Innovation and Firm Dynamics

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Productivity and firm dynamics

- In representative firm dynamics models, such as Hopenhayn (1992), idiosyncratic firm productivity shocks are exogenously given.
- In reality, at least some part of productivity is endogenous—it is affected by learning and R&D investment.
- R&D-based endogenous growth models, developed mainly from 1990s, are potentially useful in analyzing endogenous productivity.

R&D-based endogenous growth models

- There are (mainly) two variants of endogenous growth models:
  - expanding variety: Romer (1990)
  - quality ladder (creative destruction): Grossman and Helpman (1991) and Aghion and Howitt (1992).
- Recent literature tends to focus more on quality ladder models.
- Below I will talk about Klette and Kortum (2004) (KK) model, which is a variant of quality ladder model with a concept of "firm."

KK: Quality ladders

- Continuum of differentiated goods, each good indexed by $j \in [0, 1]$.
- The quality of good j at generation k: $z(j, k)$. One generation of good j is produced by one firm.
- The size of ladder:

$$q(j, k) \equiv \frac{z(j, k)}{z(j, k-1)}.$$

- One unit of good is produced by one unit of labor. Thus the unit cost is w.

KK: Goods demand

- Utility
$$U = \int_0^\infty e^{-\rho t} \ln(C_t) dt,$$
where
$$\ln(C_t) = \int_0^1 \left[\ln \sum_{k=-1}^{J_t(j)} x_t(j,k) z(j,k) \right] dj.$$
- Normalizing the income (=expenditure) at each t to 1, the demand for good j is unit elastic (after intra-temporal optimization):
$$\sum_{k=-1}^{J_t(j)} p_t(j,k) x_t(j,k) = 1.$$

KK: Firm's optimization

- Because the unit cost is constant (and the same across firms) and the demand is unit elastic, the firm with the best quality tries to maximize monopoly profit by limit pricing:

$$p_t(j) = q_t(j)w.$$

- The profit is

$$\pi_t(j) = [p_t(j) - w]x_t(j) = 1 - \frac{1}{q_t(j)}.$$

  When $q_t(j)$ is constant at q, $\pi_t(j)$ is constant at π and $x_t(j)$ is constant at $1/wq$.

- A firm is defined as the collection of products it produces. By innovating, a firm can add another product to the products it already produces. It may lose products randomly (because of other firms' innovation). A firm that produces n products earn $n\pi$.

KK: Firm dynamics

- The firm size n can increase by innovation. Innovation cost function is

  $$w\mathcal{L}_R = C(I, n),$$

  where $\mathcal{L}_R$ is the R&D input (in labor), I is the Poisson probability of innovation. C function is homogeneous of degree one. Thus

  $$w\frac{\mathcal{L}_R}{n} = C\left(\frac{I}{n}, 1\right).$$

  Let $\lambda = I/n$ and rewrite this as

  $$w\ell_R(\lambda) = c(\lambda).$$

- The firm loses one product with probability μn.

KK: Bellman equation

- The HJB equation of a firm is

$$rV(n) = \max_I \{\pi n - C(I,n) + I[V(n+1) - V(n)] \\ -\mu n[V(n) - V(n-1)]\}.$$

- Guessing $V(n) = vn$ and $I(n) = \lambda n$,

$$rv = \max_\lambda \{\pi - c(\lambda) + \lambda[v(n+1) - vn] \\ -\mu[vn - v(n-1)]\}.$$

  Thus v and λ solve

$$c'(\lambda) = v$$

  and

$$rv = \pi - c(\lambda) + \lambda v - \mu v.$$

KK: Entry and exit

- A firm exits when it has no product to produce.
- A firm can enter by paying a fixed cost $F = wh$, where h is the unit of labor that is necessary for entry. We assume free entry:

$$F = v(= wh).$$

KK: General equilibrium in balanced growth

- Total labor supply L can be distributed to L_x: production, L_R: R&D, and L_s: startup.
- Because $v = wh$ from free entry and the optimal R&D $v = w\ell'_R(\lambda)$, λ can be solved from $h = \ell'_R(\lambda)$. Given this λ, $L_R = \ell_R(\lambda)$ is a constant.
- From the normalization, $r = \rho$ and also the budget constraint is

$$1 = wL + rv = wL + \rho wh.$$

  Thus $w = 1/(L + \rho h)$. Therefore

$$L_x = \int x(j)dj = \int \frac{1}{qw}dj = \frac{1-\pi}{w} = \frac{1-\pi}{L+\rho h}.$$

  This is also a constant number.
- Thus, $L_s = L - L_x - L_R$ can be used to solve η by

$$\eta = \frac{1}{h}\left(L - \frac{1-\pi}{L+\rho h} - \ell_R(\lambda)\right).$$

KK: Properties and the growth rate

- The model allows us to calculate various statistics that can be compared to the data, such as
  - The relationship between firm size and subsequent exit probability
  - The average longevity of firm in relation to age
  - The expected size of firm for a given age
  - The size distribution of firms.
- The growth rate of C is

$$\mu \ln(q) = (\eta + \lambda) \ln(q).$$

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