Clearing up My Confusion about the Monopolistic Competition Model

Toshihiko Mukoyama

June 2020

This note is to clarify my confusion about the monopolistic competition model.

1 Setting

Consider the following setting.

1. The final good is produced by the constant-returns-to-scale production function

$$Y = \left[\int y_i^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

where Y is the final good output and $\sigma > 1$. y_i where $i \in [0, 1]$ are intermediate-good inputs. The final-good producing firms are perfectly competitive in both input and product market. Let the price of intermediate good i be p_i.

2. There is a unit mass of monopolistically competitive intermediate-good producers. Each intermediate good is produced by one firm, and indexed by i. Intermediate goods are produced by the technology

$$y_i = \ell_i^\beta,$$

where $\beta \leq 1$. The intermediate-good producers are price takers in the labor market and pay the wage w.

3. The labor supply is given by a strictly increasing function $L^s = L^s(w)$.

4. The consumer receives the wage and the profit, supply labor, consumer final goods. They are price takers.

2 My confusion

The standard procedure goes like this. The cost-minimization problem for a final good firm is, given the aggregate demand Y,

$$\min_{y_i} \int p_i y_i di$$

subject to

$$Y = \left[\int y_i^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

The first-order condition is

$$p_i = \lambda y_i^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}}, \tag{2}$$

where λ is the Lagrange multiplier. The minimized total cost is

$$\int p_i y_i di = \int \lambda y_i^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} y_i di = \lambda Y^{\frac{1}{\sigma}} \int y_i^{\frac{\sigma-1}{\sigma}} di = \lambda Y,$$

where the last equality uses (1). Thus λ can be interpreted as the unit cost of production for the final output. Let us denote $P \equiv \lambda$, where P is the competitive price of the final output, because the unit cost has to be equal to the price in the competitive environment with constant returns.

Integrating $p_i^{1-\sigma}$ using (2), and then using (1), P can be expressed as

$$P = \left[\int p_i^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}.$$

Let us choose P as a numeraire, and set $P = 1$. From (2), the inverse demand function for the intermediate good i can therefore be expressed as

$$p_i = \left(\frac{y_i}{Y}\right)^{-\frac{1}{\sigma}}. \tag{3}$$

The profit-maximization problem for an intermediate-good producer (a monopolist) is

$$\max_{p_i, y_i} \; p_i y_i - w\ell_i$$

subject to (3). This problem can be rewritten as

$$\max_{y_i} \; \left(\frac{y_i}{Y}\right)^{-\frac{1}{\sigma}} y_i - w y^{\frac{1}{\beta}}. \tag{4}$$

The first-order condition is

$$\left(1 - \frac{1}{\sigma}\right) y_i^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} = \frac{w}{\beta} y_i^{\frac{1}{\beta}-1}. \tag{5}$$

Now, let's assume that $\beta = 1$, i.e., the constant-returns case. Using (3), the equation (5) can be rewritten as

$$\left(1 - \frac{1}{\sigma}\right) p_i = w,$$

implying

$$p_i = \frac{\sigma}{\sigma - 1} w,$$

a well-known markup rule. Everything looks good so far.

Now, let's look at (5) again. Because all intermediate-good firms are homogeneous, it has to be the case that $y_i = y_j = Y$ for any i, j, in equilibrium. With $\beta = 1$, (5) implies

$$w = \left(1 - \frac{1}{\sigma}\right).$$

Here's my confusion comes in. I have not mentioned the labor market yet, said nothing about demand equals supply in labor or final-good market, and yet here w is already determined as a function of a parameter. And combining this equation with the mark-up rule says $p_i = 1$, which is consistent with the normalization $P = 1$, but now it almost looks like I am stating the obvious (when goods are homogeneous and normalize $P = 1$, of course $p_i = 1$) and I feel like I didn't achieve anything by solving up to here. I thought solving the problem (4) would give me a solution for y_i (and therefore Y). But y_i and Y disappeared in (8). I'm confused.

3 Clearing up

The confusion comes from many sources. Let us clarify first what the monopolists are doing. Above it feels like what the monopolist is doing in (4) is maximizing her profit given the aggregate demand Y. The aggregate demand, as a parameter for the monopolist, shifts the profit up and down.

This interpretation is wrong. Calling Y the aggregate demand was the first mistake. In this model, there is no fixed aggregate demand for the final good. We have already said that the final-good producers are perfectly competitive. Perfect competition means that each final-good producer perceives that it can sell *any* amount of final good for the given price P. Then what does Y represent in the problem (4), if it is not the aggregate demand? It is *what the other producers are producing.* (Note that one firm is negligible, so that we can still call Y "the other firms' production" even though in (1) it also includes the own production y_i.) To see why this interpretation is appropriate, consider the profit-maximization problem instead of cost-minimization problem for a final-good producer. (It is just easier to see from this perspective, although of course cost minimization is implied by profit maximization.)

$$\max_{y_i} \; P\left[\int y_i^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} - \int p_i y_i di.$$

Now assume that the firm is choosing only one good, i, given other y_j ($j \neq i$). The first-order condition is

$$P y_i^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} = p_i. \tag{6}$$

One can show, again, $P = \left[\int p_i^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$ has to hold, because otherwise the final-good producer either suffers from negative profit or is able to create infinite profit (by selling a lot—recall that perfect competition means the firm can sell any amount). Once again, normalize $P = 1$. Then (6) can be rewritten to obtain, once again, the inverse demand

(3). But here, it is clear that Y is *what the other intermediate-good firms are producing.* We don't have to talk about the aggregate demand. The demand for y_i in (3) increases with Y, not because the aggregate demand is high, but because when other intermediate-good producers sells more, the productivity of the good i is also higher. In other words, Y represents a technological spillover from another firm. To see the productivity effect more clearly, calculate the marginal product of y_i:

$$MP_i = \frac{\partial Y}{\partial y_i} = y_i^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}},$$

which is increasing in Y. The Y in the final part is, again, what the *other* intermediate-good firms are producing. (Recall that one firm is negligible in the entire economy so the fact that the firm i is in Y can be ignored.)

Now the correct interpretation of the problem (4): (4) is the problem of the monopolist in a strategic environment. This problem decides the best response of the producer i when the other intermediate-good producers are producing Y. And the best response function is (5).[1] To see how the Nash equilibrium of this game among the intermediate-good producers transpires, consider first the case with $\beta < 1$. In a symmetric Nash equilibrium, $y_i = y_j = Y$ has to hold. Therefore,

$$1 - \frac{1}{\sigma} = \frac{w}{\beta} Y^{\frac{1}{\beta} - 1}$$

and thus

$$(y_i =)Y = \left(\frac{(\sigma - 1)\beta}{\sigma w} \right)^{\frac{\beta}{1-\beta}} \tag{7}$$

is the Nash equilibrium output for a given w. In turn, the total labor demand is

$$L^d = Y^{\frac{1}{\beta}} = \left(\frac{(\sigma - 1)\beta}{\sigma w} \right)^{\frac{1}{1-\beta}}$$

and L^d is a decreasing function of w. Combined with the labor supply function $L^s(w)$, the equilibrium w (and therefore L and Y) is determined.

Things are a little tricker when $\beta = 1$. The symmetric Nash equilibrium for the intermediate-good producers' game requires

$$w = \left(1 - \frac{1}{\sigma} \right). \tag{8}$$

This equation looks somewhat confusing because it does not seem to specify the strategy of the players (in this case y_i). Rather, this equation just states the conditions for some parameters of the game. What the equation (8) states is that, if this equality is not satisfied,

[1] More explicitly, the game is the following: each intermediate-good firm simultaneously submit y_i. The payoff is defined as the profit (4). The Nash equilibrium is the situation where for all i, y_i is chosen to solve the problem (4) given y_j for $j \neq i$.

the symmetric Nash equilibrium of the game does not exist.[2] What will happen if (8) is not satisfied? Suppose that $w < (1 - 1/\sigma)$. In this situation, in the problem (4), the best response of y_i to any Y is to have $y_i > Y$. The best response dynamics will lead to $y_i \to \infty$. If $w > (1 - 1/\sigma)$, the best response is to have $y_i < Y$ and the best-response dynamics lead to $y_i \to 0$. In both cases, a Nash equilibrium with finite y_i (and therefore Y) does not exist. Economically, what the equation (8) represents, therefore, is a horizontal labor demand curve: if $w > (1 - 1/\sigma)$, the labor demand is infinity, whereas if $w < (1 - 1/\sigma)$, the labor demand is zero.

The wage in equilibrium, therefore, has to be determined by the horizontal labor demand curve. The quantity, L (and therefore Y), is determined by the labor supply curve $L^s(w)$ for the wage given in (8). This completes the general equilibrium.

A corollary of this process is that calculating the Nash equilibrium of the game, (7), is a required step for computing the general equilibrium. This step cannot be substituted by other conditions (such as the other market-clearing conditions). This condition tends to be forgotten, because this step is often not explicitly stated as the *Nash equilibrium condition of the game* in the $\beta = 1$ case, because (8) does not seem to specify the Nash equilibrium strategy y_i. But as one can see from above steps, $\beta = 1$ case is very special and unusual. Only covering the $\beta = 1$ case in the textbook, therefore, causes a misinterpretation of the crucial step, and adds confusion in interpreting the solution of this class of model.

4 What happened to the demand side?

One might still ask: what happened to the goods demand? Isn't there a general equilibrium of the system where the income from wage and profit go to the consumers, and the consumers demand goods? How can we guarantee that the goods market clear with the Y that we derived above? The answer is: Walras' Law. The markets in this economy are (i) final good, (ii) intermediate goods, and (iii) labor. The above procedure guarantees that (ii) and (iii) clear, and therefore (i) automatically clears.

The logic is perhaps easier to see in a perfectly-competitive economy. Consider an economy where there is only one kind of (final) good, and it is produced with the production

[2]To see this result more graphically, think of (5) as a best-response function of y_i with respect to Y. Rewriting (5),

$$y_i = \left[\frac{\beta}{w}\left(\frac{\sigma}{1-\sigma}\right)\right]^{\frac{\sigma\beta}{\sigma+\beta-\sigma\beta}} Y^{\frac{\beta}{\sigma+\beta-\sigma\beta}}.$$

When $\beta < 1$, this function crosses 45-degree line only once at $y_i > 0$. When $\beta = 1$, this function never crosses 45-degree line (other than $y_i = 0$) unless (8) is satisfied. If (8) is satisfied, this equation is 45-degree line and any $y_i = Y$ is a Nash equilibrium. Note that when $\beta > 1$, the second-order condition for the individual optimization is satisfied when $\beta < \sigma/(\sigma - 1)$ (that is, a mild increasing returns to scale is permitted in a monopoly situation), and the Nash equilibrium still exists, but the best-response function has a slope larger than 1 when it crosses the 45-degree line, and the Nash equilibrium is unstable with respect to the best response dynamics.

function

$$Y = L^{\beta}.$$

There is a unit mass of producers and the good price is normalized to one. The first-order condition for the producer's profit maximization is

$$w = \beta L^{\beta - 1}.$$

This equation, of course, is the labor demand curve. Combined with the labor supply curve, $L^s(w)$, we can determine w and L, and therefore Y. The consumer's income is wL plus the profit equals

$$\beta L^{\beta-1} \times L + [L^{\beta} - \beta L^{\beta-1} \times L] = L^{\beta}.$$

Of course this is equal to Y and the goods market clear automatically.

One might still ask: how about demand shocks? Can we consider a situation where the consumer suddenly wants to consume more? The answer is that we can. Consider the utility function

$$U = \alpha C - v(L),$$

where α is a "taste shock." v is increasing and convex. (I consider a utility linear in C to eliminate the wealth effect.) The budget constraint is $C = wL + \pi$, where π is the receipt of the profit. The first-order condition is

$$\alpha w = v'(L).$$

Thus the labor supply curve is

$$L^s = (v')^{-1}(\alpha w).$$

Thus the taste shock α influences the labor supply curve and the equilibrium computed in the steps of the previous section is influenced by the taste shock α. One can show that L and Y go up with α and w goes down with α when $\beta < 1$. Again, we don't have to talk about the "aggregate demand" as a separate entity.