Firm Dynamics and the Macroeconomy: Basics

Toshihiko Mukoyama
Georgetown University

University of Tokyo, July 2021 (Lecture 1)

Four lectures

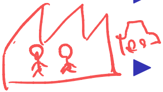
- Four lectures on Wednesdays in July 2021.
- I am not used to 105 minutes format, so I will keep adjusting slides.
- The slides (both annotated and non-annotated) will be available at my lecture notes website: https://sites.google.com/view/toshimukoyama/notes
- I will not set office hours, but I will be at the university until the end of July (8th floor of the International Academic Research Building), so please email me (tm1309@georgetown.edu) if you want to stop by.

Homogeneous versus heterogeneous firms

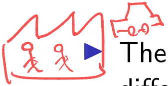
- Standard "textbook" aggregate production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \quad \alpha \in (0,1).$$

  Two interpretations, under the assumption of perfect competition:
  - Constant returns at the individual firm (establishment) level.



  - Decreasing returns at the firm (establishment) level, but one can "replicate" the same firm many times.



- The first interpretation runs into problem because a slight difference in A_{it} would shut down all firms except for one.
- The second interpretation is standard, but there is a question of whether we can "replicate" exactly the same firm. Another challenge is the fact that firms are different with each other in the data.
- It is reasonable to think that the aggregate production function is a sum of heterogeneous firms.

What heterogeneity?

- Firms are different along many dimensions.
- Here I will mostly talk about difference in productivity and size.
- The important dimensions of heterogeneity that I don't talk a lot in this lecture: age (life cycle), sectors, market power, and international dimensions.

Introducing heterogeneous firms

Two methods:

- Maintain the assumption of perfect competition and assume decreasing returns.

$$Y_t = \sum_i Y_{it} = \sum_i A_{it}(K_{it}^{\alpha} L_{it}^{1-\alpha})^{\gamma}, \quad \alpha \in (0,1), \gamma \in (0,1).$$

- Introduce product differentiation and monopolistic competition

$$Y_t = \left(\sum_i Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\sum_i (A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha})^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1$$

- An important (common) point: the reallocation of K_{it} and L_{it} across firms can affect aggregate output even when $\sum_i K_{it}$ and $\sum_i L_{it}$ are constant.$\rightarrow$ The measured total factor productivity is influenced by the distribution of productive inputs across firms. Reallocation of resources can improve the aggregate productivity.

How much does the reallocation matter for measured TFP?

- A version of Baily et al. (1992) decomposition of industry productivity change ΔP_{it}:

$$\Delta P_{it} = \sum_{e \in C} s_{et-1} \Delta p_{et} + \sum_{e \in C} (p_{et-1} - P_{it-1}) \Delta s_{et} + \sum_{e \in C} \Delta p_{et} \Delta s_{et} + \sum_{e \in N} s_{et} (p_{et} - P_{it-1}) - \sum_{e \in X} s_{et-1} (p_{et-1} - P_{it-1})$$

  where C is continuing establishments, N is entering establishments, and X is exiting establishments. The first is the “within” term, the second is the “between” term, the third is the “cross” term, and then net entry terms.
- Foster et al. (2001) measurement of U.S. manufacturing plants productivity (1977-87): within 48%, between -8%, cross 0.34, and net entry 26%.
- The reallocation accounts for more than half of productivity growth.

Measuring expansion/contraction of firms (establishments)

- How much are expanding firms expanding? Job creation:

$$JC = \frac{\sum_{n_t > n_{t-1}} (n_t - n_{t-1})}{\sum n_{t-1}}$$

- How much are contracting firms contracting? Job destruction:

$$JD = \frac{\sum_{n_t < n_{t-1}} (n_{t-1} - n_t)}{\sum n_{t-1}}$$

- The above are called "(gross) job flows." Note that the gross job flows are much larger than the net change in employment in the aggregate economy.
- Gross flows are quite large. In U.S. manufacturing (Davis, Haltiwanger, and Schuh 1996) 1973-1988, average annual JC is 9.1% and JD is 10.3%.

Some U.S. datasets: Census

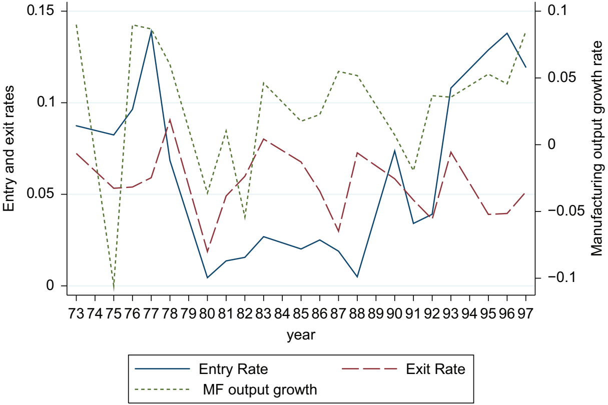
- Longitudinal Research Database (LRD): the dataset of U.S. manufacturing plants by the U.S. Census Bureau.
  - Census of Manufactures (CM): The universe of plants. Every 5 years.
  - Annual Survey of Manufactures (ASM): Subset of CM (rotated). Every year.
  - Some quarterly data is also available.
- Longitudinal Business Database (LBD): The descendent of LRD. Annual data and covers all sectors.
- Business Dynamics Statistics (BDS) is made from LBD and it is public data. It includes the numbers of firms and establishments, firm age distribution, employment distribution, entry/exit, job creation and job destruction.
- Longitudinal Employer-Household Dynamics (LEHD): Quarterly employer-household matched data.
- Statistics of U.S. Businesses (SUSB): Annual numbers of firms, establishment, employment, and annual payroll.

Some U.S. datasets: Bureau of Labor Statistics

- Quarterly Census of Employment and Wages (QCEW): Quarterly establishment-level data of employment and wages. Covers 98% of all employment.
- Business Employment Dynamics (BED, BDM): Public data made from QCEW.
- Job Openings and Labor Turnover Survey (JOLTS): Monthly data from a sample of approximately 16,000 U.S. business establishments. Asks job openings (vacancies), hires, separations, quits, layoffs.

Plant entry and exit rates from ASM

From Lee and Mukoyama (2015a)



BDS webpage

https://www.census.gov/data/datasets/time-series/econ/bds/bds-datasets.html

Economy-Wide Datasets

- Economy-wide [<1.0MB]

One-Way Datasets

- Firm Age [<1.0MB]
- Establishment Age [<1.0MB]
- Firm Size [<1.0MB]
- Establishment Size [<1.0MB]
- Initial Firm Size [<1.0MB]
- Initial Establishment Size [<1.0MB]
- Metro/Non-Metro [<1.0MB]
- State [<1.0MB]
- County [16.7MB]
- MSA [5.5MB]
- Sector [<1.0MB]
- 3-digit NAICS [<1.0MB]
- 4-digit NAICS [1.6MB]

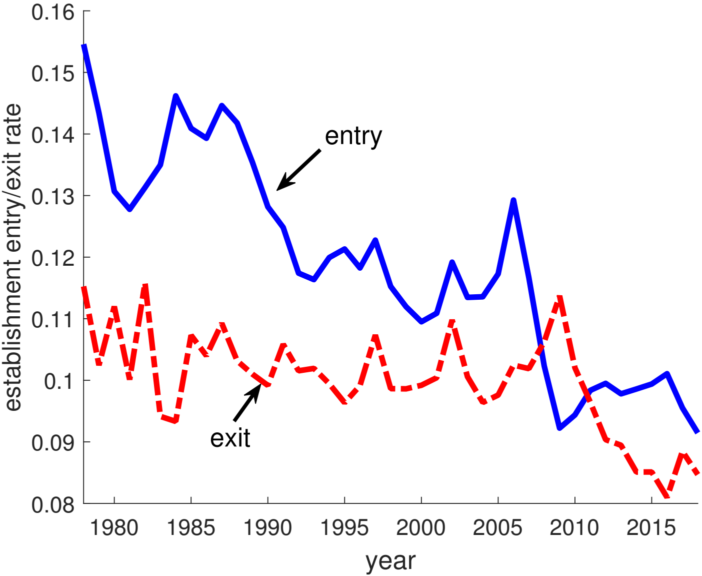
Two-Way Datasets

- Firm Age by Firm Size [<1.0MB]

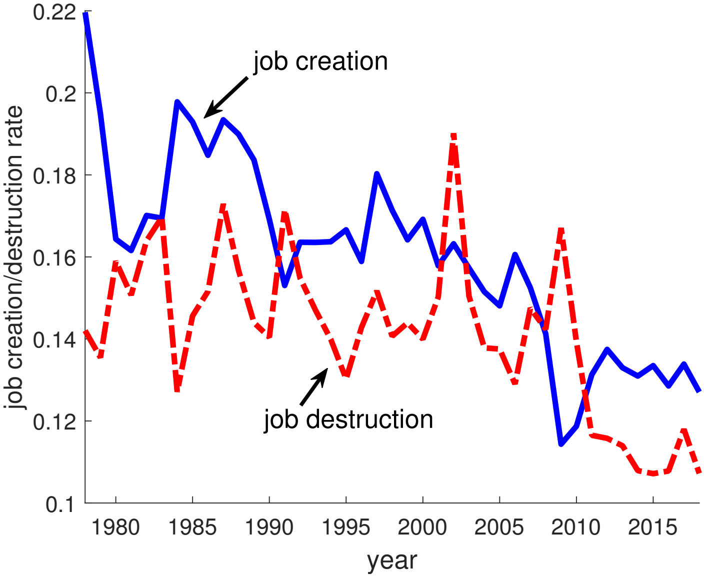
Three-Way Datasets

- MSA by Sector by Firm Age [360.2MB]
- MSA by Sector by Firm Size [232.1MB]
- MSA by Sector by Initial Firm Size [232.5MB]
- MSA by Sector by Establishment Age [362.7MB]
- State by Sector by Firm Age [52.2MB]
- State by Sector by Firm Size [48.8MB]
- State by Sector by Initial Firm Size [49.0MB]
- State by Sector by Establishment Age [52.5MB]
- Metro/Non-Metro by Sector by Firm Age [3.1MB]
- Metro/Non-Metro by Sector by Firm Size [2.9MB]
- Metro/Non-Metro by Sector by Initial Firm Size [3.0MB]
- Metro/Non-Metro by Sector by Establishment Age [3.2MB]
- Metro/Non-Metro by State by Firm Age [8.5MB]
- Metro/Non-Metro by State by Firm Size [8.1MB]
- Metro/Non-Metro by State by Initial Firm Size [8.1MB]

Establishment entry and exit rates from BDS



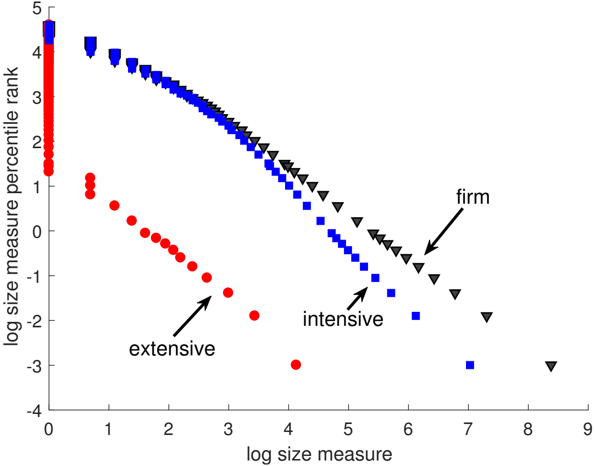
Job creation and job destruction rates from BDS



Firms versus establishments, from QCEW

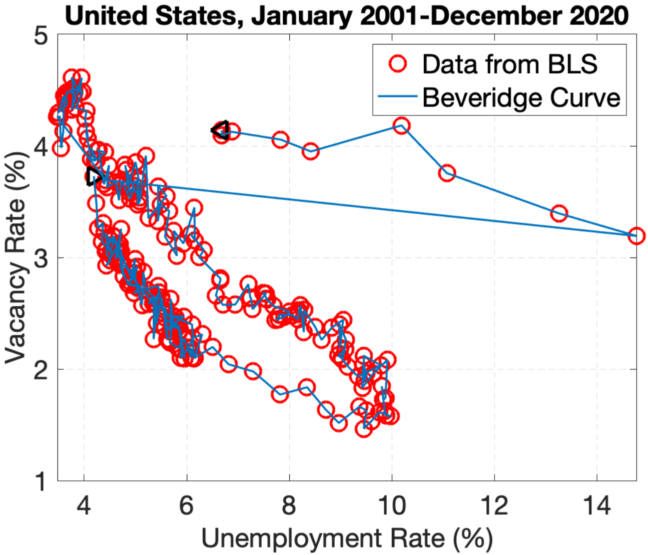
From Cao et al. (2020):

- Intensive margin: average employment per establishment in each firm
- Extensive margin: number of establishments in each firm



Beveridge curve from JOLTS

https://www.bls.gov/jlt/#data



Hopenhayn and Rogerson (1993)

- We will cover Hopenhayn and Rogerson (1993) paper as an example of the model with heterogeneous firms.
- The paper's question: What is the effect of firing taxes on employment, output, and productivity?
- Answering this question needs a framework with firing/hiring behavior of firms. The paper extends the Hopenhayn (1992) framework to general equilibrium.
- The employment effect is not obvious. A firing tax makes the firms fire less (employment goes up) but also makes the firm reluctant to hire (employment goes down). It is a quantitative question to see which effect is stronger.
- The model has many (heterogeneous) firms and a representative consumer.

Firms

- Firms act competitively in both product and labor market. The wage is numéraire. Firm's period profit:

$$pf(n', s) - n' - pc_f - g(n', n)$$

- Firing tax

$$g(n', n) = \tau \max(0, n - n')$$

- Entry ("free entry")
  - Many potential entrants
  - An entrant pays c_e units of goods when enter
  - Draws s from distribution $\nu(s)$

Timing: Incumbents

- At the beginning of period t, an incumbent firm (operated period $t-1$ has the state variables (s_{-1}, n).
- It makes the exit decision
  - if exit, pay $-g(0,n)$.
  - if stay, draw the new s from $F(s_{-1}, s)$. Denote the measure of firms at this timing as $\mu(s,n)$. Decide n', pay pc_f, and produce.

Timing: Entrants

- At the beginning of period t, pay pc_e and enter.
- Draw new s from $\nu(s)$.
- Hire n' and produce.

Consumer and government

- Representative consumer

$$\max \sum_{t=1}^{\infty} \beta^t [u(c_t) - AN_t]$$

- Government: collect firing tax and lump-sum transfer back to consumers.

Steady-state equilibrium

- Firms optimize

$$W(s,n;p) = \max_{n'\geq 0} \quad pf(n',s) - n' - pc_f - g(n',n) \\ +\beta \max[E_s W(s',n';p), -g(0,n')]$$

Decision rule for today's employment:

$$n' = N(s,n,p)$$

Decision rule for exit in the beginning of next period

$$X(s,n,p) = \begin{cases} 1 & \text{exit} \\ 0 & \text{stay} \end{cases}$$

- Free entry:

$$W^e(p) = pc_e$$

when there is positive entry, where

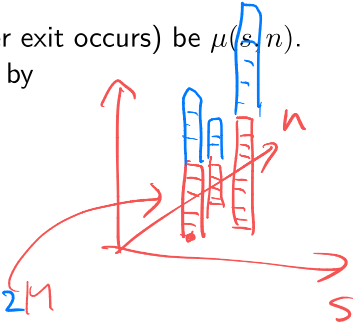
$$W^e(p) = \int W(s,0,p)d\nu(s).$$

Stationary measure (distribution)

Let the measure over the state (s, n) (after exit occurs) be $\mu(s, n)$. The next period measure μ' is determined by

- This period's measure μ
- This period's entrant mass M
- This period's price p

and thus be expressed as



$$\mu' = T(\mu, M, p).$$

An important feature of T is that it is linearly homogeneous in μ and M. In the steady state,

$$\mu = T(\mu, M, p)$$

has to hold.

Aggregate variables

Once μ is computed, various aggregate variables can be computed:

- Output

$$Y(\mu, M; p) = \int [f(N(s,n;p),s) - c_f] d\mu(s,n) + M \int f(N(s,0;p),s) d\nu(s)$$

- Total firing tax

$$R(\mu, M; p) = \int r(s,n;p) d\mu(s,n)$$

where

$$\begin{aligned} r(s,n;p) = \; & [1 - X(s,n;p)] \int g(N(s', N(s,n;p),p), N(s,n;p)) dF(s,s') \\ & + X(s,n;p) g(0, N(s,n;p)) \end{aligned}$$

- Total labor demand:

$$L^d(\mu, M, p) = \int N(s,n;p) d\mu(s,n) + M \int N(s,0;p) d\nu(s)$$

- Total profit:

$$\Pi(\mu, M; p) = pY(\mu, M, p) - L^d(\mu, M, p) - R(\mu, M, p) - Mpc_e.$$

Consumers

- In the steady state, $\beta = 1/(1+r)$ and the problem is static.

$$\max_N u(c) - AN$$

subject to

$$pc \leq N + \Pi + R$$

FOC:

$$\frac{1}{p} u' \left(\frac{N + \Pi + R}{p} \right) = A.$$

Thus the labor supply is:

$$N = L^s(p, \Pi + R).$$

Equilibrium

There are three equilibrium objects: μ, M, p. These can be derived from the conditions:

1. Free entry:

$$W^e(p) = c_e$$

2. Decision for n' and transition of s:

$$\mu = T(\mu, M, p)$$

3. Labor market equilibrium:

$$L^d(\mu, M, p) = L^s(p, \Pi(\mu, M, p) + R(\mu, M, p))$$

Equilibrium

Computation exploits the linear homogeneity of T and L^d, Π, and R as well as the "block recursive" structure (see Lee and Mukoyama (2018) and Kaas (2021))

1. Compute the optimization for a given p for various p, and find p that satisfy the free-entry condition.
2. Let $\hat{\mu}$ be the value of μ corresponding to $M = 1$. Then μ can be solved from
$$\hat{\mu} = T(\hat{\mu}, 1, p).$$
Note that with this $\hat{\mu}$, $\mu = M\hat{\mu}$ for any M because
$$M\hat{\mu} = T(M\hat{\mu}, M, p).$$
3. Find M that satisfy
$$L^d(M\hat{\mu}, M, p) = L^s(p, M(\Pi(\hat{\mu}, 1, p) + R(\hat{\mu}, 1, p)))$$

Calibration

- Functional forms

$$f(n, s) = sn^{\theta} \text{ and } u(c) = \log(c)$$

- Parameters: $\theta = 0.64$ from labor share, $\beta = 0.8$ from one period being 5 years
- Stochastic process:

$$\log(s_t) = a + \rho \log(s_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$$

- To set other parameters (A, a, ρ, σ_{ε}, c_e, c_f), assume that the US economy is with $\tau = 0$ and compare the model outcome to the US data.
  - A: target $N = 0.6$
  - a: match the average size of firm in the US
  - c_f: match the establishment exit rate in the US
  - c_e: normalize $p = 1$ (recall the wage=1: "one unit of output" is "five year wage")$\rightarrow W^e(1) = c_e$

Calibration

- Productivity process:

$$\log(s_t) = a + \rho \log(s_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- ρ, σ_ε: firm's FOC for the frictionless economy $\Rightarrow$ n is one to one with s $\Rightarrow$ match the employment dynamics at establishment level. Hopenhayn and Rogerson (1993) use $\rho = 0.93$, which is quite persistent. The subsequent micro-estimates vary substantially, and there is a discussion of whether to (i) include firm fixed effects and (ii) AR(1) is a good specification or not. See Lee and Mukoyama (2015b), Sterk et al. (2021)

Result

TABLE 3

EFFECT OF CHANGES IN τ (Benchmark Model)

	$\tau = 0$	$\tau = .1$	$\tau = .2$
Price	1.00	1.026	1.048
Consumption (output)	100	97.5	95.4
Average productivity	100	99.2	97.9
Total employment	100	98.3	97.5
Utility-adjusted consumption	100	98.7	97.2
Average firm size	61.2	61.8	65.1
Layoff costs/wage bill	0	.026	.044
Job turnover rate	.30	.26	.22
Serial correlation in log(n)	.92	.94	.94
Variance in growth rates	.55	.45	.39

Result

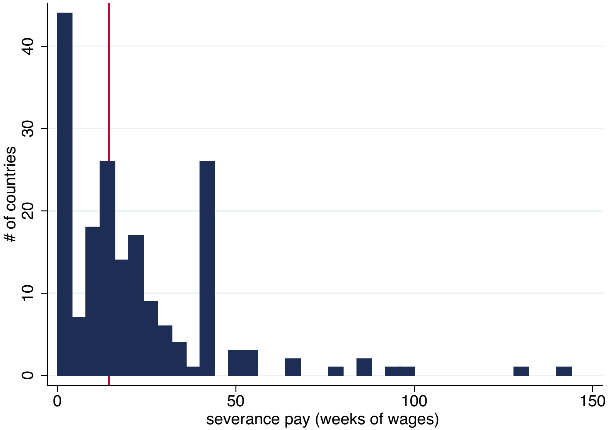
TABLE 5

ABSOLUTE DEVIATIONS FROM MPL = $1/p$

SIZE OF DEVIATION (%)	FRACTION OF FIRMS WITHIN INTERVAL	
	$\tau = .1$	$\tau = .2$
0–3	.30	.00
3–5	.45	.12
5–10	.15	.78
10–15	.00	.05
>15	.00	.05

Other countries

- From Mukoyama and Osotimehin (2019)
- Data: “Doing Business” dataset: `http://www.doingbusiness.org`
- Mandatory severance payment for a worker with 10 years of tenure



References

Baily, M. N., C. Hulten, and D. Campbell (1992). Productivity Dynamics in Manufacturing Plants. *Brookings Papers on Economic Activity, Microeconomics*, 187–249.

Cao, D., H. R. Hyatt, T. Mukoyama, and E. Sager (2020). Firm Growth through New Establishments. `https://toshimukoyama.github.io/MyWebsite/CHMS.pdf`.

Foster, L., J. Haltiwanger, and C. J. Krizan (2001). Aggregate Productivity Growth: Lessons from Microeconomic Evidence. In C. R. Hulten, E. R. Dean, , and M. J. Harper (Eds.), *New Developments in Productivity Analysis*. NBER.

Hopenhayn, H. (1992). Entry, Exit, and Firm Dynamics in Long Run Equilibrium. *Econometrica 60*, 1127–1150.

Hopenhayn, H. and R. Rogerson (1993). Job Turnover and Policy Evaluation: A General Equilibrium Analysis. *Journal of Political Economy 101*, 915–938.

Kaas, L. (2021). Block-Recursive Equilibria in Heterogeneous-Agent Models. Technical report.

Lee, Y. and T. Mukoyama (2015a). Entry and Exit of Manufacturing Plants over the Business Cycle. *European Economic Review 77*, 20–27.

Lee, Y. and T. Mukoyama (2015b). Productivity and Employment Dynamics of US Manufacturing Plants. *Economics Letters 136*, 190–193.

Lee, Y. and T. Mukoyama (2018). A Model of Entry, Exit, and Plant-level Dynamics over the Business Cycle. *Journal of Economic Dynamics and Control 96*, 1–25.

Mukoyama, T. and S. Osotimehin (2019). Barriers to Reallocation and Economic Growth: The Effects of Firing Costs. *American Economic Journal: Macroeconomics 11*, 235–270.

Sterk, V., P. Sedláček, and B. Pugsley (2021). The Nature of Firm Growth. *American Economic Review 111*, 547–579.