MIT Shocks Imply Market Incompleteness*

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Abstract

The allocation after an unanticipated event (often called an "MIT shock") is different from the allocation of a corresponding complete-market model that explicitly considers the possibility of the shock, even when the probability of the event approaches zero.

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https://toshimukoyama.github.io/MyWebsite/MIT_shock_Appendix.pdf.

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1 Introduction

Many theoretical macroeconomic studies analyze a one-time (permanent or temporary) change in a variable, starting from a steady state of a model economy, followed by a deterministic transition path. The applications have been broad, including policy analysis, business cycles, and other shocks. The change in variable is interpreted as an unanticipated "shock," called the "MIT shock" in the recent literature, and the transition dynamics are interpreted as a response to the shock. One interpretation of this exercise is that the economy with the MIT shock (henceforth, *an implicit-uncertainty economy* or *an IU economy*) corresponds to the economy with explicit uncertainty regarding the same event (henceforth, *an explicit-uncertainty economy* or *an EU economy*) when the probability of the event approaches zero.

This paper, by way of a simple example, demonstrates that for this interpretation to be correct, one has to make certain assumptions about the market structure of the EU economy. In particular, if the EU economy has a complete asset market, the equilibrium allocation of such an EU economy does not necessarily converge to the MIT-shock allocation (i.e., the response to a deterministic change in a variable) as the shock probability approaches zero.

The intuition is simple. The model has two types of consumers. I consider a shock that redistributes wealth across these two types. The complete-market EU economy features perfect risk-sharing; the marginal rates of substitution across states are equal among consumers in the complete-market EU economy. By contrast, the IU economy, where the shock is not recognized ex ante (i.e., the shock is an MIT shock), has no mechanism that equates two consumers' marginal rates of substitutions between two states, one of which involves the MIT shock and the other does not. This outcome applies even if the IU economy features complete asset markets when the MIT shock is absent. The market structure of the IU economy, in fact, better resembles an incomplete-market model in which the asset markets involving MIT shocks are missing. In this paper's example, the equilibrium allocation in the IU economy is approximated by an EU economy without one of the Arrow securities. Therefore, the allocation after the MIT shock is an outcome of two assumptions about the underlying EU economy: (i) the (vanishingly) small probability of the shock event, and (ii) an incomplete asset market. In other words, the analysis utilizing MIT shocks implicitly assumes market incompleteness.

2 Literature

Boppart et al. (2018) describe an "MIT shock" as "an unpredictable shock to the steady-state equilibrium of an economy without shocks. ... in this economy no shocks are expected to ever materialize but nevertheless a shock now occurs." I follow their description in defining the MIT shock in this paper: the probability of the shock is considered zero, and no prior (contingent) arrangement is possible for the occurrence of the MIT shock.

The dynamic analysis utilizing such shocks, either in the form of exogenous shocks or policy changes, has been fruitfully applied widely in macroeconomics literature. Earlier examples are Abel and Blanchard (1983), Auerbach and Kotlikoff (1983), and Judd (1985). Recent examples include Boppart et al. (2018), Kaplan et al. (2018), Boar and Midrigan (2020), and Guerrieri et al. (2020).

The example in Section 3 is the most relevant to the analysis of redistribution policy. In fact, the experiment here has the same structure as the recent paper by Boar and Midrigan (2020). As I show below, by assuming the policy change occurs as an MIT shock, Boar and Midrigan's (2020) experiment implicitly assumes the consumers cannot make any ex-ante arrangements to insure against the policy change. This assumption is not entirely innocuous in their policy conclusion, because the policy change affects different consumers differently.

Beyond redistribution, this paper is relevant for any shocks and policies under heterogeneous agents, especially for ones that have a scope of (ex-ante) insuring each other. When consumers are homogeneous, the asset structure is irrelevant for most of the cases, because (in a symmetric equilibrium) the consumers do not benefit from insuring each other. With heterogeneous agents, the possibility of being able to make an ex-ante arrangement against shocks and policy changes can have important consequences. For example, Mukoyama (2013) clarifies that without an ex-ante arrangement, a policy reform that makes unemployment insurance more generous contains an implicit transfer from currently employed workers to currently unemployed workers.

Guerrieri et al. (2020) highlight a different response of the economy to a supply shock depending on the market structure. Their "complete markets" framework (e.g., their Section 2.1) resembles the zero-probability limit of the model described in the Section 3.2 below: the consumers can make insurance arrangements ex ante. Their "incomplete markets" framework (e.g., their Section 2.2) is similar to the bond economy in this paper's Appendix C. Therefore, although the shock is described as "unexpected" (their p. 31), their setting (in particular their "complete markets" setting) is better understood as the probability-zero limit of an EU economy, rather than an IU economy where the shock is truly unanticipated (as in this paper's Section 3.1).

In a more general context, by distinguishing between the IU economy and the EU economy, this paper provides a clear perspective on the role of government policy. For example, it reconciles the apparent need for government intervention in the IU economy ex post, even when the IU economy appears efficient before the shock, by showing the background EU economy *is* indeed inefficient due to market incompleteness.

3 Model

Consider a two-period endowment economy with two types of consumers, Type I and Type II. Each type has a continuum of population 1. Both types are price-takers and maximize the utility

$$u(c_1) + E[u(c_2)],$$

where c_1 is the consumption in period 1 and c_2 is the consumption in period 2. The expected value $E[\cdot]$ is taken in period 1. In period 1, both types receive the endowment 1. In period 2, uncertainty exists. In a *regular state*, which occurs with probability $(1-\pi)$, where $\pi \in [0,1]$, both types receive endowment 1. In an *irregular state*, which occurs with probability π, Type I receives $(1-\tau)$, where $\tau \in (0,1)$, and Type II receives $(1+\tau)$. Thus, in the irregular state, a transfer occurs from Type I to Type II. The function $u(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable.

3.1 MIT shock

First, I construct an economy that treats the shock as an MIT shock (the IU economy). As is mentioned above, this paper follows Boppart et al. (2018) in defining the MIT shock. In the context of the current model, when the occurrence of the irregular state is considered as an MIT shock, the irregular state is not anticipated (and therefore π is considered as zero) in period 1.

Then, the problem for a Type-I consumer is

$$\max_{c_1, c_2, a} \; u(c_1) + u(c_2)$$

subject to

$$c_1 + pa = 1$$

and

$$c_2 = 1 + a,$$

where p is the price of an Arrow security that pays out one unit of the consumption good in the regular state in period 2. Because the consumers perceive only the regular state, the economy has a complete asset market with one Arrow security under this perception. Consumption in period 1 and 2 is denoted as c_1 and c_2, respectively, and the security holding is represented by a.

The Type-II consumer faces the identical problem:

$$\max_{c'_1, c'_2, a'} \; u(c'_1) + u(c'_2),$$

subject to

$$c'_1 + pa' = 1$$

and

$$c'_2 = 1 + a',$$

where prime ($\prime$) denotes variables for the Type-II consumers.

It is straightforward to show the unique competitive equilibrium is with $p = 1$, $a = a' = 0$, and $c_1 = c'_1 = c_2 = c'_2 = 1$. Now suppose the MIT shock hits the economy. Then, the ex-post allocation will be $\tilde{c}_2 = 1 - \tau$ and $\tilde{c}_2' = 1 + \tau$. A tilde $\tilde{()}$ denotes the irregular state. Thus, the entire ex-post consumption allocation ends up with

$$(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau). \tag{1}$$

3.2 Complete market

In the next two sections, we construct EU economies by treating the uncertainty explicitly. For the first EU economy, consider the presence of a complete set of Arrow securities that spans all possible states, including the irregular state. In the current example, two Arrow securities exist, and each one pays one unit of consumption good when each state takes place. Let the price of the Arrow security that pays out in the regular state be p and the price of the security that pays out in the irregular state be $\tilde{p}$.

A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, a, \tilde{a}} \; u(c_1) + (1 - \pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + pa + \tilde{p}\tilde{a} = 1,$$

$$c_2 = 1 + a,$$

and

$$\tilde{c}_2 = 1 - \tau + \tilde{a},$$

where the other notations are identical to the previous section except that a and $\tilde{a}$ denote the holdings of different Arrow securities.

A Type-II consumer's problem is (with the same notation convention as in the last section)

$$\max_{c_1', c_2', \tilde{c}_2', a', \tilde{a}'} \; u(c_1') + (1 - \pi)u(c_2') + \pi u(\tilde{c}_2'),$$

subject to

$$c_1' + pa' + \tilde{p}\tilde{a}' = 1,$$

$$c_2' = 1 + a',$$

and

$$\tilde{c}_2' = 1 + \tau + \tilde{a}'.$$

One can confirm the competitive equilibrium, where the consumers' first-order conditions are satisfied and the market-clearing conditions

$$a + a' = 0$$

and

$$\tilde{a} + \tilde{a}' = 0$$

have the following solution:

$$p = 1 - \pi$$

and

$$\tilde{p} = \pi$$

with

$$(a, a', \tilde{a}, \tilde{a}') = \left(-\frac{\pi}{2}\tau, \frac{\pi}{2}\tau, \frac{2-\pi}{2}\tau, -\frac{2-\pi}{2}\tau\right).$$

The resulting consumptions are:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau\right);$$

that is, each consumer can smooth consumption across time and state.

To see which allocation corresponds to the probability-zero situation that can be compared with the MIT shock outcome, take $\pi \to 0$ and look at the consumption when the irregular shock hits. The result is

$$\lim_{\pi \to 0}(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1, 1).$$

This finding is in contrast to (1). The ex-post allocation of the complete-market outcome, even when π approaches zero, does not approximate the ex-post allocation with the MIT shock in the IU economy. The intuition is simple. The Arrow security for the irregular state becomes increasingly cheaper as $\pi \to 0$, and thus the Type-I consumers still demand the security to hedge against the irregular state even if the state rarely occurs. The Type-II consumers are willing to sell the security at a cheap price because the probability of the state is low.

3.3 Incomplete market

For the second EU economy, consider a situation where the asset market is incomplete. In particular, suppose the Arrow security does not exist for the irregular state. A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, a} \; u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + pa = 1,$$

$$c_2 = 1 + a,$$

and

$$\tilde{c}_2 = 1 - \tau.$$

For Type-II consumers,

$$\max_{c_1', c_2', \tilde{c}_2', a'} \; u(c_1') + (1-\pi)u(c_2') + \pi u(\tilde{c}_2'),$$

subject to

$$c_1' + pa' = 1,$$

$$c_2' = 1 + a',$$

and

$$\tilde{c}_2' = 1 + \tau.$$

The competitive equilibrium is $p = 1 - \pi$, $a = a' = 0$. The resulting consumptions are:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1, 1, 1 - \tau, 1 + \tau).$$

Thus, in the limit of $\pi \to 0$,

$$\lim_{\pi \to 0} (c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau),$$

which is identical to (1). Therefore, under the setting of incomplete asset markets, the outcome of the EU economy in the limit is identical to the outcome of the IU economy.

Appendix A discusses how the results here can be generalized. It emphasizes that the assets carried into the irregular state have to be consistent between the EU economy and the IU economy. The values of these assets may change with the shock, and therefore the assets have to be reevaluated properly.

4 Conclusion

Using an analytically tractable example, this paper showed that an "unanticipated MIT shock" analysis implicitly assumes not only that the probability of the event occurring is very small, but also that the asset market is incomplete. Because of the implicit market incompleteness, the distribution of wealth upon the occurrence of the shock can be different from the limit of the complete-market counterpart.

From a practical viewpoint, three principles must be followed when an MIT-shock analysis is employed. First, one must recognize that in the MIT-shock analysis, the state-contingent claim to the unanticipated state is missing. Second, the assets that can be carried into the unanticipated state in the MIT-shock analysis have to be consistent with the contingency description in the corresponding economy that explicitly considers uncertainty. Third, when an asset is carried into the unanticipated state in the MIT-shock analysis, the values of the assets may also change and thus have to be reevaluated properly.

Appendix B discusses further implications of this paper's results. The result that the MIT-shock allocation diverges from the complete-market allocation also poses a question on the analysis of optimal policy design, especially when the policy involves distributional effects. Typically, the switch to the optimal policy is considered an unanticipated permanent change. What if, instead, the agents in the model anticipate that with some probability, the government suddenly starts imposing the optimal policy, and the agents are allowed to trade securities for this event? This type of scenario seems to be closer to the spirit of the rational-expectations hypothesis, because it allows the agents in the model to be as smart as the model solvers who can figure out the optimal policy. What does the optimal policy that is consistent with the agents' anticipation ("expectations-consistent" optimal policy) look like? Finding this type of policy is a nontrivial fixed-point problem, because the ex-ante behavior of the agents affects the nature of the optimal policy. Is the expectations-consistent optimal policy similar to the outcome of the traditional optimal-policy analysis? These questions are important topics for future research.

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Online Appendix

A Discussions on generalizations

The logic in the main text can be generalized. In particular, the discrepancy between the IU outcome (Section 3.1) and the complete-market EU outcome (Section 3.2) would generalize to broader situations. The logic is simple. The IU outcome, by construction, has no structure that makes the marginal rate of substitution (between a state that involves the MIT shock and a state that does not) equal across consumers. The complete-market EU outcome has this equivalence built in, as long as the solutions for the consumers' problems are interior, no matter how small the shock probabilities are.

The equivalence results in Section 3.3 requires some qualifications in a more general setting. First, one can generally find an incomplete-market EU setting that delivers the same outcome as the IU allocation in the probability-zero limit; one can simply construct an EU setting where all contingency on the MIT-shock state is taken away and the rest of the asset structure is the same as in the IU economy. The EU allocation constructed in this manner would converge to the IU allocation (if the limit exists), as long as the equilibrium in the constructed EU economy and the equilibrium in the IU economy are both unique and the decision rules are continuous with respect to the change in environment.[1] Second, multiple incomplete-market EU settings that are consistent with a particular IU allocation can exist. Appendix C constructs an EU economy where the asset-market structure differs from the setting of Section 3.3 and it still delivers the allocation in Section 3.1 as a limit. (Appendix C considers a bond economy.) Third, what kind of market incompleteness in the EU economy corresponds to the IU outcome depends on what kind of asset structure is assumed in the IU setting.

To see the third point more formally, Appendix D modifies the model in Section 3 and repeats the analysis. The modification in Appendix D is that the consumers' equilibrium savings are nonzero, so that carrying assets (and liabilities) into the irregular state is possible. Appendix D shows that for the limit equivalence to hold, the asset holding that can be carried into the irregular state has to be consistent between the EU economy and the IU economy.

Note the values of the assets also have to be recalculated carefully when computing each consumer's asset holding upon the occurrence of the MIT shock. In models with a simple asset structure as in Huggett (1993) and Aiyagari (1994), reevaluating the assets that the consumers can carry over to the MIT-shock state is fairly straightforward. Young (2004) is such an example, where Aiyagari's (1994) model is extended to incorporate an endogenous job search.

However, one has to be more careful in a more complex economy such as Krusell et al.'s (2010). Mukoyama (2013) is an example of an MIT-shock analysis of such a model. There,

[1] One can use a similar logic as the one used for the proof of Proposition 10 in Krusell et al. (2011).

first, reevaluating the stock (the ownership value of firms) is necessary when the government policy changes, because the stock price jumps when the policy change (which is treated as an MIT shock) occurs.

In addition, in Mukoyama (2013), making an assumption about the composition of each consumer's portfolio before the MIT shock is necessary. When consumers don't anticipate the policy change, they are indifferent between which asset to hold in an IU economy, because the return structures are identical. However, because one asset incurs capital gain/loss with the policy change whereas the other does not, each consumer's asset holding after an MIT shock is affected by the portfolio composition before the shock. Mukoyama (2013) assumes a proportional portfolio (i.e., all consumers have the same portfolio ratio) before the shock. The same consideration applies in Kaplan et al. (2018).[2] This assumption may not be appropriate in interpreting the experiment as a limit of an EU economy, if the portfolio decision in the corresponding EU economy is different from the proportional rule. Boar and Midrigan (2020) also implicitly assume a proportional portfolio, because their setting features a financial intermediary that combines different assets into one asset that consumers hold.

Another potential situation where a particular care is necessary in reevaluating the asset upon an MIT shock is when some consumers default on their debt under certain states. In such a case, the balance of asset and liability may not carry over into certain specific states.

In sum, from a practical viewpoint, three principles must be followed when an MIT-shock analysis is employed. First, one must recognize that in the background EU economy, the state-contingent claim to the MIT shock is missing. Second, the assets that can be carried into the MIT-shock state in an IU economy have to be consistent with the contingency description in the corresponding EU economy. Third, when an asset is carried into the MIT-shock state in an IU economy, the value of an asset has to be carefully reevaluated.

B Implications

In the context of policy design, if one believes a particular event is truly unanticipated, the MIT-shock analysis using an IU economy framework, as in Section 3.1, can be a useful benchmark.[3] In this case, the analysis in Section 3 has some practical implications. In such an environment, because the background EU economy is an incomplete-market economy, the IU allocation with MIT shocks suffers from an inherent inefficiency, even if the IU economy appears efficient absent MIT shocks. Ex-post government interventions, such as transfers to the most affected sectors and consumers, can be justified from an ex-ante Pareto-efficiency perspective in the corresponding EU economy. In other words, this paper reconciles the apparent need for the government intervention of the IU economy ex post, even when the IU economy appears efficient before the shock, by showing the background EU economy *is*

[2] See their footnote 23.

[3] Alternatively, one can imagine a situation where writing a contract with many contingencies is costly, as in Dye (1985).

indeed inefficient.

From a purely theoretical perspective, the lesson is that one has to be careful when conducting an analysis of an economy after an MIT shock. Recall that the outcomes after the MIT shock and the small-probability limit of a complete-market EU model are different. Given that the effect of the missing market is typically distributional, the (positive) aggregate consequences are often unaffected by the underlying assumptions on the asset market if Gorman preferences are assumed. Even with Gorman preferences, however, in an environment where the markets have other imperfections, such as market power, sticky prices, and search frictions, the positive predictions can be affected. Many macroeconomic analyses consider a complete-market outcome as a benchmark, and one has to be aware that the analysis of the MIT shock may not deliver an approximate solution to this benchmark scenario even when the probability is small. As I discussed in Appendix A, even with an analysis that explicitly treats market incompleteness, the modeling decision of what asset can be carried into the shock state would have a nontrivial effect on the ex-post outcome.

In conducting normative analysis, the asset distribution after the shock can have a profound effect on the conclusions concerning the welfare effects of the shocks to individuals. In the example in Sections 3.1 and 3.2, having an Arrow security to carry into the after-the-shock state would change the individual welfare ex post. For example, in Mukoyama (2010), after establishing general results that allows for an arbitrary wealth distribution after the shock, I provide an example under the implicit assumption that the consumers are allowed to bring the asset ("trees") into the post-shock state, whereas they cannot bring in other securities. As I argued in Appendix A (and Appendix D), the limit result can differ depending on what kind of assets can be carried into the state after the shock. In this sense, the implicit assumption in that example (that no assets other than the trees can be brought in) is not innocuous when discussing the distributional effect of policy from a normative perspective.

The fact that an MIT shock "adds" an extra state ex post could cause a conceptual dilemma in the analysis of a redistribution policy. For example, in Boar and Midrigan's (2020) setting, the financial intermediary may incur a capital loss because the value of the production firm may fall as a result of the redistribution policy. To maintain zero profit ex post, the intermediary has to pass the capital loss to the consumer. This outcome implies that, ex ante, this type of provision has to be present in the contract between the consumers and the intermediary. To write down this type of provision, however, (i) the contract has to recognize that a capital-loss event may occur in future and (ii) different types of assets have to be treated differently in the contract. These two elements in the contract are difficult to square with the fact that the intermediary is content to offer only one type of asset to the consumers, because it can, in principle, offer different mixes of assets to different consumers so that the consumers can better hedge the risk of redistribution policy.

C Incomplete market: Bond economy

Here, consider another incomplete-market EU economy. Specifically, the consumers are only allowed to trade noncontingent bonds. For the ease of computation, I assume the utility function is quadratic:

$$u(c) = \alpha c - \frac{\gamma}{2}c^2, \tag{2}$$

where $\alpha > 0$ and $\gamma > 0$. The value of α is assumed to be sufficiently large so that utility is increasing in c over the relevant range.

A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, b} \; u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + qb = 1,$$

$$c_2 = 1 + b,$$

and

$$\tilde{c}_2 = 1 - \tau + b,$$

where q is the bond price and b is the bond holding. After solving for the first-order condition, the bond demand of Type-I consumers is

$$b = \frac{q(\gamma - \alpha) + \alpha - \gamma(1 - \pi\tau)}{\gamma(q^2 + 1)}.$$

Similarly, the Type-II consumer's problem is

$$\max_{c'_1, c'_2, \tilde{c}'_2, b'} \; u(c'_1) + (1-\pi)u(c'_2) + \pi u(\tilde{c}'_2),$$

subject to

$$c'_1 + qb' = 1,$$

$$c'_2 = 1 + b',$$

and

$$\tilde{c}'_2 = 1 + \tau + b',$$

The bond demand of the Type-II consumers is

$$b = \frac{q(\gamma - \alpha) + \alpha - \gamma(1 + \pi\tau)}{\gamma(q^2 + 1)}.$$

The bond price q is set so that the excess bond demand is zero:

$$b + b' = 0.$$

It is straightforward to derive that, in equilibrium,

$$q = 1$$

and

$$(b, b') = \left(\frac{\pi}{2}\tau, -\frac{\pi}{2}\tau\right)$$

hold. The resulting consumptions are:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \left(\frac{\pi}{2} - 1\right)\tau, 1 + \left(1 - \frac{\pi}{2}\right)\tau\right),$$

which achieves some but not perfect consumption smoothing.

In the limit of $\pi \to 0$, the consumption profile when the irregular state takes place in period 2 would approach

$$\lim_{\pi \to 0}(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau),$$

which is identical to (1). In the current example, the incomplete-market structure that delivers the equivalence to the MIT-shock outcome is not unique. The following section looks at the economy with one Arrow security.

D When the type of asset structures matters

For simplicity, in the main text (Section 3), I considered a setting where in both the IU economy (Section 3.1) and the EU economies (Sections 3.3 and Appendix C), the equilibrium security holdings are zero. This assumption has two consequences: (i) For the IU economy, the outcome is identical regardless of whether the assumption is that the existing security is (a) an Arrow security for the regular state or (b) a state-noncontingent bond. (ii) The limiting outcome of the EU economies in the Arrow-security economy (Section 3.3) and the bond economy (Appendix C) are both consistent with the IU outcome. Here, I consider a setting where the equilibrium security holdings in these situations are nonzero, and show that the type of asset structures in both the IU economies and the EU economies matters.

Consider the same setting as in Section 3, except for the endowments. A Type-I consumer receives the endowment of 0 in period 1 and 2 in the period-2 regular state. In the period-2 irregular state, a Type-I consumer receives $(2 - \tau)$, where $\tau \in (0, 1)$. A Type-II consumer receives 2 in period 1 and 0 in the period-2 regular state. In the period-2 irregular state, a Type-II consumer receives τ.

Below, I show several IU outcomes can exist depending on the asset structure, because the level of asset that can be carried into the MIT-shock state can differ depending on the assumed asset structure. I show several (incomplete-market) EU allocations can also exist depending on the asset structure, and one can map an EU allocation to a corresponding IU allocation.

The lesson of this section is that, when an MIT-shock experiment is conducted, one has to use the underlying asset structure that is consistent with a particular background EU economy. If the background EU economy is an incomplete-market economy with bonds, the consumers have to be allowed to carry over the bond holdings after the MIT shock. If the background EU economy features state-by-state Arrow security, one has to think carefully about whether to allow for a similar ex-ante arrangement for the MIT-shock state.

D.1 MIT shock: Arrow security economy

First, consider an MIT shock (IU) economy. Suppose that, before the shock, the economy permits an Arrow security contingent on the regular state. The problem for a Type-I consumer is

$$\max_{c_1, c_2, a} \; u(c_1) + u(c_2),$$

subject to

$$c_1 + pa = 0$$

and

$$c_2 = 2 + a,$$

where p is the price of the Arrow security that pays out one unit of the consumption good in the regular state period 2. Because the consumers perceive only the regular state, the IU economy has a complete asset market with one Arrow security. The consumption in period 1 and 2 is denoted as c_1 and c_2, respectively, and the security holding is represented by a.

The Type-II consumer faces the problem:

$$\max_{c_1', c_2', s'} \; u(c_1') + u(c_2'),$$

subject to

$$c_1' + pa' = 2$$

and

$$c_2' = a',$$

where prime ($'$) denotes variables for the Type-II consumers.

The unique competitive equilibrium is with $p = 1$, $a = -1$, $a' = 1$, and $c_1 = c_1' = c_2 = c_2' = 1$. Now suppose the MIT shock hits the economy. Because the irregular state is not spanned by the Arrow security, the ex-post allocation will be $\tilde{c}_2 = 2 - \tau$ and $\tilde{c}_2' = \tau$. (As in the main text, a tilde ($\tilde{\ }$) denotes the irregular state.) Thus, the entire ex-post allocation ends up with

$$(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 2 - \tau, \tau). \tag{3}$$

D.2 MIT shock: Bond economy

Second, alternatively, consider another MIT shock (IU) economy with a different asset structure. Suppose, instead, the asset in the economy is a noncontingent bond. The problem for a Type-I consumer is

$$\max_{c_1,c_2,a} \; u(c_1) + u(c_2),$$

subject to

$$c_1 + qb = 0$$

and

$$c_2 = 2 + b,$$

where q is the price of the bond that pays out one unit of the consumption good in the regular state period 2. Once again, because the consumers perceive only the regular state, the IU economy's asset market is complete. The consumption in period 1 and 2 is denoted as c_1 and c_2, respectively, and the bond holding is represented by b.

The Type-II consumer faces the identical problem:

$$\max_{c_1',c_2',s'} \; u(c_1') + u(c_2'),$$

subject to

$$c_1' + pb' = 2$$

and

$$c_2' = b',$$

where prime ($'$) denotes variables for the Type-II consumers.

The unique competitive equilibrium is with $q = 1$, $b = -1$, $b' = 1$, and $c_1 = c_1' = c_2 = c_2' = 1$. Now suppose the MIT shock hits the economy. Now, in contrast to the previous case, the noncontingent bond remains in the economy, and the ex-post allocation will be $\tilde{c}_2 = 1 - \tau$ and $\tilde{c}_2' = 1 + \tau$. A tilde ($\tilde{}$) denotes the irregular state. Thus, the entire ex-post allocation ends up with

$$(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau). \tag{4}$$

Thus, comparing (3) and (4), one can see that the allocation after the MIT shock is different depending on the asset structure.

D.3 Complete market

The next three sections consider EU economies. That is, I consider a situation where the irregular state occurs with probability π, and the consumers perceive that event. First, suppose the existence of a full set of Arrow securities. The notations are the same as in Section 3.2. A Type-I consumer's problem is

$$\max_{c_1,c_2,\tilde{c}_2,a,\tilde{a}} \; u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + pa + \tilde{p}\tilde{a} = 0,$$

$$c_2 = 2 + a,$$

and

$$\tilde{c}_2 = 2 - \tau + \tilde{a},$$

where a and $\tilde{a}$ denote the holding of Arrow securities.

A Type-II consumer's problem is (with the same notation convention as in the last section)

$$\max_{c_1', c_2', \tilde{c}_2', a', \tilde{a}'} \quad u(c_1') + (1-\pi)u(c_2') + \pi u(\tilde{c}_2'),$$

subject to

$$c_1' + pa' + \tilde{p}\tilde{a}' = 2,$$

$$c_2' = a',$$

and

$$\tilde{c}_2' = \tau + \tilde{a}'.$$

One can confirm the competitive equilibrium, where the consumer's first-order conditions are satisfied and the market-clearing conditions

$$a + a' = 0$$

and

$$\tilde{a} + \tilde{a}' = 0$$

have the following solution:

$$p = 1 - \pi$$

and

$$\tilde{p} = \pi$$

with

$$(a, a', \tilde{a}, \tilde{a}') = \left(-1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, -1 + \frac{2-\pi}{2}\tau, 1 - \frac{2-\pi}{2}\tau\right).$$

The resulting consumptions is:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau\right);$$

that is, each consumer can smooth consumption across time and state.

To see whether this allocation corresponds to one of the IU allocations, take $\pi \to 0$ and look at the consumption when the irregular shock hits. The result is

$$\lim_{\pi \to 0} (c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1, 1).$$

The outcome is different from both (3) and (4).

D.4 Incomplete market: Arrow security economy

Now, consider another EU economy. Suppose, as in Section 3.3, the Arrow security does not exist for the irregular state although the consumers recognize the possibility of the irregular state in the future. A Type-I consumer's problem is

$$\max_{c_1,c_2,\tilde{c}_2,a} \; u(c_1)+(1-\pi)u(c_2)+\pi u(\tilde{c}_2),$$

subject to

$$c_1+pa=0,$$

$$c_2=2+a,$$

and

$$\tilde{c}_2=2-\tau.$$

For Type-II consumers,

$$\max_{c_1',c_2',\tilde{c}_2',a'} \; u(c_1')+(1-\pi)u(c_2')+\pi u(\tilde{c}_2'),$$

subject to

$$c_1'+pa'=2,$$

$$c_2'=a',$$

and

$$\tilde{c}_2'=\tau.$$

The competitive equilibrium is $p=1-\pi$, $a=-1$, $a'=1$. The resulting consumption is:

$$(c_1,c_1',c_2,c_2',\tilde{c}_2,\tilde{c}_2')=(1,1,1,1,2-\tau,\tau).$$

Thus, in the limit of $\pi\to 0$,

$$\lim_{\pi\to 0}(c_1,c_1',\tilde{c}_2,\tilde{c}_2')=(1,1,2-\tau,\tau),$$

which is identical to (3).

D.5 Incomplete market: Bond economy

For yet another EU economy, consider an economy with only a state-noncontingent bond. Similarly to Appendix C, I assume the utility function is quadratic:

$$u(c)=\alpha c-\frac{\gamma}{2}c^2, \tag{5}$$

where $\alpha>0$ and $\gamma>0$. The value of α is assumed to be sufficiently large so that utility is increasing in c over the relevant range.

A Type-I consumer's problem is

$$\max_{c_1,c_2,\tilde{c}_2,b} \; u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + qb = 0,$$

$$c_2 = 2 + b,$$

and

$$\tilde{c}_2 = 2 - \tau + b,$$

where q is the bond price and b is the bond holding. After solving for the first-order condition, the bond demand of Type-I consumers is

$$b = \frac{-q\alpha + \alpha - \gamma(2 - \pi\tau)}{\gamma(q^2+1)}.$$

Similarly, the Type-II consumer's problem is

$$\max_{c_1',c_2',\tilde{c}_2',b'} \; u(c_1') + (1-\pi)u(c_2') + \pi u(\tilde{c}_2'),$$

subject to

$$c_1' + qb' = 2,$$

$$c_2' = b',$$

and

$$\tilde{c}_2' = \tau + b',$$

The bond demand of the Type-II consumers is

$$b = \frac{q(2\gamma - \alpha) + \alpha - \gamma\pi\tau}{\gamma(q^2+1)}.$$

The bond price q is set so that

$$b + b' = 0,$$

and therefore, in equilibrium,

$$q = 1$$

and

$$(b, b') = \left(\frac{\pi}{2}\tau - 1, 1 - \frac{\pi}{2}\tau\right)$$

hold. The resulting consumptions are

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \left(\frac{\pi}{2} - 1\right)\tau, 1 + \left(1 - \frac{\pi}{2}\right)\tau\right).$$

In the limit of $\pi \to 0$, the consumption profile when the irregular state takes place in period 2 would approach

$$\lim_{\pi \to 0}(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau),$$

which is identical to (4).

The analysis above first confirms the conclusion of the main text: the probability-zero limit of an EU economy allocation converges to an IU economy allocation only when the asset market is missing for the MIT-shock state. A new point this section makes is that the type of incompleteness matters. Here, the IU outcome in an Arrow-security economy (Section D.1) can be approximated by an appropriate EU economy with an Arrow security (Section D.4), and the outcome in an IU economy with a bond (Section D.2) can be approximated by an incomplete-market EU economy with a bond (Section D.5).

The principle here is that the asset holding that can be carried into the irregular state has to be consistent between the EU economy and the IU economy. The value of the assets also have to be reevaluated carefully when computing each consumer's asset holding upon the occurrence of the MIT shock.

This principle may sound obvious. In some situations, however, reevaluating the asset value coming into the MIT-shock economy requires careful examination. For example, suppose the asset value of a consumer, a, is a sum of a stock px, where p is the stock price and x is the quantity of the stock holding, and a bond b; therefore,

$$a = px + b.$$

When the realization of the irregular state moves the stock price from p to $\hat{p}$, the asset has to be reevaluated as

$$\hat{a} \equiv \hat{p}x + b$$

when starting the MIT-shock state, even though both the stock and bond can be carried into the MIT-shock state. The reevaluation would not be a big issue in an economy with a simple asset structure as in Huggett (1993) and Aiyagari (1994), but would matter in a more complex economy such as Krusell et al.'s (2010) (for which Mukoyama (2013) is an example of an MIT-shock analysis), as discussed above.