta^j(t) - \sum_j \mu^j(t)\theta^j(t) q^j(\boldsymbol{\theta}(t)) - \mu^i(t)(r + \sigma^i) + \dot{\mu}^i(t) = 0. \tag{12}$$

Comparing (7) with the steady-state version of (12),

$$(r + \sigma^i)\mu^i = p^i - h - \sum_j \theta^j(q^j(\boldsymbol{\theta})\mu^j - \kappa^j), \tag{13}$$

a social efficiency can be achieved when two conditions are met:

$$\mu^i = S^i \tag{14}$$

and

$$\sum_j \theta^j(q^j(\boldsymbol{\theta})\mu^j - \kappa^j) = \sum_j \theta^j q^j(\boldsymbol{\theta})\gamma^j S^j. \tag{15}$$

The first condition is straightforward to interpret. The multiplier μ^i in (13) is the social value of a match for the social planner, and S^j is the present value of surplus from a match in equilibrium. Thus, the first condition can be interpreted as the social return being equal to the private return for each match. The second condition is the social value versus the private value of having an unemployed worker in the economy. Because an unemployed worker can be transformed into an employed worker (who has the social value of μ^j) with probability $q^j(\boldsymbol{\theta})$ with cost κ^j, the left-hand side can be viewed as a social value of an unemployed worker for a social planner. The right-hand side is the opportunity cost of moving an unemployed worker to employment in equilibrium, that is, the private value of unemployment. In the left-hand side, the net value $(q^j(\boldsymbol{\theta})\mu^j - \kappa^j)$ is multiplied by θ^j and summed across all types of jobs. This is because the social value $(q^j(\boldsymbol{\theta})\mu^j - \kappa^j)$ is measured in terms of vacancies, and thus here it must be transformed to the value per each unemployed worker. Note that (15) is always satisfied when (14) holds, because of the expression of S^i in (8). Thus, in terms of efficiency, all I need to check is the condition (14). Using (8) and (11), the equation (14) can easily be evaluated.

The equation (11) shows that the usual Hosios (1990) condition does not guarantee the efficiency when there are externalities across different types of vacancies in the matching process. In fact, the following condition can be derived:

Proposition 2 *In the steady state, the market equilibrium is socially efficient if*

$$\gamma^i = \sum_j \varepsilon_i^j \frac{\kappa^j/(1-\gamma^j)}{\kappa^i/(1-\gamma^i)} \tag{16}$$

for all i, where

$$\varepsilon_i^j \equiv -\frac{\theta^j(\partial q^j(\boldsymbol{\theta})/\partial\theta^i)}{q^j(\boldsymbol{\theta})}.$$

Proof. See Appendix A. ■

This is a generalized version of Hosios (1990) condition. Here, ε_i^j indicates the magnitude of the externality that a type-i vacancy imposes on the type-j firms. When this is negative, the creation of a type-i vacancy should be discouraged by increasing γ^i. Note that (16) will be reduced to $\gamma^i = \varepsilon_i^i$ when there is only one market i.

For a special case, which corresponds to the perfect segmentation case (2), it is straightforward to check that the following holds.

Corollary 1 *Suppose that there are no externalities across types, that is, $\partial q^j(\boldsymbol{\theta})/\partial\theta^i = 0$ for $i \neq j$. Then, in the steady state, the market equilibrium is socially efficient if $\gamma^i = \eta^i(\boldsymbol{\theta})$ for all i.*

This special case is also shown in Ljungqvist and Sargent's (2012, Section 28.4) textbook.[5]

Another notable special case of Proposition 2 is when all types are pooled in the single market, as in above Equation (3).

[5] Also see Davis (2001) for an earlier result in a static framework.

Corollary 2 *Suppose that all types are pooled in a single market, so that $q^i(\boldsymbol{\theta})$ can be expressed as, using a function $q(\cdot)$,*

$$q^i(\boldsymbol{\theta}) = q\left(\sum_k \theta^k\right).$$

In the steady state, the market equilibrium is socially efficient if

$$\gamma^i = \eta(\boldsymbol{\theta}) \frac{\sum_j [\kappa^j/(1-\gamma^j)][\theta^j/(\sum_k \theta^k)]}{\kappa^i/(1-\gamma^i)} \tag{17}$$

for all i, where

$$\eta(\boldsymbol{\theta}) \equiv -\frac{(\sum_k \theta^k)(\partial q\left(\sum_k \theta^k\right)/\partial(\sum_k \theta^k))}{q\left(\sum_k \theta^k\right)}$$

is the elasticity of $q\left(\sum_k \theta^k\right)$ with respect to $\sum_k \theta^k$.

This directly follows from Proposition 2. The condition (17) modifies the single-type Hosios condition ($\gamma^i = \eta(\boldsymbol{\theta})$) with a term that represents relative value of $\kappa^i/(1-\gamma^i)$ compared to its weighted average across i. If $\kappa^i/(1-\gamma^i)$ is larger than its average, γ^j must be below $\eta(\boldsymbol{\theta})$. In other words, a job type with a large $\kappa^i/(1-\gamma^i)$ is under-produced when the single-type condition $\gamma^i = \eta(\boldsymbol{\theta})$ is satisfied.

A further implication of Corollary 2 is that the social efficiency cannot be achieved with a common value of γ^i across types when κ^i is heterogeneous. It is straightforward to see this fact; here, the condition (17) becomes

$$\gamma = \eta(\boldsymbol{\theta}) \frac{\sum_j \kappa^j [\theta^j/(\sum_k \theta^k)]}{\kappa^i}$$

and this equality cannot be satisfied for multiple values of κ^i (because everything else is common across i). A job with a large κ^i is relatively under-produced compared to a job with a small κ^j. Closely related results are shown by Acemoglu (2001), Davis (2001), and Ljungqvist and Sargent (2012, Section 28.4) for this special case.[6]

3 The discrete-time model and the business cycle

Now I reformulate the same problem with discrete time and with productivity shocks. I consider business cycles driven by the shocks to productivities of different types of jobs. In this section, I will use a subscript to indicate a time period.

[6] Extending Davis (2001), Ljungqvist and Sargent (2012, Section 28.4) show that the *total* number of jobs in the perfect pooling case is efficient if $\gamma^i = \eta(\boldsymbol{\theta})$ holds for all i.

3.1 Model equations

The discrete-time version of (4) is

$$n_{t+1}^i = \theta_t^i q(\boldsymbol{\theta}_t) u_t + (1-\sigma^i) n_t^i. \tag{18}$$

For unemployment,

$$u_{t+1} = \sum_i \sigma^i n_t^i + \left(1 - \sum_i \theta_t^i q^i(\boldsymbol{\theta}_t)\right) u_t \tag{19}$$

governs the dynamics. The expression for the steady-state unemployment rate is the same as the continuous-time case shown in (5).

Denoting the state of the economy at time t as X_t, the Bellman equations are

$$J^i(X_t) = p_t^i - w(X_t) + \beta E[(1-\sigma^i) J^i(X_{t+1}) + \sigma^i V^i(X_{t+1})]$$

for a filled job of type i,

$$V^i(X_t) = -\kappa^i + \beta E[q^i(\boldsymbol{\theta}_t) J^i(X_{t+1}) + (1 - q^i(\boldsymbol{\theta}_t)) V^i(X_{t+1})]$$

for a vacant job of type i,

$$W^i(X_t) = w^i(X_t) + \beta E[(1-\sigma^i) W^i(X_{t+1}) + \sigma^i U(X_{t+1})]$$

for a worker who is employed in a type-i job, and

$$U(X_t) = h + \beta E\left[\sum_i \theta_t^i q^i(\boldsymbol{\theta}_t) W^i(X_{t+1}) + \left(1 - \sum_i \theta_t^i q^i(\boldsymbol{\theta}_t)\right) U(X_{t+1})\right]$$

for an unemployed worker. Most of the notations are analogous to the earlier continuous-time model. The values of the jobs and workers are similarly denoted—$J^i(X_t)$ is the value of a type-i job; $V^i(X_t)$ is the value of a type-i vacancy; $W^i(X_t)$ is a value of a worker with type-i job; and $U(X_t)$ is the value of an unemployed worker. The discount factor is represented by $\beta \in (0,1)$.

Once again, I assume free entry to vacancy posting

$$V^i(X_t) = 0$$

and the generalized Nash bargaining for wages, which results in

$$\gamma^i (J^i(X_t) - V^i(X_t)) = (1-\gamma^i)(W^i(X_t) - U(X_t)).$$

After rearranging, the above equations can be summarized by

$$\frac{\kappa^i}{1-\gamma^i} = \beta q^i(\boldsymbol{\theta}_t) E_t \left[p_t^i - h + \frac{(1-\sigma^i - \gamma^i \theta_{t+1}^i q^i(\boldsymbol{\theta}_{t+1}))\kappa^i}{(1-\gamma^i) q^i(\boldsymbol{\theta}_{t+1})} - \sum_{j \neq i} \frac{\gamma^j \theta_{t+1}^j \kappa^j}{1-\gamma^j} \right], \quad (20)$$

where $E_t[\cdot]$ represents time-t expectations.

3.2 Calibration and computation

Below I numerically solve the business cycle dynamics of the model. The goal of this exercise is to gain theoretical insights. Given the lack of appropriate data, my calculation below will be primarily illustrative. I assume that there are two types of jobs: type 1 and type 2.

A large part of the calibration is standard. One period is assumed to be one month and β is set at 0.996. In the baseline specification, I treat the type-1 job and the type-2 job symmetrically. The separation probability is assumed to be $\sigma^1 = \sigma^2 = 0.034$, following Shimer (2005). The bargaining power for the worker is $\gamma^1 = \gamma^2 = 0.72$, again following Shimer (2005). Normalizing $\bar{p}^1 = \bar{p}^2 = 1.0$, the flow value of unemployment is set at 0.71 as recommended by Hall and Milgrom (2008).

Without a good measure of the externalities across different type of jobs, here I simply assume the perfect segregation, as in (2).[7] The job matching technology is assumed to be $q^i(\boldsymbol{\theta}) = \chi(\theta^i)^{-\eta}$, where $\eta = 0.72$ as in Shimer (2005). I target $\bar{\theta}^i = 1$ as the steady state value of θ^i. Following Shimer (2005), I target the job-finding probability of an unemployed worker to be 0.49, which implies that $\chi = 0.49/2$. The overall unemployment rate is 6.4% as a result.

The steady-state equations pin down the values of κ^i. In the steady state, (20) becomes (under the assumption of $q^i(\boldsymbol{\theta}) = \chi(\theta^i)^{-\eta}$ with $\bar{\theta}^i = 1$)

$$\frac{\kappa^i}{1-\gamma^i} = \beta\chi \left[\bar{p}^i - h + \frac{(1-\sigma^i - \gamma^i\chi)\kappa^i}{(1-\gamma^i)\chi} - \frac{\gamma^j \kappa^j}{1-\gamma^j} \right],$$

for $i, j = 1, 2$. With the above calibration, $\kappa^1 = \kappa^2 = 0.051$.

The model can accommodate any Markov processes (and covariance structures) for productivity shocks. Here, for simplicity, I assume that the log-deviation of productivities from the steady-state value is identical across types (i.e., shocks are perfectly correlated across types). Let $\hat{p}_t$ be the log-deviation from the steady-state value for both p_t^1 and p_t^2 (I omit i because I assume an identical process). I assume an AR(1) structure:

$$\hat{p}_{t+1} = \rho \hat{p}_t + \epsilon_{t+1},$$

where $\rho \in (0, 1)$ and ϵ_{t+1} follows a Normal distribution with mean zero and standard deviation

[7] In the case of perfect pooling as in (3), the response of vacancies to type-specific shocks is rather extreme. For example, if there is a positive productivity shock only to type 1, type 2 vacancies completely shut down even if the shock is arbitrarily small.

β	$\bar{p}^i$	h	χ	η	σ^i	γ^i	κ^i	ρ	σ_ϵ
0.996	1.0	0.71	0.245	0.72	0.034	0.72	0.051	0.949	0.00645

Table 1: Baseline calibration

σ_ϵ. Hagedorn and Manovskii (2008) calculate the HP-filtered labor productivity process in quarterly frequency has a standard deviation of 0.013 and the autocorrelation of 0.765. With Monte Carlo simulation, I find that the corresponding values (for monthly frequency) of ρ and σ_ϵ are 0.949 and 0.00645. Table 1 summarizes the baseline calibration.

The log-linearized solutions for $\hat{\theta}_t^i$ take the form

$$\hat{\theta}_t^i = \psi^i \hat{p}_t^i + \phi^i \hat{p}_t^j. \tag{21}$$

The coefficient ψ^i is the elasticity of type-i labor market tightness with respect to the shock to the productivity of its own type. The other coefficient, ϕ^i, governs the reaction of the type-i labor market tightness to the shock to type $j \neq i$. These coefficients can be derived analytically. The solutions are detailed in Appendix B. Note that Proposition 1 suggests that $\psi^i > 0$ and $\phi^i < 0$, although here these coefficients represent the dynamic response to shocks while Proposition 1 concerns the comparison of two steady states.[8] Also note that in deriving (21), the assumption that the shocks are perfectly correlated with the same magnitude is not used. The only assumption that is necessary is that the shock for each type has an AR(1) structure and the current value of the shock depends on its own past value with persistence coefficient ρ. Therefore, the coefficients in (21) can also be interpreted as responses to type-specific shocks (with potentially different realizations). For example, one can think of an economy with only shocks to type-1 jobs by assuming that σ_ϵ for type-2 shock is zero.

The equation (21) summarizes the labor market reaction to productivity shocks. Once it is obtained, the behavior of aggregate economy can be simulated using (18) and (19).

3.3 Results

Table 2 summarizes the values for ψ^i and ϕ^i in (21) for various specifications. The columns with "total i" reports the sum of ψ^i and ϕ^i. This corresponds to the change of θ^i in response to a 1% aggregate shock (i.e., identical, perfectly-correlated shocks to both types). In the baseline specification, which is presented in the row (i) in Table 2, the reaction of θ^i to a 1% aggregate productivity shock is about 3%; this is in line with the literature that characterizes the standard model. Thus, introducing two segmented markets does not play a significant role in amplification of shocks.

[8] See Mukoyama et al. (2018, Appendix B) for a similar characterization of several variations of the DMP model, including the basic Pissarides (1985) model. Petrosky-Nadeau and Wasmer (2017) also derive a similar characterization for the basic Pissarides (1985) model. One advantage of the direct characterization of the responses to shocks, compared to the comparative steady-state analysis, is that one can see the effect of ρ.

	ψ^1	ϕ^1	total 1	ψ^2	ϕ^2	total 2
(i) Baseline	11.8	−8.6	3.2	11.8	−8.6	3.2
(ii) $\bar{p}^1 = 1.01$, $\bar{p}^2 = 0.99$	8.8	−6.3	2.5	18.1	−13.4	4.6
(iii) $\sigma^1 = 0.02$, $\sigma^2 = 0.10$	6.6	−4.0	2.6	23.0	−17.4	5.6
(iv) $\bar{p}^1 = 1.01$, $\bar{p}^2 = 0.99$; $\sigma^1 = 0.02$, $\sigma^2 = 0.10$	5.8	−3.5	2.4	49.7	−38.5	11.2

Table 2: Responses to shocks

		u	v	v/u	p
Standard Deviation		0.125	0.139	0.259	0.013
Quarterly Autocorrelation		0.870	0.904	0.896	0.765
	u	1	−0.919	−0.977	−0.302
	v	—	1	0.982	0.460
Correlation Matrix	v/u	—	—	1	0.393
	p	—	—	—	1

Table 3: US data: Hagedorn and Manovskii (2008)

		u	v	v/u	p
Standard Deviation		0.011	0.032	0.041	0.0130
Quarterly Autocorrelation		0.818	0.703	0.763	0.765
	u	1	−0.851	−0.913	−0.814
	v	—	1	0.991	0.973
Correlation Matrix	v/u	—	—	1	0.961
	p	—	—	—	1

Table 4: Simulated results for the baseline case

		u	v	v/u	p
Standard Deviation		0.020	0.071	0.089	0.0129
Quarterly Autocorrelation		0.789	0.719	0.763	0.764
	u	1	−0.892	−0.922	−0.846
	v	—	1	0.995	0.969
Correlation Matrix	v/u	—	—	1	0.959
	p	—	—	—	1

Table 5: Simulated result for case (iv) in Table 2

Table 4 presents the standard deviations, autocorrelations, and cross-correlations (all aggregated to quarterly, logged and HP-filtered with the coefficient 1,600) from the baseline model. Here, v and v/u are the total numbers in the aggregate economy, and p is the (weighted) average of the labor productivity. Comparison of this table to the U.S. data in Table 3, taken from Hagedorn and Manovskii (2008), confirms Shimer's (2005) finding that the model generates labor market fluctuations that are too small compared to the data. This issue is frequently referred to as "the labor market volatility puzzle" in the literature.[9]

Rows (ii) to (iv) in Table 2 introduce various heterogeneities in jobs. The case (ii) introduces 2% difference in the average productivity between two types. This reduces the reaction of type 1 vacancies while increasing the response of type 2 vacancies. The intuition is in line with Hagedorn and Manovskii (2008) in the case of homogeneous jobs—because the surplus from a type 2 match is now smaller, a small productivity shock induces a larger swing in a firm's profit in percentage terms.[10] The surplus from a type 2 match is smaller for two reasons: first, the lower value of $\bar{p}^2$ reduces the flow surplus from the match; and second, the higher value of $\bar{p}^1$ implies that there is a higher option value for being unemployed. This increases the outside option of the workers. The second channel would not be present if workers are heterogeneous instead of jobs being so.

Case (iii) introduces a difference in separation probabilities: $\sigma^1 = 0.02$ and $\sigma^2 = 0.10$. With this specification, the steady-state unemployment rate remains at 6.4%. This case indicates that the heterogeneity in job stability can have a similar effect on the responses as the heterogeneity in productivity—because type 1 job is more stable, it enjoys a higher surplus in expected present value. Case (iv) combines cases (ii) and (iii). In case (iv), type 2 jobs are "bad jobs" in that they are lower-paying and less stable. In this sense, this model features a dual labor market.

One notable observation is that a stronger reaction to aggregate shocks comes together with strong responses to type-specific shocks. As is consistent with Proposition 1, ψ^i is positive and ϕ^i is negative in all specifications. A 1% positive type-1 specific productivity shock in case (ii), for example, leads to 13% decline in vacancy posting of type-2 jobs. The consequences of any shocks are significantly larger for the type of jobs that have smaller surplus.

Table 5 calculates the business-cycle statistics for the case (iv). It shows that the labor market volatility in this case is about twice as large as the baseline case. Although the

[9]Mortensen and Nagypál (2007) and Pissarides (2009) argue that the appropriate target for the aggregate elasticity of v/u with respect to p (corresponding to the aggregate version of $\psi^i + \phi^i$ in our model) should be 7.56. They calculate this number by multiplying the relative standard deviation of v/u compared to p and multiplying the correlation coefficient of v/u and p. Their argument is that, given the existence of measurement errors and other shocks, it is more reasonable to target the OLS coefficient in the data rather than the relative standard deviation as the concept that corresponds to the elasticity of v/u with respect to p in the model. The corresponding number in Table 4 is $(0.259/0.013) \times 0.393 = 7.83$. This number in Table 3 is $(0.041/0.0130) \times 0.961 = 3.03$, similar to the value of $\psi^i + \phi^i$ of 3.2. In Table 5 below, this becomes $(0.089/0.0129) \times 0.959 = 6.61$. If the goal were to simply achieve the value of 7.83, the current model with $\bar{p}^1 = 1.023$, $\bar{p}^2 = 0.977$, and $\sigma^1 = \sigma^2 = 0.034$ would provide the corresponding number of 8.20. Therefore, this framework can resolve the labor market volatility puzzle once it is assumed that there are some (by the magnitude of less than 5%) differences in productivity between type 1 jobs and type 2 jobs.

[10]One of the justifications that Hagedorn and Manovskii (2008) mention in defending their calibration of a high flow utility of unemployment is that the standard DMP model is an approximation of a model with worker heterogeneity. Mortensen and Nagypál (2007) criticize this reasoning by arguing that what matters for firms' incentive in a random matching environment is the average surplus, not the surplus of a marginal worker. Our model shows that if jobs (instead of workers) are heterogeneous, the incentive of posting vacancies for marginal jobs may be affected strongly by the small change in productivity.

separation probabilities are different across jobs, the Beveridge curve is still intact in the sense that u and v are negatively correlated.[11]

4 Conclusion

This paper analyzed a simple extension of the basic DMP model. A particular focus was how heterogeneity of jobs influences the conditions for the efficiency of the equilibrium outcome and the business cycle dynamics.

On efficiency, it was shown that the well-known Hosios condition can be generalized to the case of ex ante heterogeneous jobs. The generalized condition is a simple modification of the original condition by Hosios (1990), and the generalized condition takes into account the fact that there may be externalities across different types of vacancies.

Regarding the reaction to productivity shocks, it was shown that vacancies tend to respond positively to the productivity shocks of their own type and respond negatively to the shocks of other types. The negative responses to the other types' shocks is because a positive shock to other markets drives up the wages, affecting the firms' incentive to post vacancies. The quantitative exercise shows that introducing heterogeneity can have a significant effect on business cycle dynamics because the surplus of "bad jobs" can be low and thus these jobs can respond strongly to shocks.

Properties of unemployment fluctuations are significantly different when even a relatively small productivity gap between types exist. This result may change if on-the-job search is allowed. When on-the-job search is available, the advantage of being unemployed (in terms of the ability to search for a good job) may not be much stronger than being employed in a bad job. In that case, as a result, the option value of being unemployed would be lower. In such a situation, a large productivity gap between good jobs and bad jobs would be necessary for the surplus of a bad job to be small.

The business-cycle result of this paper clarifies an important point that the chance of finding a good job influences the value of unemployment significantly. This means that the dynamics of unemployment is also influenced heavily by the possibility of moving from unemployment to *good jobs*. The fact that the value of being employed in a bad job is close to the value of unemployment means that it is likely that the separation margin would be also more important for such jobs in a model with endogenous separations.

This paper's results point to several possible directions of future research. First, the nature of the externalities in the matching market across different types of jobs can be im-

[11] The response of the type-2 jobs are even larger for a larger differences in $\bar{p}^i$ and σ^i. For example, if $\bar{p}^1 = 1.027$, $\bar{p}^2 = 0.973$ with $\sigma^1 = \sigma^2 = 0.034$, $\psi^1 = 6.2$, $\phi^1 = -4.3$, $\psi^2 = 267.4$, and $\phi^2 = -205.2$. This extreme case is not presented above because the vacancy creation of type-2 jobs shuts down completely when the value of $\hat{p}$ is very low (which is within the simulation range of the model). One has to solve the model nonlinearly in order to deal with this case. It can be done, for example, by applying the method in Krusell et al. (2010, Appendix N). Also see Petrosky-Nadeau and Zhang (2017) for another nonlinear solution method. It is also the case that adding a hiring cost (or a firing cost), in addition to vacancy cost, generates stronger responses as emphasized in Mortensen and Nagypál (2007) and Pissarides (2009). Fluctuations in discount factors, as analyzed in Mukoyama (2009) and Hall (2017), can be another source of amplification.

portant. Therefore, the measurement of such externalities is valuable. Second, heterogeneity in jobs creates interactions between good jobs and bad jobs through workers' outside options. The possibility of unemployed workers being able to find good jobs is an important factor in considering whether the surplus from a low-productivity (or unstable) match is large or small. Thus empirically investigating who fills the good jobs (whether the ones from unemployment or from bad jobs) is an important research topic for the analysis of labor market fluctuations. Third, the model indicates that a larger proportion of bad jobs are created during booms, possibly affecting the measurement of the cost of business cycle fluctuations.[12] Empirically testing this theoretical prediction is thus another important future research topic.[13]

[12]The classic reference is Lucas (1987). See, for example, Mukoyama and Şahin (2006) and Krusell et al. (2009) for the analysis with worker heterogeneity.

[13]This prediction is consistent with the empirical patterns of the entry of U.S. manufacturing plants, documented by Lee and Mukoyama (2015). They show that the productivity of entering plants (relative to incumbent plants) is lower during the booms compared to the recessions.

Acknowledgment

I thank an anonymous referee and participants at the JEA Nakahara lecture. I also thank Susan Langenkamp for editorial help. All errors are mine.

Appendix

A Proofs

Proof of Proposition 1. Here, I use the notation $q^i(\theta^i)$ instead of $q^i(\boldsymbol{\theta})$, because I assume that there are no externalities across markets. Log-linearizing (9) yields

$$\bar{p}^i \hat{p}^i = (\mathcal{A}^i + \mathcal{B}^i)\hat{\theta}^i + \sum_{j \neq i} \mathcal{B}^j \hat{\theta}^j, \tag{22}$$

where

$$\mathcal{A}^i \equiv \frac{(r+\sigma^i)\kappa^i \eta^i(\bar{\theta}^i)}{(1-\gamma^i)q^i(\bar{\theta}^i)} > 0$$

and

$$\mathcal{B}^i \equiv \frac{\gamma^i \kappa^i \bar{\theta}^i}{1-\gamma^i} > 0.$$

Here, $\eta^i(\theta^i) \equiv -q^{i\prime}(\theta^i)\theta^i/q^i(\theta^i) > 0$ is the elasticity of $q^i(\theta^i)$ with respect to θ^i.

Now, suppose that $\hat{p}^i > 0$ and $\hat{p}^j = 0$ for all $j \neq i$. Then, combining (22) for $j, k \neq i$,

$$\mathcal{A}^j \hat{\theta}^j = \mathcal{A}^k \hat{\theta}^k$$

holds. Thus, $\hat{\theta}^j$ and $\hat{\theta}^k$ have the same sign. Meanwhile, the right-hand side of (22) for $j \neq i$ has to sum up to zero, which means that $\hat{\theta}^i$ has to have the opposite sign from $\hat{\theta}^j$ (for all $j \neq i$). Combining (22) for i and $j \neq i$,

$$\bar{p}^i \hat{p}^i = \mathcal{A}^i \hat{\theta}^i - \mathcal{A}^j \hat{\theta}^j.$$

Because $\hat{\theta}^i$ and $\hat{\theta}^j$ have the opposite sign, it has to be the case that $\hat{\theta}^i > 0$ and $\hat{\theta}^j < 0$. ■

Proof of Proposition 2. The equation (11) can be rewritten as

$$-\kappa^i + \mu^i q^i(\boldsymbol{\theta}) + \sum_j \mu^j \theta^j \frac{\partial q^j(\boldsymbol{\theta})}{\partial \theta^i} = 0$$

and thus

$$-\kappa^i + \mu^i \left(1 + \frac{\sum_j \mu^j \theta^j (\partial q^j(\boldsymbol{\theta})/\partial \theta^i)}{\mu^i q^i(\boldsymbol{\theta})}\right) q^i(\boldsymbol{\theta}) = 0. \tag{23}$$

By comparing this and (8), which implies

$$-\kappa^i + S^i(1-\gamma^i)q^i(\boldsymbol{\theta}) = 0,$$

one can see that the condition (14) is satisfied if

$$\gamma^i = -\frac{\sum_j \mu^j \theta^j (\partial q^j(\boldsymbol{\theta})/\partial \theta^i)}{\mu^i q^i(\boldsymbol{\theta})}$$

holds. Then plugging the expression for S^i (which should be equal to μ^i) in (8) into this equation yields the condition (16) in Proposition 2. Now, it is straightforward to check that when (16) is satisfied, the equilibrium values of S^i and θ^i together satisfy (23) and therefore (11). Equation (13) is satisfied by the construction here. ■

B Log-linearized solutions to the discrete-time model

The task here is to utilize the log-linearized version of (20) to obtain the solutions for the response of θ^i_t on the productivity shocks $\hat{p}^i_t$ and $\hat{p}^j_t$, where $i, j = 1, 2$.

The left-hand side of (20), divided by $\beta q(\boldsymbol{\theta}_t)$, can be rewritten as

$$\frac{\kappa^i}{(1-\gamma^i)\beta\chi(\theta^i_t)^{-\eta}}$$

and thus can be log-linearized as

$$\frac{\kappa^i}{(1-\gamma^i)\beta\chi}\eta(\bar{\theta}^i)^\eta \hat{\theta}^i_t.$$

On the right-hand side, inside the expectations, there are three terms. The first term, $p^i_{t+1} - h$, can be log-linearized as $\bar{p}^i \hat{p}^i_{t+1}$. The second term can be rewritten as

$$\frac{(1-\sigma^i)\kappa^i}{(1-\gamma^i)\chi(\theta^i_{t+1})^{-\eta}} - \frac{\gamma^i \theta^i_{t+1}\kappa^i}{1-\gamma^i}.$$

This can be log-linearized as

$$\frac{(1-\sigma^i)\kappa^i}{(1-\gamma^i)\chi}\eta(\bar{\theta}^i)^\eta \hat{\theta}^i_{t+1} - \frac{\gamma^i\kappa^i}{1-\gamma^i}\bar{\theta}^i\hat{\theta}^i_{t+1}.$$

The third term in the right-hand side of (20) is

$$-\frac{\gamma^j \theta^j_{t+1}\kappa^j}{1-\gamma^j}.$$

This can be log-linearized as

$$-\frac{\gamma^j\kappa^j}{1-\gamma^j}\bar{\theta}^j\hat{\theta}^j_{t+1}.$$

Thus, the log-linearized version of (20) can be written as

$$\Omega^i \hat{\theta}^i_t = E_t[\bar{p}^i \hat{p}^i_{t+1} + \Gamma^i \hat{\theta}^i_{t+1} + \Xi^i \hat{\theta}^j_{t+1}],$$

where

$$\Omega^i = \frac{\kappa^i}{(1-\gamma^i)\beta\chi}\eta(\bar{\theta}^i)^\eta,$$

$$\Gamma^i = \frac{(1-\sigma^i)\kappa^i}{(1-\gamma^i)\chi}\eta(\bar{\theta}^i)^\eta - \frac{\gamma^i\kappa^i}{1-\gamma^i}\bar{\theta}^i,$$

and

$$\Xi^i = -\frac{\gamma^j\kappa^j}{1-\gamma^j}\bar{\theta}^j.$$

With the guess

$$\hat{\theta}^i_t = \psi^i \hat{p}^i_t + \phi^i \hat{p}^j_t$$

and using $E_t[\hat{p}^i_{t+1}] = \rho\hat{p}^i_t$,

$$\Omega^i\psi^i\hat{p}^i_t + \Omega^i\phi^i\hat{p}^j_t = \rho\bar{p}^i\hat{p}^i_t + \rho(\Gamma^i\psi^i + \Xi^i\phi^j)\hat{p}^i_t + \rho(\Gamma^i\phi^i + \Xi^i\psi^j)\hat{p}^j_t$$

holds, which implies that

$$\Omega^i\psi^i = \rho\bar{p}^i + \rho(\Gamma^i\psi^i + \Xi^i\phi^j) \tag{24}$$

and

$$\Omega^i\phi^i = \rho(\Gamma^i\phi^i + \Xi^i\psi^j) \tag{25}$$

have to hold. Equation (24) can be rewritten as

$$\psi^i = \frac{\rho\bar{p}^i + \rho\Xi^i\phi^j}{\Omega^i - \rho\Gamma^i}.$$

From (25) for j,

$$\phi^j = \frac{\rho\Xi^j}{\Omega^j - \rho\Gamma^j}\psi^i$$

holds. Therefore,

$$\psi^i = \left(1 - \frac{\rho^2\Xi^i\Xi^j}{(\Omega^i - \rho\Gamma^i)(\Omega^j - \rho\Gamma^j)}\right)^{-1}\frac{\rho\bar{p}^i}{\Omega^i - \rho\Gamma^i}$$

and

$$\phi^j = \frac{\rho\Xi^j}{\Omega^j - \rho\Gamma^j}\left(1 - \frac{\rho^2\Xi^i\Xi^j}{(\Omega^i - \rho\Gamma^i)(\Omega^j - \rho\Gamma^j)}\right)^{-1}\frac{\rho\bar{p}^i}{\Omega^i - \rho\Gamma^i}.$$

References

[1] Acemoglu, Daron (2001). "Good Jobs versus Bad Jobs," *Journal of Labor Economics* 19, 1–21.

[2] Bertola, Giuseppe and Ricardo J. Caballero (1994). "Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment," *Review of Economic Studies* 61, 435–456.

[3] Bils, Mark, Yongsung Chang, and Sun-Bin Kim (2012). "Comparative Advantage and Unemployment," *Journal of Monetary Economics* 59, 150–165.

[4] Davis, Steven J. (2001). "The Quality Distribution of Jobs and the Structure of Wages in Search Equilibrium," NBER Working Paper 8434.

[5] Elsby, Michael W. L. and Ryan Michaels (2013). "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows," *American Economic Journal: Macroeconomics* 5, 1–48.

[6] Hagedorn, Marcus and Iourii Manovskii (2008). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review* 98, 1692–1706.

[7] Hall, Robert E. (2017). "High Discounts and High Unemployment," *American Economic Review* 107, 305–330.

[8] Hall, Robert E. and Paul R. Milgrom (2008). "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review* 98, 1653–1674.

[9] Hosios, Arthur J. (1990). "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies* 57, 279–298.

[10] Kaas, Leo and Philipp Kircher (2015). "Efficient Firm Dynamics in a Frictional Labor Market," *American Economic Review* 105, 3030–3060.

[11] Krusell, Per, Toshihiko Mukoyama, and Ayşegül Şahin (2010). "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations," *Review of Economic Studies* 77, 1477–1507.

[12] Krusell, Per, Toshihiko Mukoyama, Ayşegül Şahin, and Anthony A. Smith Jr. (2009). "Revisiting the Welfare Effects of Eliminating the Business Cycles," *Review of Economic Dynamics* 12, 393–404.

[13] Lee, Yoonsoo and Toshihiko Mukoyama (2015). "Entry and Exit of Manufacturing Plants over the Business Cycle," *European Economic Review* 77, 20–27.

[14] Lise, Jeremy and Jean-Marc Robin (2017). "The Macrodynamics of Sorting between Workers and Firms," *American Economic Review* 107, 1104–1135.

[15] Ljungqvist, Lars and Thomas J. Sargent (2012). *Recursive Macroeconomic Theory,* 3rd Ed., Cambridge: MIT Press.

[16] Lucas, Robert E. Jr. (1987). *Models of Business Cycles,* New York: Basli Blackwell.

[17] Mortensen, Dale T. and Éva Nagypál (2007). "More on Unemployment and Vacancy Fluctuations," *Review of Economic Dynamics* 10, 327–347.

[18] Mueller, Andreas I. (2017). "Separations, Sorting, and Cyclical Unemployment," *American Economic Review* 107, 2081–2107.

[19] Mukoyama, Toshihiko (2009). "A Note on Cyclical Discount Factors and Labor Market Volatility," mimeo.

[20] Mukoyama, Toshihiko, Christina Patterson, and Ayşegül Şahin (2018). "Job Search Behavior over the Business Cycle," *American Economic Journal: Macroeconomics* 10, 190–215.

[21] Mukoyama, Toshihiko and Ayşegül Şahin (2006). "Costs of Business Cycles for Unskilled Workers," *Journal of Monetary Economics* 53, 2179–2193.

[22] Petrosky-Nadeau, Nicolas and Etienne Wasmer (2017). *Labor, Credit, and Goods Markets: The Macroeconomics of Search and Unemployment,* Cambridge: MIT Press.

[23] Petrosky-Nadeau, Nicolas and Lu Zhang (2017). "Solving the Diamond-Mortensen-Pissarides Model Accurately," *Quantitative Economics* 8, 611–650.

[24] Pissarides, Christopher A. (1985). "Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages," *American Economic Review* 75, 676–690.

[25] Pissarides, Christopher A. (2000). *Equilibrium Unemployment Theory,* 2nd Ed., Cambridge: MIT Press.

[26] Pissarides, Christopher A. (2009). "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77, 1339–1369.

[27] Shimer, Robert (2005). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review* 95, 25–49.