Two Extensions of the DMP Model

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Keio Lecture 2: continuing from Lecture 1

Two extensions

- Today I will briefly talk about the extensions of the Diamond-Mortensen-Pissarides (DMP) model. The DMP model has been applied to many contexts.
- The first paper is the one by Gertler, Sala, and Trigari (2008) (GST). This is an estimated DSGE model applied to the context of monetary policy. (Also see Gertler and Trigari (2009)).
- The second is Krusell, Mukoyama, and Şahin (2010) (KMS). It develops an incomplete-market version of the model, applied to the unemployment insurance policy. (Also see Mukoyama (2013)).

Gertler, Sala, and Trigari (2008)

- Adds the DMP-style labor market frictions in (Christiano-Eichenbaum-Evans/Smets-Wouters style) monetary DSGE model. Estimates the model with Baysesian methods.
  - Households: each household has many members can insure within the household for the labor market risk (Merz-Andolfatto).
  - Wholesale firms: price-takers in output market. Produces output from constant returns technology with capital and labor. Capital is rented from households in a competitive market. Labor market is frictional in the DMP manner. Wages are sticky (Calvo) with staggered Nash Bargaining.
  - Retail firms: produces differentiated goods using the wholesale goods. Monopolistically competitive with staggered price setting (Calvo).
  - Government: monetary policy follows the Taylor rule.

GST: Households

- Households own capital and labor.
- Preferences:

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \log(c_{t+s} - hc_{t+s-1}),$$

- Budget constraint:

$$c_t + i_t + \frac{B_t}{p_t r_t} = w_t n_t + (1 - n_t) b_t + r_t^k \nu_t k_{t-1}^p + \Pi_t + T_t - \mathcal{A}(\nu_t) k_{t-1}^p + \frac{B_t}{p_t},$$

$$k_t = \nu_t k_{t-1}^p,$$

$$k_t^p = (1 - \delta) k_{t-1}^p + \varepsilon_t^i \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}} \right) \right] i_t.$$

GST: Matching

- Unemployment
$$u_t = 1 - n_{t-1}.$$
- Matching
$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma}.$$
- The probability of a firm filling a vacancy
$$q_t = \frac{m_t}{v_t}.$$
The probability that a searching worker finds a job
$$s_t = \frac{m_t}{u_t}.$$

GST: Wholesale firms

- Production: constant returns

$$y_{it} = (k_{it})^{\alpha}(z_t n_{it})^{1-\alpha}$$

- k_{it} is adjustable while n_{it} is subject to frictions.
- Choosing vacancy v_t is the same thing as choosing the hiring rate x_t

$$x_{it} = \frac{q_t v_{it}}{n_{it-1}}.$$

- Then the law of motion for employment is

$$n_{it} = (\rho + x_{it}) n_{it-1}.$$

- The value of a firm (note the quadratic hiring cost)

$$\begin{aligned} F_t(w_{it}, n_{it-1}) &= p_t^w y_{it} - \frac{w_{it}^n}{p_t} n_{it} - \frac{\kappa_t}{2}(x_{it})^2 n_{it-1} - r_t^k k_{it} \\ &\quad + \beta E_t \Lambda_{t,t+1} F_{t+1}(w_{it+1}^n, n_{it}). \end{aligned}$$

GST: Wages

- Wage dynamics: with probability $1-\lambda$, the wages are renegotiated. With probability λ, it follows a partial indexation rule
$$w_{it}^{n} = \bar{\gamma} w_{it-1}^{n} \pi_{t-1}^{\gamma}$$
where $\pi_t = p_t / p_{t-1}$.
- Generalized Nash bargaining: when renegotiate, the wage w_t^{*n} solves
$$\max H_t(w_{it}^{n})^{\eta_t} J_t(w_{it}^{n})^{1-\eta_t}$$
subject to
$$w_{t+k}^{n} = \begin{cases} \bar{\gamma} w_{it+k-1}^{n} \pi_{t+k-1}^{\gamma} & \text{with probability } \lambda \\ w_{t+k}^{*n} & \text{with probability } 1-\lambda \end{cases}$$
for $k \geq 1$. $H_t(w_{it}^{n})$ is the worker's surplus and $J_t(w_{it}^{n})$ is the firm's surplus.

GST: Retailers

- Monopolistic competition. Converts one unit of wholesale good to one unit of retail good.
- Retail goods y_{jt} are aggregated into the final output y_t.
- Calvo price adjustment with probability $1 - \lambda^p$ of adjustment. Non-adjusting firms follow the indexation rule

$$p_{jt} = \bar{\gamma}^p p_{jt-1},$$

  where $\bar{\gamma}^p$ is assumed to be equal to the trend inflation rate π.
- The target price p_t^* is the solution to the maximization problem

$$\max \quad E_t \sum_{s=0}^{\infty} (\lambda^p \beta)^s \Lambda_{t,t+s} \left[\frac{p_t^*}{p_{t+s}} \left(\prod_{k=1}^{s} \bar{\gamma}^p \right) - p_{t+s}^w \right] y_{jt+s}.$$

  subject to the demand curve for y_{it}.

GST: Government

- Monetary policy follows a Taylor rule:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho^s} \left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi} \left(\frac{y_t}{y_{nt}}\right)^{r_y}\right]^{(1-\rho^s)} \varepsilon_t^r.$$

- Government spending follows

$$g_t = \left(1 - \frac{1}{\varepsilon_t^g}\right) y_t.$$

Krusell, Mukoyama, and Şahin (2010)

- Adds DMP-style labor market frictions to Bewley-Huggett-Aiyagari style incomplete market model. Also business cycle analysis (Krusell and Smith model).
  - Households: do not have an access to insurance, other than holding assets. Assets are capital stock and equity (firm ownership). Supply labor, consume.
  - Firms: produce with capital and labor. Post vacancies to hire workers.
  - "Investment firms"

KMS: Steady-state

- Two kinds of asset: capital k and equity x. The equity is the claim to the entire firms' profit. Total amount of equity is normalized to 1, and the price is p. Dividend is d. In the steady state, both capital and equity are riskless, as there is no aggregate uncertainty. No arbitrage implies

  $$1 + r - \delta = \frac{d + p}{p}.$$

  The LHS is per dollar return of capital (r is the rental rate and δ is the depreciation rate), while the RHS is per dollar return of equity.
- Using this relationship, we can aggregate the asset position using the asset level a

  $$a \equiv (1 + r - \delta)k + (p + d)x.$$

  One can think of a as the bond holding and the bond price $q \equiv 1/(1 + r - \delta)$. Each consumer is indifferent across holding k versus x.

KMS: Matching

- Matching function $M(u, v)$
- The probability that a vacancy is filled ($\theta \equiv v/u$):

$$\lambda_f = M(u, v)/v = M(1/\theta, 1)$$

- The probability that a worker finds a job:

$$\lambda_w = M(u, v)/u = \theta \lambda_f.$$

- The dynamics of u:

$$u' = (1 - \lambda_w)u + \sigma(1 - u),$$

where σ is the exogenous separation rate.

KMS: Consumers

- Employed:

$$\tilde{W}(w,a) = \max_{a'} u(c) + \beta[\sigma U(a) + (1-\sigma)W(a')]$$

  subject to

$$c + qa' = a + w \text{ and } a' \geq \underline{a}.$$

- Unemployed:

$$\tilde{U}(a) = \max_{a'} u(c) + \beta[(1-\lambda_w U(a) + \lambda_w W(a')]$$

  subject to

$$c + qa' = a + h \text{ and } a' \geq \underline{a},$$

  where h can be thought as the unemployment insurance.
- Note that even if the capital stock does not exist, the workers can consume more than w or h, because there is dividend income from firm profit. This is forgotten in the standard DMP models in the textbook, but it does not make a difference there because $W - U$ is all that matters in the standard model with linear utility.

KMS: Firms

- Vacancy:

$$V = -\xi + q\left[(1-\lambda_f)V + \lambda_f \int J(\psi_u(a))\frac{f_u(a)}{u}da\right].$$

- Filled job:

$$\tilde{J}(w,a) = \max_k zF(k) - rk - w + q\left[\sigma V + (1-\sigma)J(\tilde{\psi}_e(w,a))\right].$$

- Dividend:

$$d = \int \pi(a) f_e(a)da - \xi v.$$

  Note that $d > 0$ in the steady state, because of discounting.
- Free entry: $V = 0$.

KMS: Wages

- Generalized Nash Bargaining:

$$\max_{w} \left(\tilde{W}(w,a) - U(a)\right)^{\gamma} \left(\tilde{J}(w,a) - V\right)^{1-\gamma}.$$

- Here, the wage depends on a.

KMS: Aggregate shocks

- Suppose that there are two aggregate states: $z = g, b$.
- Given that the distribution of asset and the distribution of employment state (from the aggregate viewpoint) is predetermined, there are two possible aggregate states.
- There are two assets: k and x. Thus these assets can span these two states.
- Consider the **aggregate Arrow security** which spans these two aggregate states. Let the price of these securities be $Q_{z'}(z, S)$. (Also see Krusell, Mukoyama, and Smith (2011).)
- No-arbitrage condition

$$\sum_{z'} Q_{z'}(z, S)(1 - \delta + r(z', S')) = 1$$

$$p(z, S) = \sum_{z'} Q_{z'}(z, S)[p(z', S') + d(z', S')).$$

- We can also use $Q_{z'}(z, S)$ for discounting future profit of the firms.

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