Firm Dynamics and the Macroeconomy: Basics

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Firm dynamics with frictional labor market

- A natural question in the labor market is how the changes in labor demand, induced by the changes in firm dynamics, would affect unemployment.
- To answer that kind of questions, unemployment has to be modeled together with firm dynamics.
- Elsby and Michaels (2013) provide a basic framework with heterogeneous firms and Diamond-Mortensen-Pissarides (DMP) type labor market frictions.
- There can be other ways of modeling firm dynamics with labor market frictions. Example: Kaas and Kircher (2015).

Bellman equation for a firm

$$\Pi(n_{-1}, x) = \max_{n} \left\{ pxF(n) - w(n,x)n - \frac{c}{q}\Delta n \mathbf{1}^{+} + \beta \int \Pi(n, x')dG \right\}$$

- p is aggregate state, x is individual firm's productivity, $F(n)$ is a strictly increasing and concave production function.
- Later we'll see why the wage $w(n,x)$ is a function of n and x.
- The third term is the vacancy cost. c is the cost for posting vacancy. One unit of vacancy converts into $1/q$ units of new hires. $\Delta n = n - n_{-1}$ is the increase in employment.
- Vacancy cost is necessary only when $\Delta n > 0$. $\mathbf{1}^{+} = 1$ if and only if $\Delta n > 0$. Otherwise $\mathbf{1}^{+} = 0$.

Wage bargaining

- In a DMP model, wages are bargained because a firm and a worker are in a bilateral monopoly situation. Potentially any wage can realize between the upper bound (where the firm is indifferent between keeping the worker and separating) and the lower bound (where the worker is indifferent between staying and separating).
- In the standard model, the bargaining is one-to-one, and the most common assumption is the (generalized) Nash bargaining:

  $$\max_{w} \ (\tilde{W}(w) - U)^{\gamma}(\tilde{J}(w) - V)^{1-\gamma}.$$

  $\gamma \in (0, 1)$ represents the worker's bargaining power.
- With linear utility, the wage solves the proportional sharing rule:

  $$(1 - \gamma)(\tilde{W}(w) - U) = \gamma(\tilde{J}(w) - V).$$

Wage bargaining

- With multi-worker firms plus decreasing returns, the issue is more complex. The reason is that when one worker (threatens to) leave the bargaining, the other workers' productivity is also affected.
- A standard solution, also in Elsby and Michaels (2013), is to assume that a marginal worker Nash bargains with the firm. This assumption can be justified by considering a game between the firm and workers (similarly to justifying a one-to-one Nash bargaining with Rubinstein-Stahl game with alternating offers). The standard reference is Stole and Zwiebel (1996). Also see Brügemann et al. (2019).
- In Elsby and Michaels (2013), the wage satisfies

$$(1-\eta)[W(n,x)-\Upsilon]=\eta J(n,x),$$

where the value functions are defined at the margin.

The marginal worker's value functions

- For the firm:

$$J(n, x) = pxF'(n) - w(n, x) - w_n(n, x)n + \beta D(n, x)$$

  where $D(n, x) = \int \Pi_n(n, x')dG(x'|x)$ is the marginal (present value) future profit from one more worker. Note the term $w_n n$: this term turns out to be negative. Thus there is an extra incentive for a firm to hire more workers.

- For the worker:

$$W(n, x) = w(n, x) + \beta \mathbb{E}[s\Upsilon' + (1 - s)W(n', x')|n, x]$$

  for the employed and

$$\Upsilon = b + \beta \mathbb{E}[(1 - f)\Upsilon' + fW(n', x')].$$

  for the unemployed.

Wages

- The wage solves the differential equation

$$w(n,x) = \eta\left[pxF'(n) - w_n(n,x)n + \beta f\frac{c}{q}\right] + (1-\eta)b.$$

- In Cobb-Douglas case, we can have a closed-form solution:

$$w(n,x) = \eta\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} + \beta f\frac{c}{q}\right] + (1-\eta)b.$$

Distribution

- Given the stochastic process on x, one can compute the stationary distribution in (x, n). Let the distribution with respect to n be $H(n)$.
- Elsby and Michaels (2013) further characterizes $H(n)$ for a specific stochastic process on x.

General equilibrium

- The general equilibrium object is $\theta = V/U$. θ dictates the amount of job creation and job destruction, and it has to be consistent with the total labor force, L.
- First, for a given U, θ has to satisfy

$$\int n dH(n;\theta) + U = L$$

- Second, in the steady state,

$$S(\theta) = f(\theta)U.$$

- Therefore, in the steady state, θ has to satisfy

$$\int n dH(n;\theta) + \frac{S(\theta)}{f(\theta)} = L.$$

Entry and exit of firms

- One can extend the model to include endogenous entry and exit. The extension is fairly straightforward (I think).
- In the beginning of every period (before knowing new x), the incumbent draws a operation cost κ, which is an iid random variable. If $\kappa > \int \Pi(n_{-1}, x) dG(x|x_{-1})$, the firm exits. The continuation value will become

$$\beta \int_0^{\kappa^*} \left[\int \Pi(n, x') dG(x'|x) - \kappa \right] d\Gamma(\kappa)$$

  and the exit threshold κ^* is determined by the firm.
- For entry, a natural assumption is the entry rate being determined by the free-entry condition

$$\int \Pi(0, x) d\nu(x) = c_e,$$

  where c_e is the entry cost.

References

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