Appendix to "Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations" by Krusell, Mukoyama, and Şahin

A Related literature

The attempt of creating a unified framework in which one can study business cycles with a frictional labor market started in 1990s. After the important contributions of Merz (1995) and Andolfatto (1996) who incorporated a DMP type labor market structure in the standard RBC model, the challenge of relaxing the complete-market assumption emerged.

This extension, in principle, involves substantial complications in several aspects. First, as in the standard BHA models with aggregate shocks, the wealth distribution (an infinite-dimensional object) now enters into the decision problems as a state variable. Second, the Nash bargaining is more complex; in particular, the wage determination is no longer summarized by a simple surplus-sharing rule and the wage level depends on the asset level of individual workers. Third, due to market incompleteness, it is no longer clear who owns the firm and what the firm does with its profits.

In our paper, we deal with the first challenge by extending the computational method by Krusell and Smith (1998, 1997). We allow for individual Nash bargaining and compute the wage as a function of the asset level; thus, there is no single wage. To price the firm, we let the firm be owned by consumers and add sufficient aggregate assets so that the present-value of the profit stream is well defined.

Valdivia (1996) is an early attempt to study a model with market incompleteness, frictions, and consumer heterogeneity in a business-cycle setting. To overcome the above challenges, Valdivia (1996) makes several strong assumptions. In particular, he assumes that firms' profits are equally distributed across all households (he did not allow for trading equity which is the claim to firms' profits). In addition, he applies a simple sharing rule to determine the wages in the presence of asset heterogeneity.

Recently, various other studies have attempted to solve similar class of models. Costain and

Reiter (2005, 2007), Kallock (2006), Bils et al. (2009), Nakajima (2007), Rudanko (2006, 2009), Shao and Silos (2007), Jung and Kuester (2008) recently studied DMP models with risk aversion and imperfect consumption insurance. These papers vary considerably in the ways they deal with the aforementioned difficulties. The models of Costain and Reiter (2005) and Kallock (2006) do not have capital stock in production process and instead allow simple storage technologies as means of self-insurance. In that sense their models are not easily comparable with standard RBC models.

Costain and Reiter (2007), Bils et al. (2009), Nakajima (2007), Shao and Silos (2007), and Jung and Kuester (2008) feature capital stock for production. Jung and Kuester (2008) assume that there are two types of agents: some cannot save or borrow, while the others have access to complete insurance. This assumption makes the computation easier, but this asset structure is rather extreme as a description of reality. Bils et al. (2009), Nakajima (2007), and Shao and Silos (2007) utilize variants of Krusell-Smith method, while Costain and Reiter (2007) apply a method based on projection and perturbation techniques. Bils et al. (2009), Shao and Silos (2007), and Costain and Reiter (2007) separate agents (ex ante) into two classes: workers and entrepreneurs. They assume that firms are owned by entrepreneurs who either have linear utility or are not subject to uninsurable idiosyncratic shock. This allows the firm's profit discounting to be well defined. However, this ex-ante separation makes it difficult to use the model for evaluating the heterogeneous effect of policies, since in reality many consumers are employed as workers and also own stocks at the same time. Nakajima (2007) simplifies the wage bargaining by assuming that a "union" bargains with the firm for the single wage for all workers. He also makes a simplifying assumption that the firm's profit is discounted at the same rate as the return to the capital stock. Shao and Silos (2007) also make the "union bargaining" type assumption to eliminate the dependence of wages on asset positions of workers. Bils et al. (2009) and Costain and Reiter (2007) allow individual wage bargaining which allows the wages to depend on the workers' asset levels. Rudanko (2006, 2009) develops a model where workers and entrepreneurs are distinguished by their ability to access capital markets. While entrepreneurs have access to complete asset markets, workers are excluded from asset markets completely.

Some of these papers have features that we do not have in our model. For example, Nakajima (2007) allows for labor-leisure choice of employed workers, Bils et al. (2009) and Shao and Silos (2007) allow for heterogeneous productivity of individuals, Rudanko (2006, 2009) considers long-term wage contracts, and Jung and Kuester (2008) has wage rigidity. From this viewpoint, these papers can be considered complementary to ours.

B Consistency in the valuation of the firm

This appendix establishes that the valuation of the firm is consistent across the individual job level and the aggregate level (equity price).

- Aggregate level (equity price):

  $$\tilde{p} = d + q\tilde{p} \tag{30}$$

  holds, where

  $$d = \int \pi(a) f_e(a) da - \xi v$$

  is dividend (same as the text) and q is the discount factor for the firm.

  Note that $\tilde{p}$ is different from p in the text. $\tilde{p}$ is the value of the firm *before* the dividend is paid, and p is the value of the firm *after* the dividend is paid. They are related by $\tilde{p} = p + d$. In fact, (30) implies

  $$p + d = d + q(p + d),$$

  and therefore $p = q(p + d)$. From (1), the discount factor q is equal to $1/(1 + r - \delta)$.

- Individual job level:

  Define $J(a)$ and V with

  $$J(a) = \pi(a) + q(\sigma V + (1 - \sigma)J(\psi_e(a))) \tag{31}$$

  and

  $$V = -\xi + q\left[(1 - \lambda_f)V + \lambda_f \int J(\psi_u(a))\frac{f_u(a)}{u}da\right]$$

and note that

$$\tilde{p} = \int J(a) f_e(a) da. \tag{32}$$

In addition, $V = 0$ must hold in equilibrium.

Note also that in steady state, assuming that $\psi_u(a)$ and $\psi_e(a)$ are increasing,

$$\int_{\underline{a}}^{a} f_e(a')da' = \lambda_w \int_{\underline{a}}^{\psi_u^{-1}(a)} f_u(a')da' + (1-\sigma) \int_{\underline{a}}^{\psi_e^{-1}(a)} f_e(a')da'$$

holds. Differentiating with respect to a, using Leibniz's rule, we obtain

$$f_e(a) = \lambda_w f_u(\psi_u^{-1}(a))\gamma_u(a) + (1-\sigma) f_e(\psi_u^{-1}(a))\gamma_e(a), \tag{33}$$

where

$$\gamma_u(a) = \frac{d\psi_u^{-1}(a)}{da} = \frac{1}{\psi_u'(\psi_u^{-1}(a))} \tag{34}$$

and

$$\gamma_e(a) = \frac{d\psi_e^{-1}(a)}{da} = \frac{1}{\psi_e'(\psi_e^{-1}(a))}. \tag{35}$$

Inserting (31) into (32), we obtain

$$\tilde{p} = \int \pi(a)da + q \int (1-\sigma) J(\psi_e(a)) f_e(a) da.$$

(Note that we used $V = 0$.) Adding vV (from (7)), which is equal to zero, to the right-hand side, we arrive at

$$\tilde{p} = \int \pi(a)da - \xi v + q \left[\lambda_f v \int J(\psi_u(a)) \frac{f_u(a)}{u} da + \int (1-\sigma) J(\psi_e(a)) f_e(a) da \right].$$

The first two terms equal d; thus, it is sufficient to show that

$$\lambda_f v \int J(\psi_u(a)) \frac{f_u(a)}{u} da + \int (1-\sigma) J(\psi_e(a)) f_e(a) da = \tilde{p}. \tag{36}$$

To show this, inserting (33) into (32) we obtain

$$\tilde{p} = \int J(a) \lambda_w f_u(\psi_u^{-1}(a))\gamma_u(a) da + \int J(a)(1-\sigma) f_e(\psi_u^{-1}(a))\gamma_e(a) da.$$

In the first term, replace the variable a by $a = \psi_u(a')$. Then $a' = \psi_u^{-1}(a)$. The first term will become

$$\lambda_w \int J(\psi_u(a')) f_u(a') \gamma_u(\psi_u(a')) \psi_u'(a') da',$$

where the final $\psi_u'(a')$ is the Jacobian. Using $\lambda_w u = \lambda_f v$ and (34), this is equal to the first term in (36). The second term is similar, using (35).

C Recursive stationary equilibrium

Definition 1 (Recursive stationary equilibrium) *The recursive stationary equilibrium consists of a set of value functions* $\{\tilde{W}(w,a),\ \tilde{J}(w,a),\ W(a),\ J(a),\ U(a),\ V\}$*, a set of decision rules for asset holdings* $\{\tilde{\psi}_e(w,a),\ \psi_e(a),\ \psi_u(a)\}$*, prices* $\{r,\ p,\ \omega(a)\}$*, vacancy* v*, matching probabilities* λ_f *and* λ_w*, dividend* d*, and the distribution of employment and asset* μ *(which contains the information of* $f_e(a)$*,* $f_u(a)$*, and the unemployment rate* u*) which satisfy*

1. *Consumer optimization:*

   Given the job-finding probability λ_w*, prices* $\{r,\ p\}$*, and wage* w*, the individual decision rules* $\tilde{\psi}_e(w,a)$ *and* $\psi_u(a)$ *solve the optimization problems (3) and (6), with the value functions* $\tilde{W}(w,a)$ *and* $U(a)$*. Given the wage function* $\omega(a)$*,* $W(a)$ *and* $\psi_e(a)$ *satisfy (4) and (5).*

2. *Firm optimization:*

   Given prices r *and* w*, distribution* μ*, and the employed consumer's decision rule* $\tilde{\psi}_e(w,a)$*, the firm solves the optimization problem (8), with the value function* $\tilde{J}(w,a)$*. Given the wage function* $\omega(a)$*,* $J(a)$ *satisfies (9). Given the worker-finding probability* λ_f*,* r*, the unemployed consumer's decision rule* $\psi_u(a)$*, and* μ*,* V *satisfies (7).*

3. *Free entry:*

   The number of vacancy posted, v*, is consistent with the firm free-entry:* $V = 0$*.*

4. *Asset market:*

The asset-market equilibrium condition

$$\int \psi_e(a) f_e(a) da + \int \psi_u(a) f_u(a) da = (1 - \delta + r)\bar{k} + p + d$$

holds: the left-hand side is the total asset supply and the right-hand side is the total asset demand. The no-arbitrage condition (1) holds. Dividend d satisfies (10). $\bar{k}$ satisfies $\tilde{k} = \bar{k}/(1-u)$, where $\tilde{k}$ satisfies the firm's first-order condition: $r = zF'(\tilde{k})$ for given r.

5. *Matching:*

   λ_f and λ_w are functions of v and u as in Section 3.2.

6. *Nash bargaining:*

   The wage function $\omega(a)$ is determined through Nash bargaining between the firms and the consumers by solving (11).

7. *Consistency:*

   μ is the invariant distribution generated by λ_w, σ, and the consumer's decision rules.

D Solution of the model without aggregate shocks

1. We use a discrete grid on a; the grid is fine for the value function and coarser for the wage function. For value function, we use 1000 grid points with equal distance over $[0, 500]$. For the wage function, we use 125 grid points.[37] In between the grid points, the values of the functions are interpolated using cubic splines.[38]

2. Guess an $\omega(a)$.

3. Guess a θ. Note that this will give us λ_w and λ_f. From the steady-state condition

   $$u\lambda_w = (1-u)\sigma,$$

   we know u. Since $\nu/u = \theta$, we also know ν.

[37] In some experiments, we use 50 grid points to gain stability.

[38] When this interpolation does not perform well, we used linear interpolation.

4. Guess $\bar{k}$. Since $r = zf'(\tilde{k})$, where $\tilde{k} = \bar{k}/(1-u)$, we know r.

5. In the worker's problem, note that since

$$p = \frac{p+d}{1+r-\delta}, \tag{37}$$

we can define

$$a = (1+r-\delta)(k+px) \tag{38}$$

and write the employed worker's budget constraint as

$$a' = (1+r-\delta)(a+w-c).$$

We also know w as a function of a. So let us solve the worker's problem as

$$W(a) = \max_{a'} u(c) + \beta\left[\sigma U(a') + (1-\sigma)W(a')\right]$$

subject to

$$a' = (1+r-\delta)(a+\omega(a)-c)$$

and

$$U(a) = \max_{a'} u(c) + \beta\left[(1-\lambda_w)U(a') + \lambda_w W(a')\right]$$

subject to

$$a' = (1+r-\delta)(a+h-c).$$

If necessary, we can interpolate on $\omega(a)$.

6. Given this, we know the asset and employment decision rules for workers, so we can easily calculate the invariant distribution for the worker's idiosyncratic states. These are calculated by iterating over the density functions, $f_u(a)$ and $f_e(a)$, until these converge. (The initial condition that we used for this iteration is that everyone holds the amount $\bar{k}$ of the asset.)

7. In the firm's problem, first iterate on

$$J(a) = zF(\tilde{k}) - r\tilde{k} - \omega(a) + \frac{1}{1+r-\delta}(1-\sigma)J(\psi_e(a))$$

until convergence. (We already used the fact that $V = 0$.)

8. Calculate the right-hand side of V as

$$-\xi + \frac{1}{1+r-\delta}\lambda_f \int J(\psi_w(a))\frac{f_u(a)}{u}.$$

This quantity should be zero in equilibrium; if it is positive, our θ is too low, and if it is negative, our θ is too high.

9. We can calculate d from

$$d = \int \pi(\omega(a)) f_e(a) da - \xi\nu.$$

Here, $\pi(\omega(a)) = zf(\tilde{k}) - r\tilde{k} - \omega(a)$. From (37), we calculate p. Summing up (38) for each individual i,

$$\int a_i di = (1+r-\delta)\left[\int k_i di + p\int x_i di\right].$$

Since $\int k_i di = \bar{k}$ and $\int x_i di = 1$, $\bar{k}$ should satisfy

$$\bar{k} = \frac{1}{1+r-\delta}\int a_i di - p.$$

If this $\bar{k}$ is different from the initial guess, we have to update.

10. Finally, we can calculate the new $\omega(a)$ for each a from Nash bargaining:

$$\max_w \left(\tilde{W}(w,a) - U(a)\right)^\gamma \left(\tilde{J}(w,a) - V\right)^{1-\gamma}.$$

Here, we can use $V = 0$. $\tilde{W}(w,a)$ is the solution to

$$\tilde{W}(w,a) = \max_{a'} u(c) + \beta\left[\sigma U(a') + (1-\sigma)W(a')\right]$$

subject to

$$a' = (1+r-\delta)(a+w-c).$$

$J(w,a)$ is the solution of

$$\tilde{J}(w,a) = zF(\tilde{k}) - r\tilde{k} - w + \frac{1}{1+r-\delta}(1-\sigma)J(\tilde{\psi}_e(w,a)).$$

11. Repeat until convergence.

E Higher risk aversion

Table 14 presents the summary statistics for different utility functions with the Shimer calibration. One is log utility, and the other is $u(c) = c^{1-\zeta}/(1-\zeta)$ with $\zeta = 5$ (we kept the other parameters constant, except that the vacancy cost ξ is adjusted so that $\theta = 1.0$ holds in equilibrium). Larger ζ is associated with higher precautionary savings and thus with higher $\bar{k}$. Higher $\bar{k}$ leads to larger profitability of each vacancy: v increases, θ increases, and u decreases. Naturally, p and d increase.

	ξ	θ	u	v	$\bar{k}$	p	d	w
log utility	0.5315	1.00	7.69%	0.0769	66.54	0.82	0.004	2.48
$\zeta = 5$	0.5390	1.00	7.69%	0.0769	66.80	0.83	0.004	2.48

Table 14: Summary statistics for the model without aggregate shocks. w is the average wage in the economy.

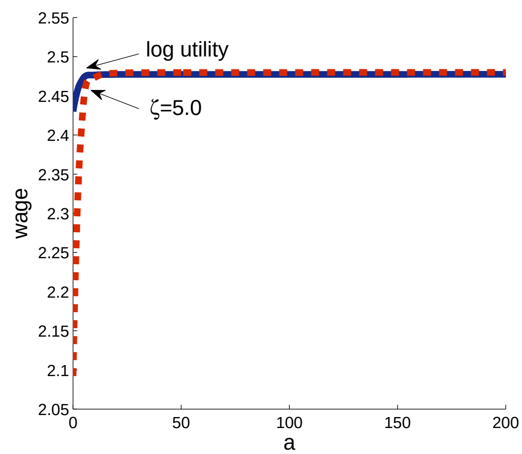

Figure 10: Wages for log utility and $u(c) = c^{1-\zeta}/(1-\zeta)$ with $\zeta = 5$.

Figure 10 compares the wage functions across different utility functions (log and $\zeta = 5$). The change in wages close to the borrowing constraint is larger with $\zeta = 5$. This is because the outside option $U(a)$ is very low for a nearly constrained worker, when the utility function has large curvature. The resulting wage dispersion is higher in this case (mean-min wage ratio is 1.185) compared to the log utility case.

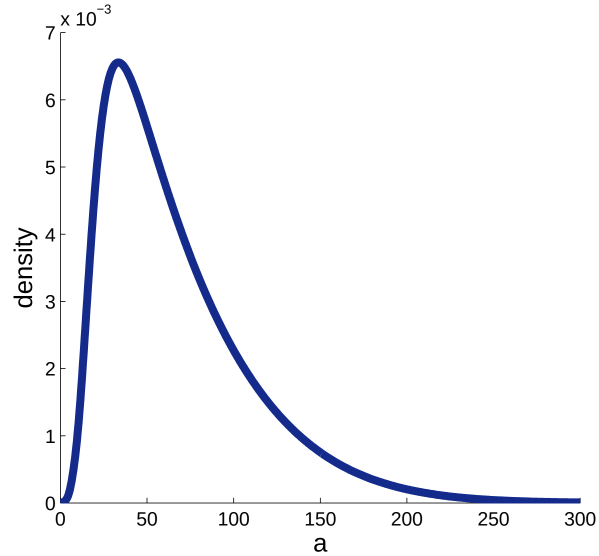

Figure 11: Asset distribution for log utility.

Figure 11 plots the asset holding density for the log utility case. (Asset distributions are similar for $\zeta = 5$ case.) As can be seen from the asset distribution, consumers avoid to be at the very lower tail of the asset distribution. This is due to the additional savings incentive that arises in our model. The wealth distribution here, clearly, is not realistic, but this is probably not a major shortcoming; in the present setting, the only source of wealth inequality is unemployment shocks (there is no ex-ante consumer/worker heterogeneity, and there are no other shocks, such as wage shocks).

The additional incentive for saving can be seen easily from the Euler equations of the consumers.[39] For employed consumers, the Euler equation is (when they are not borrowing-constrained)

$$u'(c_t) = \beta(1 + r - \delta) \left[\sigma u'(c^u_{t+1}) + (1 - \sigma)(1 + \omega'(a_{t+1}))u'(c^e_{t+1}) \right],$$

where c_t is the current consumption and c^u_{t+1} is the future consumption in the case of unemployment, c^e_{t+1} is the future consumption in the case of employment. For unemployed consumers, the Euler

[39]For the purposes of illustration, we assume that the wage function $\omega(a)$ is differentiable. In the numerical calculations below, we do not make this assumption.

equation is

$$u'(c_t) = \beta(1 + r - \delta)\left[(1 - \lambda_w)u'(c^u_{t+1}) + \lambda_w(1 + \omega'(a_{t+1}))u'(c^e_{t+1})\right].$$

These Euler equations are standard except for $\omega'(a_{t+1})$, which turns out to be positive in our calibration.

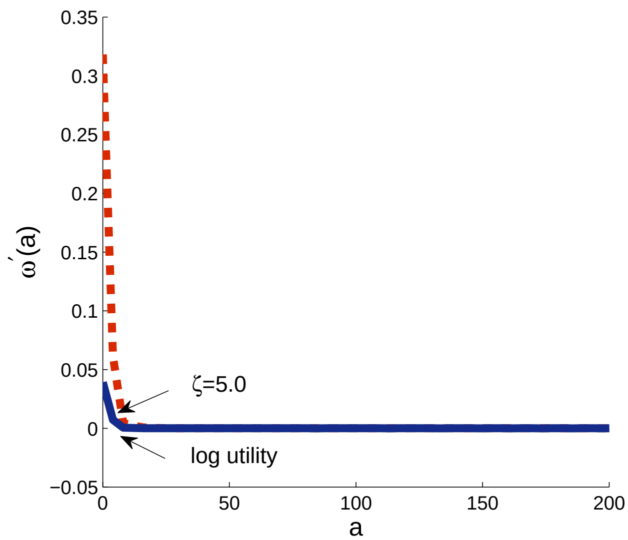

Figure 12: Derivative of the wage function for log utility and $u(c) = c^{1-\zeta}/(1-\zeta)$ with $\zeta = 5$.

Figure 12 plots $\omega'(a_{t+1})$ for the log utility case and $\zeta = 5$ case. $\omega'(a_{t+1})$ is large for the consumers with low asset holdings. Consumers with low asset holdings cannot insure themselves from being unemployed—since they have no other resources to consume (they are constrained by the borrowing limit), they suffer from low consumption when they are unemployed. This makes their bargaining position weaker. As the asset level increases, the consumers are better insured, and their value function becomes closer to linear. As a consequence, $\omega(a)$ becomes flatter. Note that when the utility function is linear (as in standard DMP model), the wage doesn't depend on a. We will see later that the aggregate behavior of our model is very similar to the linear utility model.

In this model, a positive $\omega'(a_{t+1})$ provides an extra incentive to save. When the wage is Nash bargained, the consumers try to escape from the low asset holding level. As a result, most of the populations stay in the part where $\omega(a)$ is flat. Thus, the homogeneity of the wages across the population is generated by the endogenous choice of asset by the consumers.

Now, we change z in our incomplete markets model with $\zeta = 5.0$ and compare it with the linear model. Table 15 summarizes the results. The results are remarkably similar in both economies.

	y	u	v	θ	$\bar{k}$
$z = 0.98$-linear	-2.94%	7.94%	-3.5%	-4.7%	-2.95%
$z = 0.98$-incomplete	-2.94%	7.79%	-3.5%	-4.7%	-2.96%
$z = 1.02$-linear	$+2.97\%$	7.75%	$+3.5\%$	$+4.7\%$	$+2.97\%$
$z = 1.02$-incomplete	$+2.97\%$	7.60%	$+3.7\%$	$+5.0\%$	$+2.99\%$

Table 15: Comparative statics for the linear model and for the incomplete-markets model with $\zeta = 5$, all in % deviations from the $z = 1.00$ case, except for u.

F Comparison to the linear model

In this section, we compare our result to the linear DMP model. In it, the consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t c_t$$

subject to

$$c + \frac{a'}{1+r-\delta} = a + w$$

when working and

$$c + \frac{a'}{1+r-\delta} = a + h$$

when unemployed. Here, $a = k + px$ holds, as in the original model. Because utility is linear, $1 + r - \delta = 1/\beta$ holds. Now, since the workers are indifferent in terms of the timing of the consumption, without loss of generality, we can let $a' = a$ for everyone, and set

$$c_t = \frac{r-\delta}{1+r-\delta} a + w$$

for employed and

$$c_t = \frac{r-\delta}{1+r-\delta}a + h$$

for unemployed. Since the first terms of the right-hand sides are constant and common across the employment states, we can factor them out in the utility function and consider $c_t = w$ and $c_t = h$ without loss of generality.

The value functions become

$$W = w + \frac{1}{1+r-\delta}[\sigma U + (1-\sigma)W]$$

and

$$U = h + \frac{1}{1+r-\delta}[(1-\lambda_w)U + \lambda_w W].$$

On the firm side, the value of a filled job is

$$J = y - w + \frac{1}{1+r-\delta}[\sigma V + (1-\sigma)J],$$

where y is defined as

$$y = \arg\max_{\tilde{k}} \ z\tilde{k}^{\alpha} - r\tilde{k},$$

that is,

$$y = z\left(\frac{r}{\alpha z}\right)^{\frac{\alpha}{\alpha-1}} - r\left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}},$$

since

$$\tilde{k} = \left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}}.$$

The value of vacancy is

$$V = -\xi + \frac{1}{1+r-\delta}[\lambda_f J + (1-\lambda_f)V]. \tag{39}$$

From free entry, $V = 0$.

Now, since $W - U$ and $J - V$ is linear in w, the Nash bargaining solution results in the simple surplus-sharing rule:

$$W - U = \gamma S$$

and

$$J - V = (1-\gamma)S, \tag{40}$$

where

$$S = (W - U) + (J - V) \tag{41}$$

is the total surplus.

From (39), (40), and the free-entry condition,

$$S = \frac{(1+r-\delta)\xi}{(1-\gamma)\lambda_f}$$

holds. From (41) and the value functions,

$$S = \frac{(1+r-\delta)(y+\xi-h)}{r-\delta+\sigma+(1-\gamma)\lambda_f+\gamma\lambda_w}$$

holds. Combining these two, simplifying, and using the definitions of λ_f and λ_w, the following holds:

$$\frac{y-h}{r-\delta+\sigma+\gamma\chi\theta^{1-\eta}} = \frac{\xi}{(1-\gamma)\chi\theta^{-\eta}}.$$

This is the same as equation (21) in Hornstein et al. (2006). This equation is shown as equation (12) in the main text.

G The role of wealth dispersion

In this appendix we look at a slightly different model in order to be able to accommodate more larger dispersion in wealth. In particular, we generate wealth dispersion that is such that a much larger fraction of workers are in the upward-sloping part of the wage curve. To do this, we employ a very simple version of the setting with heterogeneity in patience studied in Krusell and Smith (1998).[40] We assume that there are two types of agents with permanently different discount factors: one group has $\beta_h = 0.997$ and the other $\beta_l = 0.995$. For simplicity, we moreover assume that patient agents do not have labor income (which would be a small part of their earnings anyway, given that they

[40]One could also pursue the possibility that wages are highly dispersed, and random; see Castañeda, Díaz-Giménez and Ríos-Rull (2003). The approach taken here was guided merely by ease of computation.

will accumulate large amounts of wealth). Thus, their income is perfectly deterministic and their equilibrium consumption in steady state will be constant, implying that the equilibrium interest rate in steady state must be given by $r = 1/\beta_h - 1 + \delta$. We solve and compare three different models: the linear model, the incomplete-markets model with log utility and the incomplete-markets model with $\zeta = 5$. We calibrate the vacancy cost to $\xi = 0.632$ (constant across economies) by targeting $\theta = 1$ in the incomplete-markets model with logarithmic utility. For the linear model, we use the same discount rate for the firm and for the consumers.

Figure 13 shows the wage function and the asset distribution of the unemployed. As can be seen from the asset distributions, there is now a much larger mass of agents in the curved part of the wage distributions.

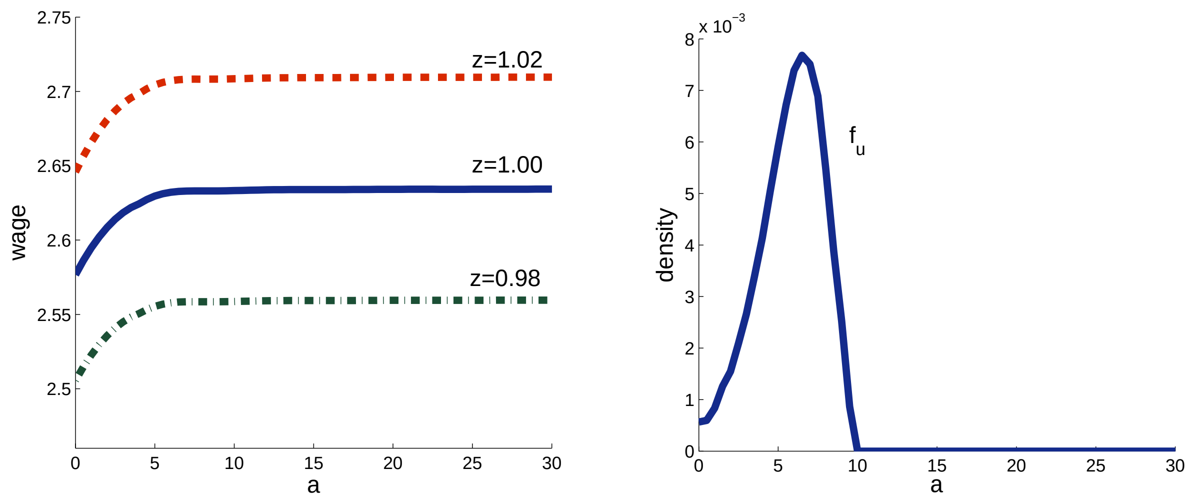

Figure 13: Wage functions (left panel) and asset distribution for the unemployed (right panel).

Table 16 presents the comparative statics with respect to productivity (z) for all three specifications. Though the unemployment levels differ between three models, the responses of the economy to changes in z are remarkably similar.

	y	u	v	θ	$\bar{k}$
$z = 0.98$-linear	-2.94%	7.94%	-3.5%	-4.7%	-2.95%
$z = 0.98$-incomplete	-2.94%	7.79%	-3.5%	-4.7%	-2.95%
$z = 1.02$-linear	$+2.97\%$	7.75%	$+3.5\%$	$+4.7\%$	$+2.97\%$
$z = 1.02$-incomplete	$+2.96\%$	7.60%	$+3.5\%$	$+4.7\%$	$+2.97\%$

Table 16: Comparative statics for the linear model and for the incomplete-markets, all in % deviations from the $z = 1.00$ case, except for u.

H Recursive equilibrium

The following defines the recursive equilibrium of the model with aggregate shocks.

Definition 2 (Recursive equilibrium) *The recursive equilibrium consists of a set of value functions* $\{\tilde{W}(w, a; z, S),\ \tilde{J}(w, a; z, S),\ W(a; z, S),\ J(a; z, S),\ U(a; z, S),\ V(z, S)\}$*, a set of decision rules for asset holdings* $\{\tilde{\psi}_e^{z'}(w, a; z, S),\ \psi_e^{z'}(a; z, S),\ \psi_u^{z'}(a; z, S)\}$*, prices* $\{r(z, S),\ p(z, S),\ Q_g(z, S),\ Q_b(z, S),\ \omega(a; z, S)\}$*, vacancy* $v(z, S)$*, matching probabilities* $\lambda_f(z, S)$ *and* $\lambda_w(z, S)$*, dividend* $d(z, S)$*, and a law of motion for the distribution,* $S' = \Omega(z, S)$*, which satisfy*

1. *Consumer optimization:*

   Given the aggregate states, $\{z, S\}$*, job-finding probability* $\lambda_w(z, S)$*, prices* $\{r(z, S),\ p(z, S),\ Q_g(z, S),\ Q_b(z, S)\}$*, wage* w*, and the law of motion for the distribution,* $S' = \Omega(z, S)$*; the individual decision rules* $\tilde{\psi}_e^{z'}(w, a; z, S)$ *and* $\psi_u^{z'}(a; z, S)$ *solve the optimization problems (19) and (22), with the value functions* $\tilde{W}(w, a; z, S)$ *and* $U(a; z, S)$*. Given the wage function* $\omega(a; z, S)$*,* $W(a; z, S)$ *and* $\psi_e^{z'}(a; z, S)$ *satisfy (20) and (21).*

2. *Firm optimization:*

   Given the aggregate states, $\{z, S\}$*, prices* $\{r(z, S),\ Q_g(z, S),\ Q_b(z, S)\}$*, wage* w*, the law of motion for the distribution,* $S' = \Omega(z, S)$*, and the employed consumer's decision rule* $\tilde{\psi}_e^{z'}(w, a; z, S)$*, the firm solves the optimization problem (24) with (25), with the value functions* $\tilde{J}(w, a; z, S)$*. Given the wage function* $\omega(a; z, S)$*,* $J(a; z, S)$ *satisfies (26). Given the* *aggregate states, $\{z, S\}$, the worker-finding rate $\lambda_f(z, S)$, prices $Q_g(z, S)$ and $Q_b(z, S)$, and the unemployed consumer's decision rule $\psi_u^{z'}(a; z, S)$, $V(z, S)$ satisfies (23).*

3. *Free entry:*

   The number of vacancy posted, $v(z, S)$, is consistent with the firm free-entry: $V(z, S) = 0$.

4. *Asset markets clear:*

   The asset-market equilibrium condition (29) holds for each z' and the asset prices satisfy (17) and (18). Dividends $d(z, S)$ are given by (27). $\bar{k}$ satisfies $\tilde{k} = \bar{k}/(1-u)$, where $\tilde{k}$ satisfies the firm's first-order condition $r(z, S) = zF'(\tilde{k})$ for given $r(z, S)$.

5. *Matching:*

   $\lambda_f(z, S)$ and $\lambda_w(z, S)$ are functions of $v(z, S)$ and u as in Section 3.2.

6. *Nash bargaining:*

   The wage function $\omega(a; z, S)$ determined through Nash bargaining between the firms and the consumers by solving (28).

7. *Consistency:*

   The transition function $\Omega(z, S)$ is consistent with $\lambda_w(z, S)$, σ, and the consumer's decision rules.

I Resource balance/goods-market equilibrium

In this subsection, we show that the resource balance condition (goods-market equilibrium condition)

$$\bar{c} + \bar{k}' = (1-\delta)\bar{k} + zF(\tilde{k})(1-u) - \xi v + hu$$

holds, where

$$\tilde{k} = \frac{\bar{k}}{1-u},$$

$$\bar{c} \equiv \int c_e(a; z, S) f_e(a; S) da + \int c_u(a; z, S) f_u(a; S) da,$$

$$c_e(a;z,S) \equiv a + \omega(a;z,S) - Q_g(z,S)\psi_e^g(a;z,S) - Q_b(z,S)\psi_e^b(a;z,S), \tag{42}$$

and

$$c_u(a;z,S) \equiv a + h - Q_g(z,S)\psi_u^g(a;z,S) - Q_b(z,S)\psi_u^b(a;z,S). \tag{43}$$

Note that the asset-market equilibrium condition (29) holds.

The first step is to show that

$$\int af_e(a;S)da + \int af_u(a;S)da = (1-\delta+r(z,S))\bar{k} + p(z,S) + d(z,S) \tag{44}$$

is implied by the asset-market equilibrium in the last period and the law of motion for the individual states.

Assuming that the decision rules for a' are increasing in a, the law of motion for the asset distribution is as follows:

$$\int_{\underline{a}}^{a'} f_e(\tilde{a};S')d\tilde{a} = \lambda_w \int_{\underline{a}}^{(\psi_u^{z'})^{-1}(a';z,S)} f_u(a;S)da + (1-\sigma)\int_{\underline{a}}^{(\psi_e^{z'})^{-1}(a';z,S)} f_e(a;S)da \tag{45}$$

$$\int_{\underline{a}}^{a'} f_u(\tilde{a};S')d\tilde{a} = (1-\lambda_w)\int_{\underline{a}}^{(\psi_u^{z'})^{-1}(a';z,S)} f_u(a;S)da + \sigma\int_{\underline{a}}^{(\psi_e^{z'})^{-1}(a';z,S)} f_e(a;S)da.$$

Here, $(\psi_u^{z'})^{-1}(a';z,S)$ denotes the value of a that satisfies $a' = \psi_u^{z'}(a;z,S)$.

To derive (44), we use a one-period-forwarded version:

$$\int a'f_e(a';S')da' + \int a'f_u(a';S')da' = (1-\delta+r(z',S'))\bar{k}' + p(z',S') + d(z',S').$$

From (29), we need to show that

$$\int a'f_e(a';S')da' + \int a'f_u(a';S')da' = \int \psi_e^{z'}(a;z,S)f_e(a;S)da + \int \psi_u^{z'}(a;z,S)f_u(a;S)da. \tag{46}$$

Differentiating (45) with respect to a',

$$f_e(a';S') = \lambda_w f_u((\psi_u^{z'})^{-1}(a';z,S);S)\rho_e(a';z,S) + (1-\sigma)f_e((\psi_e^{z'})^{-1}(a';z,S);S)\rho_u(a';z,S), \tag{47}$$

where

$$\rho_u(a';z,S) = \frac{d(\psi_u^{z'})^{-1}(a';z,S)}{da'} = \frac{1}{(\psi_u^{z'})'((\psi_u^{z'})^{-1}(a';z,S);z,S)} \tag{48}$$

and

$$\rho_e(a'; z, S) = \frac{d(\psi_e^{z'})^{-1}(a'; z, S)}{da'} = \frac{1}{(\psi_e^{z'})'((\psi_e^{z'})^{-1}(a'; z, S); z, S)}.$$

Now, multiply both sides of (47) with a' and integrate to obtain

$$\begin{aligned}&\textstyle\int a' f_e(a'; S')da' \\ &\quad = \lambda_w \textstyle\int a' f_u((\psi_u^{z'})^{-1}(a'; z, S); S)\rho_e(a'; z, S)da' \\ &\qquad +(1-\sigma) \textstyle\int a' f_e((\psi_e^{z'})^{-1}(a'; z, S); S)\rho_u(a'; z, S)da'.\end{aligned} \tag{49}$$

Changing variables by using $a' = \psi_u^{z'}(a; z, S)$ (implying $a = (\psi_u^{z'})^{-1}(a'; z, S)$), the first term on the right-hand side becomes

$$\lambda_w \int \psi_u^{z'}(a; z, S) f_u(a; S)\rho_e(\psi_u^{z'}(a; z, S); z, S)(\psi_u^{z'})'(a; z, S)da,$$

where $(\psi_u^{z'})'(a; z, S)$ is a Jacobian. From (48), this is equal to

$$\lambda_w \int \psi_u^{z'}(a; z, S) f_u(a; S)da.$$

Similarly, the second term on the right-hand side of (49) is equal to

$$(1-\sigma) \int \psi_e^{z'}(a; z, S) f_e(a; S)da.$$

Therefore, (49) becomes

$$\int a' f_e(a'; S')da' = \lambda_w \int \psi_u^{z'}(a; z, S) f_u(a; S)da + (1-\sigma) \int \psi_e^{z'}(a; z, S) f_e(a; S)da.$$

Similarly, we can show that

$$\int a' f_u(a'; S')da' = (1-\lambda_w) \int \psi_u^{z'}(a; z, S) f_u(a; S)da + \sigma \int \psi_e^{z'}(a; z, S) f_e(a; S)da.$$

Summing up, we obtain (46).

Next, integrating (42) and (43) for everyone gives us

$$\begin{aligned}&-\bar{c} + \textstyle\int a f_e(a; S)da + \int a f_u(a; S)da + \int \omega(a; z, S) f_e(a; S)da + hu \\ &\quad = \textstyle\int Q_g(z, S)\psi_e^g(a; z, S) f_e(a; S)da + \int Q_b(z, S)\psi_e^b(a; z, S) f_e(a; S)da \\ &\qquad + \textstyle\int Q_g(z, S)\psi_u^g(a; z, S) f_u(a; S)da + \int Q_b(z, S)\psi_u^b(a; z, S) f_u(a; S)da.\end{aligned}$$

From (44), the left-hand side of this expression is equal to

$$-\bar{c} + (1 - \delta + r(z,S))\bar{k} + p(z,S) + d(z,S) + \int \omega(a;z,S) f_e(a;S) da + hu. \tag{50}$$

In equilibrium, there are $1-u$ jobs in the economy and each job employs $\tilde{k} = \bar{k}/(1-u)$ units of capital. Thus, (25) becomes

$$\pi(a;z,S) = zF(\tilde{k}) - r(z,S)\tilde{k} - \omega(a;z,S).$$

From (27) and $\int f_e(a;S)da = (1-u)$, (50) is equal to

$$-\bar{c} + (1-\delta)\bar{k} + p(z,S) + zF(\tilde{k})(1-u) - \xi v + hu.$$

Therefore, we have to show that

$$\begin{aligned}
&\bar{k}' + p(z,S) \\
&\quad = \int Q_g(z,S)\psi_e^g(a;z,S) f_e(a;S)da + \int Q_b(z,S)\psi_e^b(a;z,S) f_e(a;S)da \\
&\qquad + \int Q_g(z,S)\psi_u^g(a;z,S) f_u(a;S)da + \int Q_b(z,S)\psi_u^b(a;z,S) f_u(a;S)da.
\end{aligned}$$

From (29), the right-hand side is equal to

$$Q_g(z,S)[(1-\delta+r(g,S'))\bar{k}' + p(g,S') + d(g,S')] + Q_b(z,S)[(1-\delta+r(b,S'))\bar{k}' + p(b,S') + d(b,S')].$$

From the asset-pricing equations (17) and (18), this is equal to $\bar{k}' + p(z,S)$.

J Solution of the model with aggregate shocks

Since we have many state variables, we use a relatively small number of grid points: 60 points in a direction for the value functions, 15 points in a direction for the wage function, 4 points in the $\bar{k}$ as well as the u direction. For a, we use more grids close to 0 to accommodate more curvature. We use cubic splines in a direction and linear interpolation in other directions.

1. Assume a law of motion for aggregate capital,

$$\log \bar{k}' = a_0 + a_1 \log \bar{k} + a_2 \log u + a_3 \log z, \tag{51}$$

prediction rules for the current aggregate variables as functions of the aggregate state,

$$\log\theta = b_0 + b_1 \log\bar{k} + b_2 \log u + b_3 \log z, \tag{52}$$

where we note that u' can be calculated once θ is given as

$$u' = (1 - \lambda_w(\theta))u + \sigma(1 - u), \tag{53}$$

and asset-price functions,

$$\log(p(z,\bar{k},u) + d(z,\bar{k},u)) = c_0 + c_1 \log\bar{k} + c_2 \log u + c_3 \log z \tag{54}$$

$$\log Q_z(z,\bar{k},u) = \begin{cases} d_0 + d_1 \log\tilde{Q}_z(z,\bar{k},u) + d_2\log\bar{k} + d_3\log u & \text{if } z = g \\ e_0 + e_1 \log\tilde{Q}_z(z,\bar{k},u) + e_2\log\bar{k} + e_3\log u & \text{if } z = b, \end{cases} \tag{55}$$

where $\tilde{Q}_z(z,\bar{k},u) \equiv \pi_{zz}/(1-\delta+r(z,\bar{k}',u'))$ (note that $\bar{k}'$ and u' are obtained as functions of z, $\bar{k}$, and u by the above equations). $\tilde{Q}_z$ is the exact value of Q_z when $g = b$. We expect Q_z not to be too different from $\tilde{Q}_z$ when shocks are not too large. When $z = g$, we can calculate Q_b using

$$Q_g(z,\bar{k},u)(1-\delta+r(g,\bar{k}',u')) + Q_b(z,\bar{k},u)(1-\delta+r(b,\bar{k}',u')) = 1. \tag{56}$$

When $z = b$, this can be used to calculate Q_g, given Q_b. In total, we have 20 coefficients to iterate on.

2. Start the loop on individual optimization and Nash bargaining.

   (a) Outside loop: assume an initial wage function $\omega(a;z,\bar{k},u)$ for each aggregate grid point $(z,\bar{k},u)$.

   (b) Give the initial values for the value functions, $W(a;z,\bar{k},u)$ and $U(a;z,\bar{k},u)$.

   (c) Inside loop: for each value (grid point) of a, $\bar{k}$, u, z, perform the worker's individual optimization.

   (d) Repeat until $W(a;z,\bar{k},u)$ and $U(a;z,\bar{k},u)$ converge.

   (e) Calculate $J(a;z,\bar{k},u)$ by using the worker's decision rule and noting that $V(a;z,\bar{k},u) = 0$.

(f) Based on the above functions, we calculate $\tilde{W}(w, a; z, \bar{k}, u)$ and $\tilde{J}(w, a; z, \bar{k}, u)$, and perform Nash bargaining for each aggregate state. Thus, the Nash bargaining delivers the wage function $\omega(a; z, \bar{k}, u)$ based on $\tilde{W}(w, a; z, \bar{k}, u)$, $U(a; z, \bar{k}, u)$, $\tilde{J}(w, a; z, \bar{k}, u)$ and $V(a; z, \bar{k}, u)(= 0)$.

(g) Revise the wage function $\omega(a; z, \bar{k}, u)$ by taking a weighted average of the original wage function and the new wage function obtained in the previous step. Repeat until convergence.

Note that with aggregate shocks, the wage depends not only on a but also on the aggregate state, $(z, \bar{k}, u)$. The wage function, $\omega(a; z, \bar{k}, u)$, is defined on $15 \times 2 \times 4 \times 4$ grid points. (The Nash bargaining is performed at each of these points.) For each $(z, \bar{k}, u)$, the wage function (as a function of a) is interpolated using the cubic splines between grid points. In the simulation (next step), we need to compute the wages for $(\bar{k}, u)$ that are not on the grid. In that case, we first compute a wage function $\omega_{z^*\bar{k}^*u^*}(a)$ for a specific $(z^*, \bar{k}^*, u^*)$ by linearly interpolating in $\bar{k}$ and u directions. Then $\omega_{z^*\bar{k}^*u^*}(a)$ is used to compute the wage at each a (interpolated by cubic spline between a grids).

3. Simulation.

(a) Give initial values for a, employment status, $\bar{k}$, u, and z. (Note that the sum of a is equal to $[(1 - \delta + r)\bar{k} + p + d]$, so once we have $\bar{k}$, u, and z, we know the sum of a.)

(b) Using the condition that

$$\begin{aligned} V(z, S) = -\xi \quad &+Q_g(z, S)((1 - \lambda_f(\theta))V(g, S') + \lambda_f(\theta) \textstyle\int J(\psi_u^g(a; z, S); g, S')[f_u(a; S)/u]da) \\ &+Q_b(z, S)((1 - \lambda_f(\theta))V(b, S') + \lambda_f(\theta) \textstyle\int J(\psi_u^b(a; z, S); b, S')[f_u(a; S)/u]da) \end{aligned}$$

must equal zero (using the prediction rules (51), (52), and (53) for $\bar{k}'$ and u' in evaluating the future value function), we can calculate the equilibrium value of θ. Then we know $v = u\theta$. Record this value of θ as “data” (for later use).

(c) Calculate u' using the computed value of θ and (53). Note that this u' may not be the same as the u' predicted using (53), if the prediction rules are incorrect. Record this u' as data.

(d) Calculate the sums of a'_gs and a'_bs from the consumer's decisions. Call them A'_g and A'_b. From asset-market equilibrium,

$$A'_g = (1-\delta + r(g,\bar{k}',u'))\bar{k}' + p(g,\bar{k}',u') + d(g,\bar{k}',u') \tag{57}$$

and

$$A'_b = (1-\delta + r(b,\bar{k}',u'))\bar{k}' + p(b,\bar{k}',u') + d(b,\bar{k}',u') \tag{58}$$

have to hold. These may not hold if the consumers' prediction rules are incorrect. Here, we search for the values of Q_g, Q_b, and $\bar{k}'$ so that these two equations and (56) hold. To this end, consider the case of $z = g$. (When $z = b$, g and b are reversed everywhere.) The main idea here follows Krusell and Smith (1997).

Note that (57) can be rewritten as

$$\bar{k}' = \frac{A'_g - p(g,\bar{k}',u') - d(g,\bar{k}',u')}{1-\delta + r(g,\bar{k}',u')} \tag{59}$$

and that (58) can be rewritten as

$$\bar{k}' = \frac{A'_b - p(b,\bar{k}',u') - d(b,\bar{k}',u')}{1-\delta + r(b,\bar{k}',u')}.$$

Therefore,

$$\frac{A'_g - p(g,\bar{k}',u') - d(g,\bar{k}',u')}{1-\delta + r(g,\bar{k}',u')} = \frac{A'_b - p(b,\bar{k}',u') - d(b,\bar{k}',u')}{1-\delta + r(b,\bar{k}',u')} \tag{60}$$

holds. We will search for a Q_g that satisfies (60); we can expect that A'_g is decreasing in Q_g and A'_b is decreasing in Q_b.[41] Note that Q_b can be calculated as a (decreasing) function of Q_g, from (56). To calculate A'_g and A'_b for each Q_g, we re-calculate the optimization problem for a given Q_g: for employed consumers

$$\begin{aligned}\hat{W}(Q_g, a; z, \bar{k}, u) = \max_{a'_g, a'_b} \quad u(c) + \beta[\quad & \pi_{zg}(\sigma U(a'_g; g, \bar{k}', u') + (1-\sigma)W(a'_g; g, \bar{k}', u')) \\ & +\pi_{zb}(\sigma U(a'_b; b, \bar{k}', u') + (1-\sigma)W(a'_b; b, \bar{k}', u'))]\end{aligned}$$

[41] We use (51), (52), (53), (54), and $r = zF'(\tilde{k})$ to calculate $p(z',\bar{k}',u') + d(z',\bar{k}',u')$ and $r(z',\bar{k}',u')$. Thus, they are not functions of our unknowns.

subject to

$$c + Q_g a'_g + Q_b(Q_g) a'_b = a + \omega(a; z, \bar{k}, u),$$

$$a'_g \geq \underline{a},$$

$$a'_b \geq \underline{a},$$

and $\bar{k}'$ and u' given, and for the unemployed

$$\begin{aligned}\hat{U}(Q_g, a; z, \bar{k}, u) = \max_{a'_g, a'_b} \quad u(c) + \beta[\quad & \pi_{zg}((1-\lambda_w)U(a'_g; g, \bar{k}', u') + \lambda_w W(a'_g; g, \bar{k}', u')) \\ & + \pi_{zb}((1-\lambda_w)U(a'_b; b, \bar{k}', u') + \lambda_w W(a'_b; b, \bar{k}', u'))]\end{aligned}$$

subject to

$$c + Q_g a'_g + Q_b(Q_g) a'_b = a + h,$$

$$a'_g \geq \underline{a},$$

$$a'_b \geq \underline{a},$$

and $\bar{k}'$ and u' given.

Now calculate A'_g and A'_b with this method for different values of Q_g until we find a Q_g that makes (60) hold with equality. If we are in a rational-expectations equilibrium, this Q_g has to equal the Q_g from the prediction rule (55).

Then we calculate $\bar{k}'$ from (59) and Q_b from (56). Record the values of Q_g, Q_b, and $\bar{k}'$ as data.

(e) From

$$d(z, \bar{k}, u) = \int \pi(a; z, \bar{k}, u) f_e(a; \bar{k}, u) da - \xi v,$$

where $\pi(a; z, \bar{k}, u) = \tilde{\pi}(\omega(a; z, \bar{k}, u); z, \bar{k}, u)$, we can obtain data on $d(z, \bar{k}, u)$. (We already know all the values determining the first term, and v was obtained in an earlier step.)

(f) From

$$p(z, \bar{k}, u) = Q_g(z, \bar{k}, u)[p(g, \bar{k}', u') + d(g, \bar{k}', u')] + Q_b(z, \bar{k}, u)[p(b, \bar{k}', u') + d(b, \bar{k}', u')],$$

we can obtain data on $p(z, \bar{k}, u)$. Here, Q_g and Q_b were obtained in an earlier step, and for $p(z', \bar{k}', u') + d(z', \bar{k}', u')$, we use the prediction rules (51), (52), (53), and (54).

(g) With a random number generator, obtain z' and move to the next period. The individual's employment and asset status are also forwarded to the next period. The individual asset holdings are represented by a density function.[42] Repeat from step (b) for N periods. Discard the first n periods from the sample. We set $N = 2000$ and $n = 500$ in our program.

(h) Using all the above data ($\bar{k}$, z, u, θ, Q_z, p, d), we can revise the laws of motion, labor-market tightness functions, and pricing functions by running ordinary least squares regressions.

(i) Repeat until the prediction rules (the laws of motion, labor market tightness functions, and pricing functions) predict the simulated data with sufficient accuracy (high R^2). As stated in the main text, we find prediction rules that are very accurate. For the Shimer calibration, all R^2s are larger than 0.9999. For the HM calibration, all R^2s are larger than 0.999. R^2 here is defined as

$$R^2 \equiv 1 - \frac{\sum_{t=501}^{2000}(m_t - m_t^p)^2}{\sum_{t=501}^{2000}(m_t - \bar{m})^2},$$

where m_t is the simulated value of a variable, m_t^p is the predicted value of m_t using the prediction rule (law of motion) that the consumers used in the optimization step and the simulated values of the right-hand-side variables, and $\bar{m}$ is the average of m_t.

K Laws of motion and prediction rules

K.1 The Shimer calibration

The law of motion for capital stock is

$$\log \bar{k}' = 0.06851 + 0.9823 \log \bar{k} - 0.0022 \log u + 0.0451 \log z, \quad R^2 = 0.99999.$$

[42] The period-0 density is given exogenously, but we discard a sufficient number of initial periods from the sample in the regressions below in order to remove the influence of the initial distribution. For the Shimer calibration, we used a uniform distribution on $[0, 2\bar{k}]$. For the HM calibration, since the distribution moves slowly, we used the stationary distribution from the no-aggregate-shocks model as the initial distribution. In both cases, we checked that the results are not sensitive to the choice of the initial distribution.

The prediction rules for the other aggregate variables are

$$\log\theta = -2.1080 + 0.5071\log\bar{k} + 0.0079\log u + 1.3912\log z, \quad R^2 = 0.99999,$$

$$\log(p+d) = -1.8735 + 0.3902\log\bar{k} - 0.0544\log u + 1.0495\log z, \quad R^2 = 0.99969,$$

$$\log Q_g = -0.5830 - 4.1619\log\tilde{Q}_g + 0.0538\log\bar{k} + 0.0015\log u, \quad R^2 = 0.99994,$$

$$\log Q_b = 0.5138 + 5.6423\log\tilde{Q}_b - 0.0465\log\bar{k} - 0.0013\log u, \quad R^2 = 0.99994,$$

where $\tilde{Q}_g$ and $\tilde{Q}_b$ are functions of z, $\bar{k}$, and u (see Appendix J). Thus, almost all the variation in the left-hand-side variables $(\theta, p+d, Q_g, Q_b)$ can be explained by the predicted value in the right-hand side.

K.2 The HM calibration

The law of motion for capital stock is

$$\log\bar{k}' = 0.0847 + 0.9785\log\bar{k} - 0.0021\log u + 0.0327\log z, \quad R^2 = 0.99999.$$

The prediction rules for the other aggregate variables are

$$\begin{aligned}\log\theta \quad = \quad & -159.9317 + 73.4443\log\bar{k} - 2.7952\log u + 197.4511\log z \\ & -8.4340(\log k)^2 + 0.0043(\log u)^2 - 45.5953\log k\log u \\ & -1.1617\log u\log z - 0.6528\log k\log u, \quad R^2 = 0.99999,\end{aligned}$$

$$\log(p+d) = -4.9175 + 1.3456\log\bar{k} - 0.0549\log u + 3.2854\log z, \quad R^2 = 0.99997,$$

$$\begin{aligned}\log Q_g \quad = \quad & 0.3631 + 1.7347\log\tilde{Q}_g - 0.1415\log\bar{k} - 0.0008\log u \\ & +0.0160(\log k)^2 - 0.0001(\log u)^2 + 0.0000\log k\log u, \quad R^2 = 0.99992,\end{aligned}$$

and

$$\begin{aligned}\log Q_b \quad = \quad & -0.1182 - 0.1234\log\tilde{Q}_b + 0.0100\log\bar{k} + 0.0004\log u \\ & +0.000002(\log k)^2 + 0.0000(\log u)^2 + 0.000004\log k\log u, \quad R^2 = 0.99914,\end{aligned}$$

where $\tilde{Q}_g$ and $\tilde{Q}_b$ are functions of z, $\bar{k}$, and u.

L Accuracy of the computational algorithm

The agents in our model use a linear law of motion to forecast next period's capital stock ($\bar{k}'$). Similarly, they use linear (or quadratic) prediction rules to predict the other aggregate variables ($\theta, (p+d), Q_g$ and Q_b) for the current period. To evaluate the agents' forecasting and prediction abilities we compute the errors that they make in forecasts 1 period ahead and 25 years (200 periods) ahead.

We assume that the agents in the economy have the knowledge of the the current period's capital stock and the unemployment rate. By only using this information and next period's technology shock, agents can predict aggregate variables 1 period ahead by using the linear prediction rules. We start from period 501 of our simulation and compute the capital stock, interest rate, and unemployment rate 1 period ahead and the current period's ($\theta, (p+d), Q_g$ and Q_b). Then we compare these predicted values with values that we observe from the simulations. We report two measures of accuracy: the correlation between the values implied by the linear rules and the simulations and the maximum percentage deviation from the value implied by the simulation. We also report the forecasting and prediction errors for 25-years-ahead forecasts.

	1 period ahead		25 years ahead	
Variable	$corr(x, \hat{x})$	max % error	$corr(x, \hat{x})$	max % error
$\bar{k}'$	1.000000	0.0052	0.999947	0.0474
θ	0.999992	0.0335	0.999986	0.0461
$p+d$	0.999998	0.0474	0.999995	0.0545
Q_g	1.000000	0.0058	1.000000	0.1061
Q_b	1.000000	0.0060	1.000000	0.1125
r	1.000000	0.0036	0.999984	0.0325
u'	0.999996	0.0059	0.999984	0.0121

Table 17: Forecasting and prediction accuracy for the Shimer calibration where x is the value of the variable from the simulations and $\hat{x}$ is the predicted value.

	1 period ahead		25 years ahead	
Variable	$corr(x, \hat{x})$	max % error	$corr(x, \hat{x})$	max % error
$\bar{k}'$	1.000000	0.0064	0.999968	0.0605
θ	0.999990	0.5906	0.999985	0.5971
$p + d$	0.999992	0.2235	0.999989	0.1918
Q_g	1.000000	0.0296	1.000000	0.2355
Q_b	1.000000	0.0243	1.000000	0.1645
r	1.000000	0.0109	0.999995	0.0373
u'	0.999996	0.1946	0.999991	0.2784

Table 18: Forecasting and prediction accuracy for the HM calibration where x is the value of the variable from the simulations and $\hat{x}$ is the predicted value.

M Data for the U.S. economy and calculation of the cyclical statistics of the model

Output (data): We compute the logarithm of quarterly real GDP per capita and detrend it by using an HP filter. The smoothing parameter that we use is 1600. The standard deviation of the cyclical component is 0.0158.

Stock prices and dividends (data): We compute the standard deviation of the cyclical component of the stock market prices and dividends for 1951-2004 period. This data set is constructed by Robert Shiller. See `http://www.irrationalexuberance.com/index.htm`. We use monthly data for stock prices and dividends which are normalized by CPI. We adjust the monthly data to quarterly frequency. For stock prices we select the monthly value for the 3rd, 6th, 9th, and 12th month of each year. For the dividends, we sum up the monthly flows for three months. The standard deviation of the cyclical component of the natural logarithm of stock prices is 0.1012 and for dividends it is 0.0286. Dividends fluctuate far less than stock prices. The finding that a typical economic model produces much less fluctuation in the stock prices (compared to the dividend fluctuations) has been labeled the “stock-market volatility puzzle” in the literature (Shiller (1981)).

Vacancy-unemployment ratio (data): The vacancy-unemployment ratio is constructed by calculating the ratio of the Help Wanted Advertising Index to the rate of unemployment, measured in index units per thousand workers. This data set was constructed by Robert Shimer. See `http://home.uchicago.edu/ shimer/data/mmm/`. The data are quarterly and span the period of 1951 to 2005. First we detrend the natural logarithm of the vacancy-unemployment ratio and then compute the standard deviation of the cyclical component of the series.

Investment, consumption and wage (data): We report these statistics from Andolfatto (1996).

Labor share (data): Ríos-Rull and Santaeulàlia-Llopis (2007) find that the standard deviation of the labor share is 43% of that of output. Andolfatto (1996) reports this number to be 68% of that of output.

Model: We simulate our model for 2000 periods, discard the first 500 periods, and then adjust the generated data to a quarterly frequency. We detrend the series by using an HP filter with a smoothing parameter of 1600 for output, investment, consumption, the average wage, the labor share, the stock price, the dividend, and the vacancy-unemployment ratio. The values of these variables at time t are calculated using the simulated data as follows: output is calculated as $z_t\bar{k}_t^{\alpha}(1-u_t)^{1-\alpha}-\xi v_t$, investment is $\bar{k}_{t+1}-(1-\delta)\bar{k}_t$, consumption is $z_t\bar{k}_t^{\alpha}(1-u_t)^{1-\alpha}-\bar{k}_{t+1}+(1-\delta)\bar{k}_t-\xi v_t+hu_t$, the average wage is the average wage of the employed agents in the economy at time t, the labor share is the average wage divided by $z_t(\bar{k}_t/(1-u_t))^{\alpha}$, and the vacancy-unemployment ratio is v_t/u_t.

Tables 19 and 20 report additional statistics from our model. We consider two different measures of the stock price either p or $p+\bar{k}$, and two different measure of dividends either d or $d+(1+r-\delta)\bar{k}-\bar{k}')$.

	U.S. Economy	Model Shimer	Model HM
Stock price (p)	6.41	0.98	2.88
Stock price ($p+\bar{k}$)	6.41	0.27	0.25
Dividend (d)	1.81	31.24	82.57
Dividend ($d+(1+r-\delta)\bar{k}-\bar{k}'$)	1.81	5.59	1.87

Table 19: Standard deviation of detrended series divided by the standard deviation of output. All variables are logged and HP-filtered. Note that standard deviation of output is 0.0158 for the U.S. data, 0.0138 for the Shimer calibration and 0.0159 for the HM calibration.

	U.S. Economy	Model Shimer	Model HM
Stock price (p)	0.34	0.95	0.89
Stock price ($p + \bar{k}$)	0.34	0.24	0.57
Dividend (d)	0.36	0.99	0.45
Dividend ($d + (1 + r - \delta)\bar{k} - \bar{k}'$)	0.36	-0.98	-0.83

Table 20: Correlation with output. All variables are logged and HP-filtered.

N The linear model with aggregate shocks

We consider the "large family" construction as in Merz (1995). The consumer ("family") maximizes

$$E\left[\sum_{t=0}^{\infty} \beta^t c_t\right]$$

subject to

$$c_t + \bar{k}_{t+1} = (1 + r_t - \delta)\bar{k}_t + w_t(1 - u_t) + hu_t + d_t.$$

From the first-order condition for capital accumulation (Euler equation),

$$E_t[(1 + r_{t+1} - \delta)] = \frac{1}{\beta} \tag{61}$$

holds.

The net output per match, y_t, is defined as

$$y_t = \arg\max_{\tilde{k}_t} \; z_t \tilde{k}_t^{\alpha} - r_t \tilde{k}_t, \tag{62}$$

where $\tilde{k}_t$ is the capital-output ratio ($\bar{k}_t/(1 - u_t)$) at time t. From the first-order condition,

$$r_t = \alpha z_t \tilde{k}_t^{\alpha - 1} \tag{63}$$

holds. Using this and (61), $\tilde{k}_{t+1}$, is solved as a function of z_t and solution to

$$\pi_{zg}\alpha[\tilde{k}'(z)]^{\alpha-1} + \pi_{zb}\alpha[\tilde{k}'(z)]^{\alpha-1} = \frac{1}{\beta} + \delta - 1,$$

where $\tilde{k}_{t+1}$ is now denoted as $\tilde{k}'(z)$. Solving this, we obtain

$$\tilde{k}'(z) = \left(\frac{1/\beta + \delta - 1}{\pi_{zg}\alpha g + \pi_{zb}\alpha b}\right)^{\frac{1}{\alpha-1}}.$$

From (63), the interest rate can be solved as:

$$r(z_{-1}, z) = \alpha z[\tilde{k}'(z_{-1})]^{\alpha-1},$$

where z_{-1} is the value of z_{t-1}. From (62), the net output per match is:

$$y(z_{-1}, z) = z[\tilde{k}'(z_{-1})]^{\alpha} - r(z_{-1}, z)\tilde{k}'(z_{-1}).$$

It will turn out (verified later) that w_t is a function of z_{t-1} and z_t. Thus we denote it as $w(z_{-1}, z)$. On the firm side, the value of a filled job is

$$J(z_{-1}, z) = y(z_{-1}, z) - w(z_{-1}, z) + \beta E[\sigma V(z') + (1-\sigma)J(z, z')|z].$$

The value of a vacancy is

$$V(z) = -\xi + \beta E[\lambda_f J(z, z') + (1-\lambda_f)V(z, z')|z]. \tag{64}$$

Note that V depends only on z. From free entry, $V(z) = 0$. This condition determines θ, and thus θ is a function of z: $\theta(z)$. Thus λ_f and λ_w are also a function of z.

The value functions for the consumers become

$$W(z_{-1}, z) = w(z_{-1}, z) + \beta E[\sigma U(z, z') + (1-\sigma)W(z, z')|z]$$

and

$$U(z_{-1}, z) = h + \beta E[(1-\lambda_w)U(z, z') + \lambda_w W(z, z')|z].$$

Since $W(z_{-1}, z) - U(z_{-1}, z)$ and $J(z_{-1}, z) - V(z)$ are linear in $w(z_{-1}, z)$, the Nash bargaining solution results in the simple surplus-sharing rules

$$W(z_{-1}, z) - U(z_{-1}, z) = \gamma S(z_{-1}, z)$$

and

$$J(z_{-1}, z) - V(z) = (1-\gamma)S(z_{-1}, z), \tag{65}$$

where

$$S(z_{-1}, z) = (W(z_{-1}, z) - U(z_{-1}, z)) + (J(z_{-1}, z) - V(z)) \tag{66}$$

is the total surplus. Thus, w is indeed a function of z_{-1} and z.

From (64), (65), and the free-entry condition $V(z) = 0$,

$$\xi = \beta[\pi_{zg}\lambda_f(z)(1-\gamma)S(z,g) + \pi_{zb}\lambda_f(z)(1-\gamma)S(z,b)]. \tag{67}$$

There are two equations (for each z) here, which determine $\theta(z)$ given $S(z,z')$. From (66) and the value functions,

$$S(z_{-1},z) = y(z_{-1},z) - h + \beta(\pi_{zg}S(z,g) + \pi_{zb}S(z,g))(1-\sigma-\gamma\lambda_w(z)). \tag{68}$$

There are four equations here (for each z_{-1} and z). Recall that

$$\lambda_f(z) = \chi\theta(z)^{-\eta}$$

and

$$\lambda_w(z) = \chi\theta(z)^{1-\eta}.$$

Thus, (67) and (68) can be solved for six unknowns: $S(z_{-1},z)$ and $\theta(z)$.

Once these are found, we can calculate the wage as

$$w(z_{-1},z) = y(z_{-1},z) - (1-\gamma)S(z_{-1},z) + \beta(1-\sigma)(\pi_{zg}(1-\gamma)S(z,g) + \pi_{zb}(1-\gamma)S(z,b)).$$

Unemployment follows

$$u' = u + \sigma(1-u) - \lambda_w(z)u.$$

Vacancies can be calculated as

$$v = \theta(z)u.$$

Since the sum of $J(z_{-1},z)$s across firms, $(1-u)J(z_{-1},z)$, is equal to the stock price before the dividend payment, $p+d$, we have

$$p = (1-u)J(z_{-1},z) - d = (1-u)(1-\gamma)S(z_{-1},z) - d.$$

Here, d is the sum of profits across firms:

$$d = (1-u)(y(z_{-1},z) - w(z_{-1},z)) - \xi v.$$

The capital stock, $\bar{k}_t$, is

$$\bar{k}_t = (1 - u_t)\tilde{k}_t(z_{t-1}).$$

GDP, Y_t, is $(1 - u_t)y_t(z_{t-1}, z_t) - \xi v_t$, investment is $\bar{k}_{t+1} - (1 - \delta)\bar{k}_t$, and consumption is

$$c_t = Y_t + (1 - \delta)\bar{k}_t - \bar{k}_{t+1}.$$

Tables 21 and 22 summarize the statistics for each state from our simulations. All the values are shown as percentage deviations of the average value in each state from the total average, except for u. We can see that all variables are more volatile than in our baseline model. Especially in the HM calibration, the volatility of labor market variables (u, v, θ) are more pronounced in the linear model. This is a result of the consumption smoothing process. Capital adjusts slowly in our baseline model where the adjustment is instantaneous in the linear model.

	u	v	θ	$\bar{k}$	p	d
$z = b$	7.78%	−3.2%	−4.1%	−3.0%	−3.0%	−82.9%
$z = g$	7.63%	3.1%	+3.9%	+2.9%	+2.9%	+80.0%

Table 21: Summary statistics of the simulated data (Shimer calibration).

	u	v	θ	$\bar{k}$	p	d
$z = b$	10.53%	−14.0%	−29.9%	−4.3%	−10.1%	−238.5%
$z = g$	7.42%	+15.7%	+33.7%	+4.8%	+11.4%	+268.2%

Table 22: Summary statistics of the simulated data (HM calibration).

O Complete-markets model with aggregate shocks

As in Merz (1995), we can think of the economy as consisting of many large families. Each family insures the workers from idiosyncratic shocks. There are many such families, so that each family takes the aggregate states $(z, \bar{k}, u)$ as given. The family's utility function is

$$E\left[\sum_{t=0}^{\infty} \beta^t \log(c_t)\right].$$

The optimization problem is given from

$$R(k,X) = \max_{c,k'} \quad \log(c) + \beta[\pi_{zg} R(k', X'_g) + \pi_{zb} R(k', X'_b)]$$

subject to

$$c + k' = (1 + r(X) - \delta)k + (1-u)w(X) + uh + d(X),$$

where $X \equiv (z, \bar{k}, u)$ is the aggregate state. k is the individual capital stock for the family. In equilibrium, $k = \bar{k}$ holds. X_g represents $(g, \bar{k}, u)$ and X_b represents $(b, \bar{k}, u)$. For the family, the vacancy-unemployment ratio $\theta(X)$, dividend $d(X)$, and the wage function $w(X)$ are given. Unemployment evolves following $u' = u + \sigma(1-u) - \lambda_w(\theta)u$. The interest rate $r(X)$ is given from the firm's optimization as

$$r(X) = \alpha z \left(\frac{\bar{k}}{1-u}\right)^{\alpha-1}.$$

Thus, given $w(X)$, $d(X)$, and $\theta(X)$, this optimization can be carried out. Note that it will turn out that only aggregate state variables appear in the Nash bargaining, so that w is only a function of X (changing k does not affect wage).

This optimization will result in the individual decision rules $k' = \tilde{\kappa}(k, X)$ and $c = \tilde{\zeta}(k, X)$. The equilibrium values for $\bar{k}'$ and c are given by $\bar{k}' = \kappa(X) = \tilde{\kappa}(\bar{k}, X)$ and $c = \zeta(X) = \tilde{\zeta}(\bar{k}, X)$. A one-period Arrow security which gives one unit of consumption goods conditional on z' (note that $\bar{k}'$ and u' are predetermined) can be priced as

$$Q_{z'}(X) = \beta \pi_{zz'} \frac{\zeta(X)}{\zeta(X')}.$$

The matched workers and the unemployed workers can be viewed as "assets" from the viewpoint of the family. The former generate $w(X)$ every period and the latter generate h per period. Thus, the value of these assets, respectively, are

$$W(X) = w(X) + Q_g(X)[\sigma U(X'_g) + (1-\sigma)W(X'_g)] + Q_b(X)[\sigma U(X'_b) + (1-\sigma)W(X'_b)]$$

and

$$U(X) = h + Q_g(X)[(1-\lambda_w(X))U(X'_g) + \lambda_w(X)W(X'_g)] + Q_b(X)[(1-\lambda_w(X))U(X'_b) + \lambda_w(X)W(X'_b)].$$

The value of a filled job is

$$J(X) = y(X) - w(X) + Q_g(X)[\sigma V(X'_g) + (1-\sigma)J(X'_g)] + Q_b(X)[\sigma V(X'_b) + (1-\sigma)J(X'_b)]$$

and the value of a vacancy is

$$V(X) = -\xi + Q_g(X)[\lambda_f(X)J(X'_g) + (1-\lambda_f(X))V(X'_g)] + Q_b(X)[\lambda_f(X)J(X'_b) + (1-\lambda_f(X))V(X'_b)].$$

From free entry, $V(X) = 0$. This condition determines $\theta(X)$.

The surplus per match, $y(X)$, can be calculated by

$$y(X) = z\left(\frac{\bar{k}}{1-u}\right)^{\alpha} - r(X)\left(\frac{\bar{k}}{1-u}\right).$$

Since $W(X)-U(X)$ and $J(X)-V(X)$ are linear in $w(X)$, the Nash bargaining solution results in the simple surplus-sharing rules

$$W(X) - U(X) = \gamma S(X)$$

and

$$J(X) - V(X) = (1-\gamma)S(X),$$

where

$$S(X) = (W(X) - U(X)) + (J(X) - V(X)) \tag{69}$$

is the total surplus. Thus, w is indeed a function of X.

From (69) and the value functions, $S(X)$ can be computed using the mapping

$$S(X) = y(X) + \xi - h + (Q_g(X)S(X'_g) + Q_b(X)S(X'_b))(1 - \sigma - (1-\gamma)\lambda_f(X) - \gamma\lambda_w(X)).$$

This gives $J(X) = (1-\gamma)S(X)$.

Given $J(X)$ and $V(X) = 0$,

$$0 = -\xi + Q_g(X)\lambda_f(X)J(X'_g) + Q_b(X)\lambda_f(X)J(X'_b).$$

Thus

$$\lambda_f(X) = \frac{\xi}{Q_g(X)(1-\gamma)S(X'_g) + Q_b(X)(1-\gamma)S(X'_b)}$$

will solve for $\theta(X)$, since $\lambda_f(X) = \chi\theta(X)^{-\eta}$.

One can then calculate the wage from

$$w(X) = y(X) + \xi - (1-\gamma)S(X) + (1-\sigma-\lambda_f(X))(Q_g(X)(1-\gamma)S(X'_g) + Q_b(X)(1-\gamma)S(X'_b)),$$

and thus[43]

$$w(X) = y(X) + \xi - (1-\gamma)S(X) + \frac{(1-\sigma-\lambda_f(X))\xi}{\lambda_f(X)}.$$

Unemployment follows

$$u' = u + \sigma(1-u) - \lambda_w(X)u$$

and vacancies are given by

$$v = \theta(X)u.$$

Since the sum of $J(X)$, $(1-u)J(X)$, is equal to the stock price before the dividend payment, $p+d$,

$$p = (1-u)J(X) - d = (1-u)(1-\gamma)S(X) - d.$$

Here, d is the sum of profits across firms:

$$d(X) = (1-u)(y(X) - w(X)) - \xi v.$$

Tables 23, 24, 25, and 26 summarize the properties of the model. These are very similar to the incomplete-markets outcome reported in the text.

[43]This corresponds to Andolfatto's (1996) equation (23).

	u	v	θ	$\bar{k}$	p	d
$z = b$	7.75%	−2.6%	−3.3%	−0.4%	−2.5%	−50.0%
$z = g$	7.65%	+2.2%	+2.8%	+0.3%	+2.1%	+41.8%

Table 23: Summary statistics of the simulated data (Shimer calibration).

	u	v	θ	$\bar{k}$	p	d
$z = b$	8.79%	−10.2%	−20.6%	−0.4%	−7.6%	−132.2%
$z = g$	7.18%	+8.5%	+17.2%	+0.3%	+6.3%	+110.5%

Table 24: Summary statistics of the simulated data (HM calibration).

	U.S. economy	Complete market model: Shimer	Complete market model: HM
Investment	3.14	3.82	2.97
Consumption	0.56	0.30	0.20
Labor share	0.43	0.04	0.35
Wage	0.44	0.94	0.30
Vacancy-unemployment ratio	16.27	1.46	8.5

Table 25: Standard deviation of detrended series divided by the standard deviation of output. All variables are logged and HP-filtered. Note that standard deviation of output is 0.0158 for the U.S. data, 0.0138 for the Shimer calibration, and 0.0159 for the HM calibration.

	U.S. Economy	Model Shimer	Model HM
Investment	0.90	0.99	0.98
Consumption	0.74	0.93	0.92
Labor share	-0.13	-1.00	-0.98
Wage	0.04	1.00	0.96
Vacancy-unemployment ratio	0.90	1.00	0.93

Table 26: Correlation with output for the complete market model.

P Comparison of models

In this section, we provide a detailed comparison of our model with the linear and complete-market versions of the model. Note that in the linear model, consumption and investment are allowed to be negative. For that reason, to make the comparison with the linear model possible, we apply the H-P filter without taking the natural logarithm of the model-generated time series throughout this section.

Tables 27, 28, and 29 compare the three models for the Shimer calibration. These tables reveal that all three models are very similar in terms of the labor market outcomes. The vacancy-unemployment ratio is more volatile in the linear model. Because of consumption smoothing, capital stock moves slowly in the models with concave utility, making θ's movement smoother. As is expected, the linear model has much more volatile investment and consumption since the consumption smoothing motive is not present in the linear setting. The complete- and incomplete-markets models behave very similarly with respect to the labor market. However, the complete-markets model has less volatile investment and more volatile consumption in relative terms. In the incomplete-markets model, some consumers are not well insured. For these consumers, an increase in income does not necessarily lead to an immediate increase in consumption. This effect can also be seen in the relatively low correlation between output and consumption. In the linear model, output and investment are not strongly correlated, since investment jumps when the aggregate state changes and it takes one period for investment to have an effect. Output and consumption are not strongly correlated because when the aggregate state switches from bad to good, consumption falls initially because of the spike in investment.

Tables 30, 31, and 32 compare the three models for the HM calibration. The comparison of the three models is qualitatively very similar to the Shimer case. Quantitatively, the differences are more evident.

Incomplete	u	v	θ	$\bar{k}$	p	d
$z = b$	7.73%	−2.6%	−3.3%	−0.5%	−2.2%	−58.3%
$z = g$	7.64%	+2.1%	+2.7%	+0.4%	+1.9%	+48.7%
Linear	u	v	θ	$\bar{k}$	p	d
$z = b$	7.78%	−3.2%	−4.1%	−3.0%	−3.0%	−82.9%
$z = g$	7.63%	3.1%	+3.9%	+2.9%	+2.9%	+80.0%
Complete	u	v	θ	$\bar{k}$	p	d
$z = b$	7.75%	−2.6%	−3.3%	−0.4%	−2.5%	−50.0%
$z = g$	7.65%	+2.2%	+2.8%	+0.3%	+2.1%	+41.8%

Table 27: Comparison of models, Shimer calibration.

	Incomplete markets model	Complete markets model	Linear model
Investment	0.91	0.77	9.14
Consumption	0.15	0.25	9.05
Labor share	0.005	0.004	0.005
Wage	0.70	0.71	0.70
Vacancy-unemployment ratio	0.43	0.44	0.45

Table 28: Standard deviation of detrended series divided by the standard deviation of output for the Shimer calibration. All variables are HP-filtered. Note that standard deviation of output is 0.09 for the incomplete- and complete-markets models, and 0.13 for the linear model.

	Incomplete markets model	Complete markets model	Linear model
Investment	0.99	0.99	0.13
Consumption	0.62	0.93	-0.02
Labor share	-1.00	-1.00	-0.75
Wage	1.00	1.00	1.00
Vacancy-unemployment ratio	1.00	1.00	0.98

Table 29: Correlation with output for the Shimer calibration.

Incomplete	u	v	θ	$\bar{k}$	p	d
$z = b$	8.75%	-10.2%	-20.5%	-0.4%	-7.5%	-134.7%
$z = g$	7.17%	+8.5%	+17.1%	+0.4%	+6.3%	+112.6%
Linear	u	v	θ	$\bar{k}$	p	d
$z = b$	10.53%	-14.0%	-29.9%	-4.3%	-10.1%	-238.5%
$z = g$	7.42%	+15.7%	+33.7%	+4.8%	+11.4%	+268.2%
Complete	u	v	θ	$\bar{k}$	p	d
$z = b$	8.79%	-10.2%	-20.6%	-0.4%	-7.6%	-132.2%
$z = g$	7.18%	+8.5%	+17.2%	+0.3%	+6.3%	+110.5%

Table 30: Comparison of models, HM calibration.

	Incomplete markets model	Complete markets model	Linear model
Investment	0.75	0.61	9.54
Consumption	0.10	0.17	9.58
Labor share	0.04	0.04	0.03
Wage	0.22	0.22	0.20
Vacancy-unemployment ratio	1.82	1.83	1.72

Table 31: Standard deviation of detrended series divided by the standard deviation of output for the HM calibration. All variables are HP-filtered. Note that standard deviation of output is 0.10 for the incomplete- and complete-markets models, and 0.20 for the linear model.

	Incomplete markets model	Complete markets model	Linear model
Investment	0.98	0.98	−0.11
Consumption	0.37	0.92	0.17
Labor share	−0.98	−0.98	−0.99
Wage	0.96	0.96	0.89
Vacancy-unemployment ratio	0.93	0.93	0.82

Table 32: Correlation with output for the HM calibration.