

# Firm Dynamics and the Macroeconomy: Basics

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From today, we focus on  $L$

- If the capital is fixed,

$$\gamma \in (0, 1)$$

$$A_i (\underbrace{K_i^\alpha}_{\text{fixed}} L_i^{1-\alpha})^\gamma = \underbrace{B_i}_{\text{fixed}} L_i^\beta$$

where

$$\underbrace{B_i}_{\text{fixed}} = \underbrace{A_i K_i^{\alpha\gamma}}_{\text{fixed}}$$

and

$$\beta = \underbrace{\gamma(1-\alpha)}_{\text{fixed}} \in (0, 1).$$

## From today, we focus on $L$

- ▶ If the rental market for capital is perfectly competitive,

$$\max_{K_i} A_i (K_i^\alpha L_i^{1-\alpha})^\gamma - rK_i$$

imply

$$K_i = \left(\frac{\alpha\gamma}{r}\right)^{\frac{1}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}} L_i^{\frac{\gamma-\alpha\gamma}{1-\alpha\gamma}}$$

Plugging this solution into the production function

$$A_i (K_i^\alpha L_i^{1-\alpha})^\gamma = \left(\frac{\alpha\gamma}{r}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}} L_i^{\frac{\gamma-\alpha\gamma}{1-\alpha\gamma}}$$

Thus we can write a new production function

$$B_i L_i^\beta$$

where

$$B_i = \left(\frac{\alpha\gamma}{r}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}}$$

and

$$\beta = \frac{\gamma - \alpha\gamma}{1 - \alpha\gamma} \in (0, 1)$$

## Misallocation

types of

First, continuing with the discussion in the last class, let us talk about the misallocation. An example:

- ▶ There are two firms, firm 1:  $Y_1 = A_1 L_1^\alpha$  and firm 2:  $Y_2 = A_2 L_2^\alpha$  where  $\alpha \in (0, 1)$ .
- ▶ Let  $A_1 = 1$  and  $A_2 = 2$ .  $\alpha = 1/2$ .
- ▶ Assume that there are firm-specific distortions  $\tau_i$  ( $i = 1, 2$ ). We can think of  $\tau_i$  as a tax.
- ▶ Firm  $i$  maximizes the profit

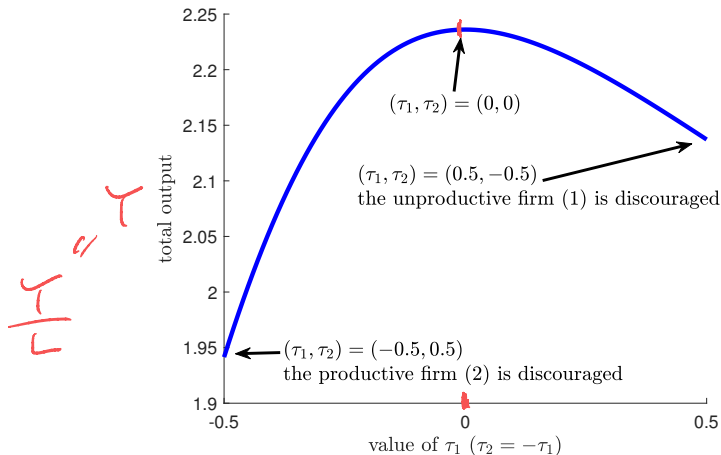
$$(1 - \tau_i)A_i L_i^\alpha - w L_i.$$

Because the actual output is  $A_i L_i^\alpha$ , the firm's decision problem is distorted.

- ▶ Assume that the total labor is fixed at 1. Thus  $w$  is determined by  $L_1 + L_2 = 1$ .
- ▶ The total output is computed as

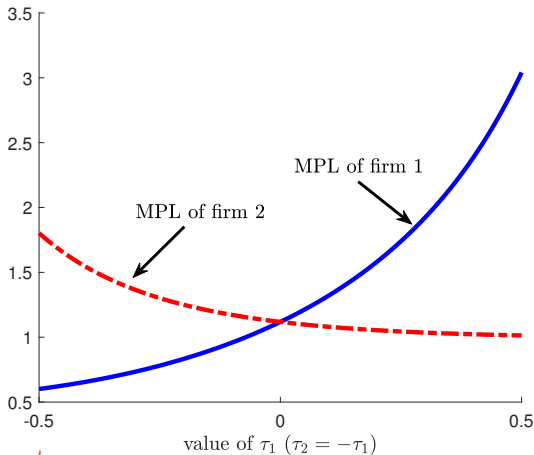
$$Y = Y_1 + Y_2 = A_1 L_1^\alpha + A_2 L_2^\alpha$$

# Misallocation



- ▶ The total output is reduced the most with positive correlation between the distortion (discouragement) and productivity (Restuccia and Rogerson, 2008)

# Misallocation



- ▶ MPL ( $\alpha A_i L_i^{\alpha-1}$ ) dispersion is the source of the productivity loss.

## Situations where misallocations can occur: Examples

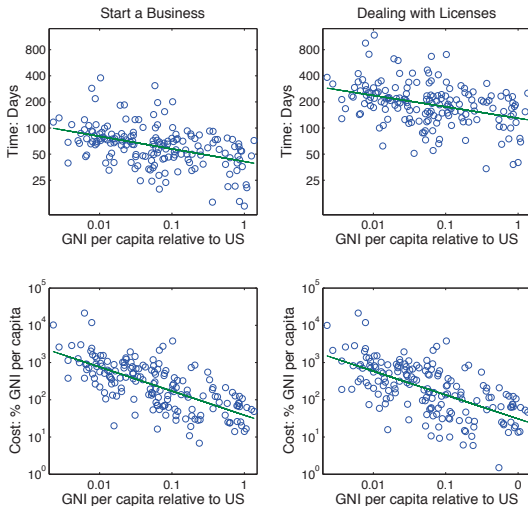
- ▶ Tax rates are different depending on firm identity/characteristics
- ▶ Regulations that depend on firm size
- ▶ Firing/hiring taxes/subsidies
- ▶ Entry/exit taxes/subsidies, some other frictions
- ▶ Financial frictions
- ▶ Contract enforcement

### Notes:

- ▶ The cost of reduced entry depends on the post-entry importance of entrants.
- ▶ The importance of financial frictions depends on the persistence of shocks (whether the firm can overcome the friction by self-financing); see Moll (2014). The shock does seem to be persistent; see Lee and Mukoyama (2015). There still can be effects for young firms and potential entrants.
- ▶ The importance of contract enforcement at the industry level positively correlates with industry productivity (Mukoyama and Popov, 2020)

# Entry barriers

From Moscoso Boedo and Mukoyama (2012)



- ▶ Why? Political economy considerations (Mukoyama and Popov, 2014)



## Misallocation as a theory of TFP

- ▶ Misallocation can change the measured TFP (measured by  $Y/L^\alpha$ , for example) without changing  $A_1$  and  $A_2$ .
- ▶ The effect can be sizable but not as much as 10-folds differences between rich and poor countries.
- ▶ For the development questions, the determination of  $A_i$  (growth of productivity at the firm level) is still important.

**Firm growth**

# On firm growth

$$\tau = \int_0^{\infty} A_{\tau} e^{\gamma a} da \quad e^{-\delta a}$$

*(Note:  $A_{\tau} e^{\gamma a}$  is circled in blue, and  $e^{-\delta a}$  is circled in red. Arrows point from the integrand to the integral symbol.)*

Two small points first:

- ▶ First, note that individual firm growth is not necessary or sufficient for aggregate growth.
- ▶ Second, the loss from missing entry can be large if we take firm growth into account.

An example:

- ▶ Labor supply is elastic (employment is demand-determined). One firm hires one worker.
- ▶ The production of a firm who enters at time  $\tau$  and age  $a$  (today is  $t = a + \tau$ ) is  $A_{\tau} e^{\gamma a}$ .  $\gamma \geq 0$  is the firm growth rate. Assume that  $A_{\tau} = A_0 e^{g\tau}$ .

- ▶ The surviving firms at age  $a$  is  $e^{-\delta a}$ . Assume  $\delta > \gamma$ .
- ▶ The mass of entrants is 1.
- ▶ Outcome: The total employment is  $\int_0^{\infty} e^{-\delta a} da = 1/\delta$ . The aggregate production is  $A_0 e^{gt} / (\delta + g - \gamma)$ .
- ▶ If  $\Delta$  units of entrants are lost, the immediate loss is  $\Delta A_{\tau} dt$  but the present value of loss is  $\Delta A_{\tau} / (\rho + \delta - \gamma)$ , where  $\rho$  is the discount rate.

*cohort age*

*time.*

$T$

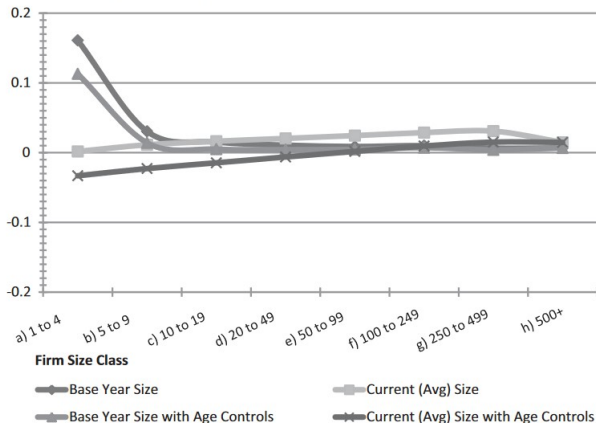
*•  $g=0, \delta>0$   
 $T$  const  
firms grow*

*•  $g>0, \delta=0$   
 $T$  grow  
firms stay the same size*

# Some facts on firm growth

Figures from Haltiwanger et al. (2013)

B. Continuing Firms only

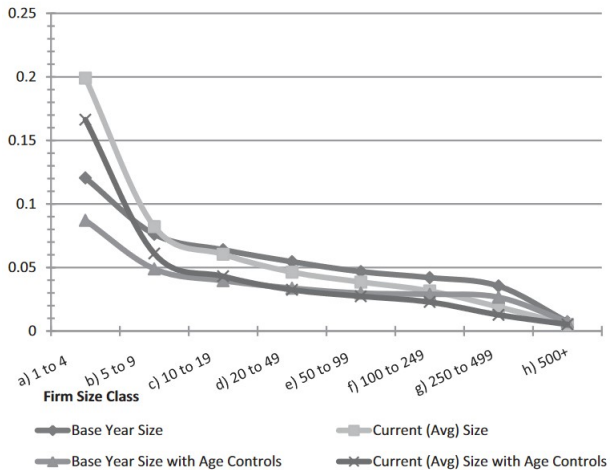


- ▶ The growth rate of a firm is independent of size: “Gibrat’s Law” (mixed supports in the data)

# Some facts on firm growth

Figures from Haltiwanger et al. (2013)

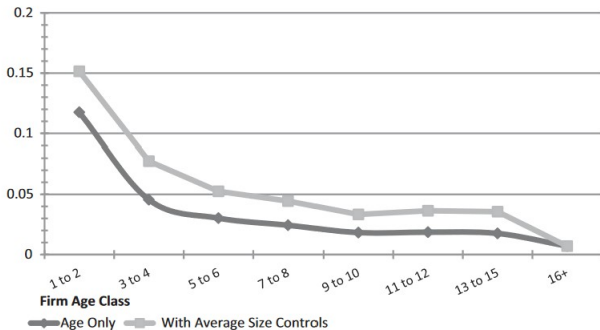
FIGURE 3.—FIRM EXIT BY FIRM SIZE



# Some facts on firm growth

Figures from Haltiwanger et al. (2013)

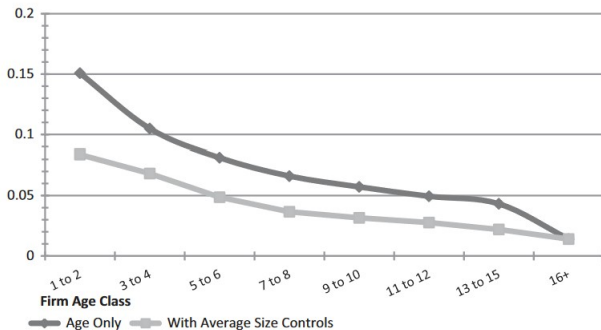
B. Continuing Firms Only



# Some facts on firm growth

Figures from Haltiwanger et al. (2013)

FIGURE 5.—FIRM EXIT BY FIRM AGE



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