

Barriers to Reallocation and Economic Growth: the Effects of Firing Costs*

Toshihiko Mukoyama
Georgetown University
tm1309@georgetown.edu

Sophie Osotimehin
University of Virginia
sophie.osotimehin@virginia.edu

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Abstract

We study how factors that hinder the reallocation of inputs across firms influence aggregate productivity growth. We extend Hopenhayn and Rogerson's (1993) general equilibrium firm dynamics model to allow for endogenous innovation. We calibrate the model using US data, and then evaluate the effects of firing taxes on reallocation, innovation, and aggregate productivity growth. In our baseline specification, we find that firing taxes reduce overall innovation and productivity growth. We also show that firing taxes can have opposite effects on the entrants' innovation and the incumbents' innovation, and thus the overall outcome depends on the relative strengths of these forces.

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1 Introduction

Recent empirical studies have underlined the existence of large flows of productive resources across firms and their important role for aggregate productivity. Production inputs are constantly being reallocated as firms adjust to changing market environments, and new products and techniques are developed. As documented recently by Micco and Pagés (2007) and Haltiwanger et al. (2014), labor market regulations may dampen this reallocation of resources. Using cross-country industry-level data, these studies show that restrictions on hiring and firing reduce the pace of both job creation and job destruction. In a similar vein, Davis and Haltiwanger (2014) find that the introduction of common-law exceptions that limit firms' ability to fire their employees at will has a negative impact on job reallocation in the United States.

The objective of this paper is to study the implications of firing regulations for aggregate productivity growth. By reducing job reallocation across firms, firing costs may affect not only the level of aggregate productivity, but they are also likely to modify the firms' incentives to innovate. We investigate the consequences of firing costs on job reallocation and productivity growth using a model of innovation-based economic growth. We extend Hopenhayn and Rogerson's (1993) model of firm dynamics by introducing an innovation decision. Firms can invest in research and development (R&D) and improve the quality of products. Hence, in contrast to Hopenhayn and Rogerson's (1993) model (and the Hopenhayn (1992) model that it is based on) where the productivity process is entirely exogenous, job creation and job destruction in our model are the result of both idiosyncratic exogenous productivity shocks and *endogenous* innovation.

Following the seminal work of Grossman and Helpman (1991) and Aghion and Howitt (1992), we model innovation as a process of *creative destruction*: entrants displace the incumbent producers when they successfully innovate on an existing product. In addition to this Schumpeterian feature, we incorporate the innovations developed by incumbent firms. We allow incumbent firms to invest in R&D to improve the quality of their own product. The model is parsimonious and can be characterized analytically in the absence of firing costs. In particular, we show how the innovation rate of entrants and incumbents shape the growth rate of the economy and the firm size distribution. The frictionless model highlights the crucial role of reallocation for economic growth. As products of higher quality are introduced into the market, labor is reallocated towards these high-quality firms.¹ By limiting the reallocation of labor across firms, firing costs change the

¹Aghion and Howitt (1994) is an earlier study that highlights this aspect of the Schumpeterian growth model in their analysis of unemployment.

firms' incentives to innovate and hence change the growth rate of the economy.

We model firing costs as a tax and study its effect on innovation and growth. We find that the effects of firing taxes on aggregate productivity growth depend on the interaction between the innovation of entrants and incumbents. In fact, a firing tax can have opposite effects on entrants' and incumbents' innovation: while a firing tax tends to reduce entrants' innovation, it may raise the innovation incentives of incumbent firms. A firing tax reduces the entrants' innovation because the tax itself represents an additional cost that reduces expected future profits (direct effect). In addition, the misallocation of labor further reduces expected future profits (misallocation effect). For incumbents, the consequences of a firing tax are less clear-cut. In particular, a firing tax has an ambiguous impact on the incumbents' incentive to innovate. Firms that are larger than their optimal size have additional incentives to invest in R&D in the presence of a firing tax. For those firms, innovating has the added benefit of allowing them to avoid paying the firing tax, as they would no longer need to downsize if the quality of their product were higher (tax-escaping effect). By contrast, for firms that are smaller than their optimal size, the direct effect and the misallocation effect tend to discourage innovation. In addition, the incumbents' incentive to innovate is affected by the rate at which entrants innovate. By reducing the entry rate, firing costs lower the incumbent's probability of being taken over by an entrant. This decline in the rate of creative destruction raises the expected return of R&D investments and therefore tends to raise the incumbents' innovation (creative-destruction effect). In the quantitative analysis, we find that the positive effects dominate and the incumbents' innovation increases as a result of a firing tax in our baseline case.

With the fall in the entrants' innovation rate and the increase in the incumbents' innovation rate, the overall effect of a firing tax on growth depends on the importance of the two types of innovation for growth.² In our baseline calibration, in which entry is the main driver of aggregate productivity growth, the negative effect on entrants dominates, and the firing tax leads to a fall in the rate of growth of aggregate productivity. Our results illustrate the importance of including the incumbents' innovation in the analysis: the fall in the growth rate is dampened by the response of the incumbents' innovation, and ignoring this dimension would have led to overestimating the decline in the growth

²Saint-Paul (2002) makes a related argument that countries with a rigid labor market tend to produce relatively secure goods at a late stage of their product life cycle, so that these countries tend to specialize in 'secondary' innovations. A country with a more flexible labor market tends to specialize in 'primary' innovations. Thus increasing firing costs may encourage 'secondary' innovations, and the effect on aggregate growth depends on which type of innovation is more important. Bartelsman et al. (2016) propose a related model and provide evidence that countries with higher firing costs have relatively smaller high-risk innovation sectors.

rate. This result has implications beyond the study of firing costs. Regulations or market imperfections that reduce the entry rate are likely to have a weaker impact on growth once the incumbents' innovation is accounted for.

Since the overall outcome on growth depends on the relative strength of two opposing forces, we have conducted various additional analyses not only to uncover the key features behind our baseline result but also to evaluate the size of the growth effect. We find that the entrants' contribution to aggregate productivity growth (in the absence of firing costs) is crucial for the overall outcome. If the entrants' contribution to growth is small, then firing taxes can raise aggregate productivity growth. Given that the entrants' role for growth differ across economies, the general message of the paper is more nuanced than that of studies focusing on the level effect, in which firing taxes are always detrimental to the level of aggregate productivity. We find that the overall negative effect in the baseline case is similar to the growth effect of a 5% labor tax. This, however, does not mean that the welfare consequences of the reduction in growth, caused by firing taxes, is moderate. A back-of-the-envelope calculation suggests that the growth effect lowers consumer welfare more than the level effect does.

The negative effect of the firing tax on growth, suggested by our baseline calibration, can be found in recent empirical studies on the topic. In some recent studies, firing regulations have been shown to have a negative effect on the level but also on the growth rate of aggregate productivity. For example, Autor et al. (2007) estimate how common-law restriction that limits firms' ability to fire (the "good faith exception") in the US had a detrimental effect on state total factor productivity in manufacturing. Bassanini et al. (2009) find that firing costs tend to reduce total factor productivity growth in industries where firing costs are more likely to be binding.³ Meanwhile, some studies, such as Acharya et al. (2013) and Ueda and Claessens (2016), find that there are situations in which employment protection regulations can have positive effects on innovation and growth. These conflicting results are not inconsistent with our theoretical model, which uncovers various forces that affect the aggregate growth rate in opposite directions. The message of our model is that, in evaluating the growth effects of employment protection, we have to carefully analyze the strengths of the different forces that shape both the composition of innovation and the aggregate growth rate.

Our paper is related to several theoretical papers that study the consequences of firing costs on aggregate productivity. The existing literature, however, has mainly focused on the effects of firing costs on the *level* of aggregate productivity. Using a general equilibrium model of firm dynamics, Hopenhayn and Rogerson (1993) and more

³See also Andrews et al. (2015) and Cetto et al. (2016) for similar results.

recently Moscoso Boedo and Mukoyama (2012) and Da-Rocha et al. (2016) have shown that firing costs hinder job reallocation and reduce allocative efficiency and aggregate productivity.⁴ In line with these papers, we find that the *level* of employment and labor productivity falls. We show that, in addition to the level effects, employment protection also affects the *growth rate* of aggregate productivity.

In focusing on the consequences of barriers to labor reallocation on aggregate productivity growth, our analysis goes one step beyond the recent literature on misallocation that focuses on the level effects, following the seminal work of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Empirical studies that evaluate the contribution of reallocation to productivity changes, such as Foster et al. (2001) and Osotimehin (2016), are designed to analyze the sources of productivity growth, rather than the level; in that sense, our analysis is more comparable to that literature. We highlight that barriers to reallocation affect not only the allocation of resources across firms with different productivity levels, but also the productivity *process* itself as it modifies the firms' incentives to innovate. The additional effect of barriers to reallocation when productivity is endogenous is also the focus of Gabler and Poschke (2013) and Bento and Restuccia (forthcoming).⁵ In contrast to our study, their focus is, as in the studies cited above, exclusively on the level of aggregate productivity. Samaniego (2006b) highlights the effects of firing costs in a model with productivity growth. He considers, however, only exogenous productivity growth and studies how the effects of firing costs differ across industries.⁶ Poschke (2009) is one of the few exceptions that studies the effects of firing costs on aggregate productivity growth. In Poschke (2009), firing costs act as an exit tax, which lowers the exit rate of low productivity firms. We focus on a different channel and show that firing costs may also affect aggregate productivity growth through their effects on R&D and innovation.⁷

⁴Hopenhayn and Rogerson (1993) find that a firing cost that amounts to one year of wages reduces aggregate total factor productivity by 2%. Moscoso Boedo and Mukoyama (2012) consider a wider range of countries, and show that firing costs calibrated to match the level observed in low income countries can reduce aggregate total factor productivity by 7%. Da-Rocha et al. (2016) analyzes a stylized continuous-time model where firm-level employment can only take two different values, and also find that the firing cost reduces aggregate productivity.

⁵In Gabler and Poschke (2013), firms grow by engaging in risky experimentation, and firing costs lead to a small *increase* in experimentation. Bento and Restuccia (forthcoming) show that policy distortions that are positively correlated to establishment-level productivity imply larger reductions in aggregate productivity when productivity is endogenous.

⁶He finds that firing costs have a stronger negative impact in industries where the rate of technical change is rapid. In a related paper, Samaniego (2008) finds that the increase in aggregate employment induced by embodied technical change is smaller in the presence of firing costs.

⁷Bertola (1991) is an earlier paper that analyzes the growth effect of firing costs. His analysis is mostly qualitative.

Our paper is also related to the growing literature on innovation and firm dynamics that follows the contribution by Klette and Kortum (2004). In particular, our paper is related to Acemoglu et al. (2013) that study the consequences of R&D subsidies and the allocation of R&D workers across firms. By contrast, our paper studies the effect of the allocation of production workers across firms. Also related are models by Akcigit and Kerr (2015), Acemoglu and Cao (2015), and Peters (2016) that consider quality-ladder firm dynamics models in which incumbents are allowed to innovate on their own products.⁸ Our model also exhibits this feature but focuses on a distinct question. Compared to these models, one important difference of our approach is that we use labor market data to discipline the model parameters, consistently with our focus on labor market reallocation and labor market policy.⁹ Methodologically, while these models are typically written in continuous time, we use a discrete-time framework.¹⁰ This modeling strategy allows us to solve the model with firing taxes using a similar method to those used for standard heterogeneous-agent models (such as Huggett (1993) and Aiyagari (1994)) and standard firm-dynamics models (such as Hopenhayn and Rogerson (1993) and Lee and Mukoyama (2008)). Using this method is particularly important for our model, since firing taxes introduce a kink in the return function and makes it difficult to fully characterize the model analytically. The solution method also allows us to easily extend the model and to introduce several features that improve the model’s fit.

The paper is organized as follows. Section 2 sets up the model. Section 3 provides an analytical characterization of the model. Section 4 describes the quantitative analysis. Section 5 analyzes two extensions of the baseline model and also discusses the robustness of baseline results. Section 6 concludes.

2 Model

We build a model of firm dynamics in the spirit of Hopenhayn and Rogerson (1993). We extend their framework to allow for endogenous firm-level productivity. The innovation process is built on the classic quality-ladder models of Grossman and Helpman (1991) and Aghion and Howitt (1992), and also on the recent models of Acemoglu and Cao (2015) and Akcigit and Kerr (2015).

⁸Earlier papers that analyze incumbents’ innovations in the quality-ladder framework include Segerstrom and Zolnierok (1999), Aghion et al. (2001), and Mukoyama (2003).

⁹Garcia-Macia et al. (2016) also utilizes labor market data to quantify their model, innovation is however exogenous in their model.

¹⁰Ates and Saffie (2016) is another recent contribution based on a discrete-time formulation of the Klette-Kortum model.

There is a continuum of differentiated intermediate goods on the unit interval $[0, 1]$ and firms, both entrants and incumbents, innovate by improving the quality of these intermediate goods. Final goods are produced from the intermediate goods in a competitive final good sector. We first describe the optimal aggregate consumption choice. We then describe the final good sector and the demand for each intermediate good. We then turn to the decisions of the intermediate goods firms which constitute the core of the model. Finally, we present the balanced growth equilibrium.

2.1 Consumers

The utility function of the representative consumer has the following form:

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t [\log(C_t) - \xi L_t],$$

where C_t is consumption at time t , L_t is labor supply at time t , $\beta \in (0, 1)$ is the discount factor, and $\xi > 0$ is the parameter of the disutility of labor. Similarly to Hopenhayn and Rogerson (1993), we adopt the indivisible-labor formulation of Rogerson (1988), and L_t represents the fraction of individuals who are employed at time t .

The consumer's budget constraint is

$$A_{t+1} + C_t = (1 + r_t)A_t + w_t L_t + T_t,$$

where

$$A_t = \int_{\mathcal{N}_t} V_t^j dj$$

is the asset holding. The representative consumer owns all the firms; V_t^j indicates the value of a firm that produces product j at time t , and \mathcal{N}_t is the set of products that are actively produced at time t .¹¹ In the budget constraint, r_t is the net return of the asset; w_t is the wage rate; and T_t is a lump-sum transfer used to transfer the income from the firing tax to the consumer.

The consumer's optimization results in two first-order conditions. The first is the Euler equation:

$$\frac{1}{C_t} = \beta(1 + r_{t+1})\frac{1}{C_{t+1}}, \tag{1}$$

¹¹We do not distinguish firms and establishments in this paper. Later we use establishment-level data in our calibration. Using firm-level data yields similar results.

and the second is the optimal labor-leisure choice:

$$\frac{w_t}{C_t} = \xi. \quad (2)$$

2.2 Final good firms

The final good Y_t is produced by the technology

$$Y_t = \left(\int_{\mathcal{N}_t} \mathbf{q}_{jt}^\psi y_{jt}^{1-\psi} dj \right)^{\frac{1}{1-\psi}}.$$

The price of Y_t is normalized to one, y_{jt} is the amount of intermediate product j used at time t , and \mathbf{q}_{jt} is the *realized quality* of intermediate product j .¹² The realized quality is the combination of the *potential quality* q_{jt} , which depends on the innovation decision of intermediate-good firms, and an exogenous transitory shock α_{jt} :

$$\mathbf{q}_{jt} = \alpha_{jt} q_{jt}.$$

We assume that α_{jt} is i.i.d. across time and products.¹³ We also assume that the transitory shock is a product-specific shock rather than a firm-specific shock, so that the value of α_{jt} does not alter the ranking of the realized quality compared to the potential quality.¹⁴

Let the *average potential quality* of intermediate goods be

$$\bar{q}_t \equiv \frac{1}{N_t} \left(\int_{\mathcal{N}_t} q_{jt} dj \right),$$

where N_t is the number of actively produced products, and the *quality index* Q_t be

$$Q_t \equiv \bar{q}_t^{\frac{\psi}{1-\psi}}.$$

Note that the quality index grows at the same rate as aggregate output Y_t along the balanced-growth path.

The final good sector is perfectly competitive, and the problem for the representative

¹²Similar formulations are used by Luttmer (2007), Acemoglu and Cao (2015), and Acikigit and Kerr (2015), among others.

¹³The i.i.d. assumption across time is relaxed in Section 5.1.1.

¹⁴If the shock is at the firm level, it is possible that the incumbent firm i 's realized quality $\alpha_{it} q_{it}$ is larger than the new firm j 's realized quality $\alpha_{jt} q_{jt}$ even if $q_{jt} > q_{it}$.

final good firm is

$$\max_{y_{jt}} \left(\int_{\mathcal{N}_t} \mathbf{q}_{jt}^\psi y_{jt}^{1-\psi} dj \right)^{\frac{1}{1-\psi}} - \int_{\mathcal{N}_t} p_{jt} y_{jt} dj.$$

The first-order condition leads to the inverse demand function for y_{jt} :

$$p_{jt} = \mathbf{q}_{jt}^\psi y_{jt}^{-\psi} Y_t^\psi. \quad (3)$$

Final-good firms are introduced for ease of exposition; as in the standard R&D-based growth models, one can easily transform this formulation into a model without final goods, assuming that the consumers and the firms engaging in R&D activities combine the intermediate goods on their own.¹⁵ In this sense, the final-good sector is a veil in the model, and we will ignore the final-good firms when we map the model to the firm dynamics data.

2.3 Intermediate-good firms

The core of the model is the dynamics of the heterogeneous intermediate-good firms. Each intermediate-good firm produces one differentiated product and is the monopolist producer of that product. Intermediate-good firms enter the market, hire workers, and produce. Depending on the changes in the quality of their products, they expand or contract over time, and they may be forced to exit. Compared to standard firm dynamics models, the novelty of our model is that these dynamics are largely driven by endogenous innovations.

We consider two sources of innovations. One is the *innovation by incumbents*: an incumbent can invest in R&D to improve the potential quality of its own product. The other is the *innovation by entrants*: an entrant can invest in R&D to innovate on a product that is either (i) not currently produced, or (ii) currently produced by another firm.¹⁶ If the entrant is successful at innovating, the entrant becomes the monopolist for that product and displaces the incumbent monopolist whenever the product is currently

¹⁵See, for example, Barro and Sala-i-Martin (2004).

¹⁶In our model, the only way incumbents can innovate is by improving the quality of the products they are currently producing. While we do not explicitly model the creative destruction by incumbents, one can interpret that this margin as being captured by the entry component. In this model, a unit of production (“a firm”) is a single product line. Creative destruction from entry here can therefore be interpreted as the displacement caused by the innovation of both new firms and incumbent firms (when they innovate on products they are not currently producing). Assuming that incumbent firms open new establishments when they add a product line, our calibration is consistent with this interpretation. In fact, below we calibrate the size of the creative destruction effect using the job creation by new *establishments*, which includes the opening of new establishments by incumbent firms.

produced by an incumbent. The previous producer is, as a result, forced to exit.¹⁷

2.3.1 Production of intermediate goods

Each product j is produced by the leading-edge monopolist who produces the highest quality for that particular product. The firm's production follows a linear technology

$$y_{jt} = \ell_{jt},$$

where ℓ_{jt} is the labor input. Our main policy experiment is to impose a firing tax on intermediate-good firms. We assume that the firm must pay the tax τw_t for each worker fired,¹⁸ including when the firm exits.¹⁹

2.3.2 Innovation by incumbents

The incumbent producer can innovate on its own product. The probability that an incumbent innovates on its product at time t is denoted x_{Ijt} . A successful innovation increases the potential quality of the product from q_{jt} to $(1 + \lambda_I)q_{jt}$, where $\lambda_I > 0$, in the following period. The cost of innovation, \mathbf{r}_{Ijt} , is assumed to be

$$\mathbf{r}_{Ijt} = \theta_I Q_t \frac{q_{jt}}{q_t} x_{Ijt}^\gamma,$$

where $\gamma > 1$ and θ_I are parameters.²⁰

2.3.3 Innovation by entrants

A potential entrant enters after having successfully innovated on an intermediate good that is either currently produced by an incumbent or not currently produced. In order

¹⁷Instead of assuming that the lower-quality producer automatically exits, we could resort to a market participation game with price competition as in Akcigit and Kerr (2015).

¹⁸Following the literature (e.g. Hopenhayn and Rogerson (1993)), we assume that the firing costs are incurred only when the firm contracts or exits (that is, only when job destruction occurs). As is well documented (see, for example, Burgess et al. (2000)), worker flows are typically larger than job flows. The implicit assumption here is that all worker separations that are not counted as job destruction are voluntary quits that are not subject to the firing tax.

¹⁹An alternative specification is to assume that the firm does not need to incur firing costs when it exits. See Samaniego (2006a) and Moscoso Boedo and Mukoyama (2012) for discussions.

²⁰The assumption that the innovation cost increases with productivity is frequently used in endogenous growth literature. See, for example, Segerstrom (1998), Howitt (2000), and Akcigit and Kerr (2015). Kortum (1997) provides empirical support for this assumption in a time-series context.

to innovate, a potential entrant must spend a fixed cost ϕQ_t and a variable cost

$$\mathbf{r}_{Ejt} = \theta_E Q_t x_{Ejt}^\gamma$$

to innovate with probability x_{Ejt} .²¹ Here, ϕ , γ and θ_E are parameters. A successful innovation increases the quality of product j from q_{jt} to $(1+\lambda_E)q_{jt}$ in the following period. The innovation step for the entrants, λ_E , is allowed to be different from the incumbents' innovation step λ_I . We assume that the entrants' innovation is not targeted: each entrant innovates on a randomly selected product. The entrants choose their innovation probability before learning the quality of the product they will innovate upon. An entrant innovates on an existing product with probability N_t , and on an inactive product with probability $1 - N_t$. We assume that innovating over a vacant line improves the quality of the product over a quality drawn from a given distribution $h(\hat{q})$. We denote by m_t the mass of potential entrants.

2.3.4 Exit

Firms can exit for two reasons: (i) the product line is taken over by an entrant with a better quality; (ii) the firm is hit by an exogenous, one-hoss-shay depreciation shock (exit shock). While exit is an exogenous shock from the viewpoint of the incumbent firm in both cases, the first type of exit is endogenously determined in equilibrium.²²

The probability that an incumbent is taken over by an entrant is denoted μ_t . As we will see, this probability, which we also call the rate of creative destruction, depends on the mass of potential entrants and on the innovation intensity of each entrant. The probability of the depreciation shock, assumed to be constant across firms, is denoted by $\delta \in (0, 1)$. After this shock, the product becomes inactive until a new entrant picks up that product. From a technical viewpoint, the depreciation shock enables the economy to have a stationary distribution of (relative) firm productivity.²³

2.4 Timing of events and value functions

The timing of events in the model is the following. Below, we omit the firm subscript j when there is no risk of confusion.

²¹Bollard et al. (2016) provide empirical support for the assumption that entry costs increase with productivity.

²²Note that, under the assumptions above, a firm never find it optimal to voluntarily exit. Even when the firing tax exists, the strategy of operating in a small scale today and exiting tomorrow dominates exiting immediately.

²³See, for example, Gabaix (2009).

At the beginning of period t , all innovations from last period's R&D spending realize. Incumbent firms must exit if an entrant has innovated on their product line, including when the incumbent and the entrant innovate at the same time. Then the transitory productivity shock realizes. The firms (including new entrants) receive the depreciation shock with probability δ . Exiting firms pay the firing cost. Potential entrants and incumbents decide on their innovation rate, and at the same time, incumbents choose their employment level and pay the firing costs whenever they contract. The labor market clears and production takes place. The consumer chooses his consumption and saving.

We now express the firm's optimization problem as a dynamic programming problem. The expected value for the firm at the beginning of the period (after receiving the transitory shock and before receiving the depreciation shock) is

$$Z_t(q_t, \alpha_t, \ell_{t-1}) = (1 - \delta)V_t^s(q_t, \alpha_t, \ell_{t-1}) + \delta V_t^o(\ell_{t-1}).$$

The first term on the right-hand side is the value from surviving and the second term is the value from exiting due to the exogenous exit shock. When exiting, the firm has to pay a firing tax on all the workers fired. The value of exiting is then

$$V_t^o(\ell_{t-1}) = -\tau w_t \ell_{t-1}.$$

The value of survival is

$$V_t^s(q_t, \alpha_t, \ell_{t-1}) = \max_{\ell_t, x_{It}} \left\{ \Pi_t(q_t, \alpha_t, \ell_{t-1}, \ell_t, x_{It}) + \frac{1}{1 + r_{t+1}} \left((1 - \mu_t) S_{t+1}(x_{It}, q_t, \ell_t) - \mu_t \tau w_{t+1} \ell_t \right) \right\}.$$

Here, $S_{t+1}(x_{It}, q_t, \ell_t)$ is the value of not being displaced by an entrant and μ_t is the probability of being displaced by an entrant. The value of not being displaced by an entrant is

$$S_{t+1}(x_{It}, q_t, \ell_t) = (1 - x_{It}) E_{\alpha_{t+1}}[Z_{t+1}(q_t, \alpha_{t+1}, \ell_t)] + x_{It} E_{\alpha_{t+1}}[Z_{t+1}((1 + \lambda_I)q_t, \alpha_{t+1}, \ell_t)],$$

where $E_{\alpha_{t+1}}[\cdot]$ is the expected value with respect to α_{t+1} and the period profit is

$$\Pi_t(q_t, \alpha_t, \ell_{t-1}, \ell_t, x_{It}) = ([\alpha_t q_t]^\psi \ell_t^{-\psi} Y_t^\psi - w_t) \ell_t - \theta_I Q_t \frac{q_t}{\bar{q}_t} x_{It}^\gamma - \tau w_t \max(0, \ell_{t-1} - \ell_t),$$

where the inverse demand function is obtained from equation (3).

We assume free entry, that is, anyone can become a potential entrant by paying the entry costs. The free entry condition for potential entrants is

$$\max_{x_{E_t}} \left\{ -\theta_E Q_t x_{E_t}^\gamma - \phi Q_t + \frac{1}{1+r_t} x_{E_t} \bar{V}_{E,t+1} \right\} = 0, \quad (4)$$

where $\bar{V}_{E,t+1}$ is the expected value of an entrant at time $t+1$. Because the entrant decides on its innovation probability before learning its quality draw, the expected value $\bar{V}_{E,t+1}$ is constant across potential entrants as is the innovation probability. The optimal value of the innovation probability, $x_{E_t}^*$, is determined by

$$\frac{1}{1+r_t} \bar{V}_{E,t+1} - \gamma \theta_E Q_t x_{E_t}^{*\gamma-1} = 0. \quad (5)$$

Note that x_E^* is not affected by the firing tax. The response of the entry rate to changes in the firing tax occurs through variation in the mass of potential entrants m_t . From (4) and (5), $x_{E_t}^*$ satisfies

$$-\theta_E x_{E_t}^{*\gamma} - \phi + \gamma \theta_E x_{E_t}^{*\gamma} = 0$$

and thus $x_{E_t}^*$ is a constant number x_E^* that can easily be solved as a function of parameters. The solution is

$$x_E^* = \left(\frac{\phi}{\theta_E(\gamma-1)} \right)^{\frac{1}{\gamma}}. \quad (6)$$

2.5 Balanced growth equilibrium

Because the economy exhibits perpetual growth, we first need to transform the problem into a stationary one before applying the usual dynamic programming techniques. From this section, we focus on the balanced-growth path of the economy, where w_t , C_t , Y_t , Q_t grow at a common rate g . Note that the average quality \bar{q}_t grows at rate $g_q = (1+g)^{\frac{1-\psi}{\psi}} - 1$ along this path. Let us normalize all variables except q_t by dividing by Q_t . For q_t , we normalize with \bar{q}_t . All normalized variables are denoted with a hat ($\hat{\cdot}$): for example, $\hat{Y}_t = Y_t/Q_t$, $\hat{C}_t = C_t/Q_t$, $\hat{q}_t = q_t/\bar{q}_t$, and so on.

2.5.1 Normalized Bellman equations

From the consumer's Euler equation (1),

$$\beta(1+r_{t+1}) = \frac{C_{t+1}}{C_t} = 1+g$$

holds. Therefore, $(1+g)/(1+r) = \beta$ holds along the stationary growth path. We rewrite the firm's value functions using this expression. Below, we use the hat notation for the stationary value functions to distinguish from the previous section. The time subscripts are dropped, and we denote by ℓ the previous period employment and by ℓ' the current period employment. The value at the beginning of the period is

$$\hat{Z}(\hat{q}, \alpha, \ell) = (1 - \delta)\hat{V}^s(\hat{q}, \alpha, \ell) + \delta\hat{V}^o(\ell), \quad (7)$$

where

$$\hat{V}^o(\ell) = -\tau\hat{w}\ell.$$

The value of survival is

$$\hat{V}^s(\hat{q}, \alpha, \ell) = \max_{\ell', x_I} \left\{ \hat{\Pi}(\hat{q}, \alpha, \ell, \ell', x_I) + \beta \left((1 - \mu)\hat{S} \left(x_I, \frac{\hat{q}}{1 + g_q}, \ell' \right) - \mu\tau\hat{w}\ell' \right) \right\}, \quad (8)$$

where

$$\hat{S} \left(x_I, \frac{\hat{q}}{1 + g_q}, \ell' \right) = (1 - x_I)E_{\alpha'} \left[\hat{Z} \left(\frac{\hat{q}}{1 + g_q}, \alpha', \ell' \right) \right] + x_I E_{\alpha'} \left[\hat{Z} \left(\frac{(1 + \lambda_I)\hat{q}}{1 + g_q}, \alpha', \ell' \right) \right].$$

The period profit can be rewritten as

$$\hat{\Pi}(\hat{q}, \alpha, \ell, \ell', x_I) = ([\alpha\hat{q}]^\psi \ell'^{-\psi} \hat{Y}^\psi - \hat{w})\ell' - \theta_I \hat{q} x_I^\gamma - \tau\hat{w} \max\langle 0, \ell - \ell' \rangle. \quad (9)$$

Note that the Bellman equation (8) can be solved for given \hat{Y} , \hat{w} , g_q , and μ .

For the entrants, the free entry condition can be rewritten as:

$$\max_{x_E} \left\{ -\theta_E x_E^\gamma - \phi + \beta x_E \hat{V}_E \right\} = 0.$$

2.5.2 General equilibrium under balanced growth

Let the decision rule for x_I be $\mathcal{X}_I(\hat{q}, \alpha, \ell)$, and the decision rule for ℓ' be $\mathcal{L}'(\hat{q}, \alpha, \ell)$. Denote the stationary measure of the (normalized) individual state variables as $f(\hat{q}, \alpha, \ell)$ before the innovation and hiring decisions. Innovating over a vacant line improves the quality of the product over a quality drawn from a given distribution $h(\hat{q})$. Let Ω denote the cumulative distribution function of α , and let ω denote the corresponding density function. Given these functions, we can solve for the stationary measure as the fixed point of the mapping $f \rightarrow \mathbf{T}f$, where \mathbf{T} is given in Appendix A. The total mass of active

product lines is

$$N \equiv \int \int \int f(\hat{q}, \alpha, \ell) d\hat{q} d\alpha d\ell.$$

From the steady-state condition, the mass of active product lines can be computed easily as²⁴

$$N = \frac{\mu(1 - \delta)}{\delta + \mu(1 - \delta)}. \quad (10)$$

The average innovation probability of incumbents is

$$\bar{x}_I = \int \int \int \mathcal{X}_I(\hat{q}, \alpha, \ell) (f(\hat{q}, \alpha, \ell)/N) d\hat{q} d\alpha d\ell.$$

The probability that an incumbent is displaced by an entrant, μ , is equal to the aggregate innovation by entrants:

$$\mu = m x_E^*.$$

Let us denote \bar{f} the marginal “density” (measure) of relative productivity:

$$\bar{f}(\hat{q}) \equiv \int \int f(\hat{q}, \alpha, \ell) d\alpha d\ell.$$

Then the normalized value of entry in the stationary equilibrium can be calculated as:

$$\hat{V}_E = \int \left[\int \hat{Z} \left(\frac{(1 + \lambda_E)\hat{q}}{1 + g_q}, \alpha, 0 \right) (\bar{f}(\hat{q}) + (1 - N)h(\hat{q})) d\hat{q} \right] \omega(\alpha) d\alpha.$$

In the goods market, the final goods are used for consumption and R&D; and therefore,

$$\hat{Y} = \hat{C} + \hat{R}$$

holds, where $\hat{R} = \int \int \int \theta_I \hat{q} \mathcal{X}_I(\hat{q}, \alpha, \ell)^\gamma f(\hat{q}, \alpha, \ell) + m(\theta_E x_E^\gamma + \phi)$ is the normalized R&D spending, which includes the potential entrants’ fixed cost, and \hat{C} is given by the labor-leisure decision (2).

3 Characterization of the model

In the absence of firing taxes, the model’s solution can be characterized analytically. The frictionless case provides a useful benchmark and gives some insight into the determinants of innovation and growth in the model. The economy without firing taxes is used also to calibrate the model in the quantitative analysis. Characterizing the economy with the

²⁴The equation is derived from the equality of inflows and outflows: $\delta N = \mu(1 - \delta)(1 - N)$.

firing tax is less straightforward. In this section, we also provide a partial characterization of the model with the firing tax, which facilitates the numerical computation of the equilibrium.

3.1 Analytical characterization of the frictionless economy

The solution of the economy without the firing tax boils down to a system of nonlinear equations. The full characterization is in Appendix B. Here, we present several key results.

The first proposition characterizes the value function and the innovation probability of incumbents.

Proposition 1 *Given \hat{Y} , μ , and g_q , the value function for the incumbents is of the form*

$$\hat{Z}(\hat{q}, \alpha) = \mathcal{A}\alpha\hat{q} + \mathcal{B}\hat{q},$$

and the optimal decision for x_I is

$$x_I = \left(\frac{\beta(1-\mu)\lambda_I(\mathcal{A} + \mathcal{B})}{(1+g_q)\gamma\theta_I} \right)^{\frac{1}{\gamma-1}},$$

where

$$\mathcal{A} = (1-\delta)\psi\frac{\hat{Y}}{N}$$

and \mathcal{B} solves

$$\mathcal{B} = (1-\delta)\beta(1-\mu) \left(1 + \frac{\gamma-1}{\gamma}\lambda_I x_I \right) \frac{\mathcal{A} + \mathcal{B}}{1+g_q}.$$

Proof. See Appendix B. ■

This result shows that x_I is constant across firms regardless of the values of α and \hat{q} . This result hence implies that the expected growth of a firm is independent of its size, which is consistent with *Gibrat's law*.²⁵ The independence between the firm's growth rate and its size implies that the endogenous productivity process is a *stochastic multiplicative process with reset events*.²⁶ This process allows us to characterize the right tail of the firm productivity distribution as follows.

²⁵Various studies have found that Gibrat's law holds for large firms, while many document important deviations for young and small firms (e.g. Evans (1987) and Hall (1987)). See Sutton (1997) for a survey.

²⁶See, for example, Manrubia and Zanette (1999).

Proposition 2 *Suppose that the distribution of the relative productivity of vacant lines, $h(\hat{q})$, is bounded. Then the right tail of the relative firm productivity \hat{q} follows a Pareto distribution with shape parameter κ (that is, the density has the form $F\hat{q}^{-(\kappa+1)}$), which solves*

$$1 = (1 - \delta) [(1 - \mu)x_I\gamma_I^\kappa + \mu\gamma_E^\kappa + (1 - \mu)(1 - x_I)\gamma_N^\kappa].$$

where $\gamma_I \equiv (1 + \lambda_I)/(1 + g_q)$, $\gamma_E \equiv (1 + \lambda_E)/(1 + g_q)$, and $\gamma_N \equiv 1/(1 + g_q)$.

Proof. See Appendix B. ■

Because the firm size (in terms of employment) is log-linear in \hat{q} for a given α , the right-tail of the firm size also follows the Pareto distribution with the same shape parameter κ .

Finally, we are able to characterize the growth rate of average productivity

Proposition 3 *The growth rate of average productivity is given by*

$$g_q = (1 - \delta)[(1 + \lambda_I x_I)(1 - \mu) + (1 + \lambda_E)\mu] + \delta(1 + \lambda_E)\bar{q}^h - 1,$$

where \bar{q}_h is the average relative productivity of inactive product lines.

Proof. See Appendix B. ■

Once the firing tax is introduced, x_I is no longer constant across firms, and therefore this formula is not valid. It is still useful, however, to think of the policy's effect on growth through these three components: the incumbents' innovation, the entrants' innovation on active products, and the entrants' innovation on inactive products.

3.2 A characterization of the economy with the firing tax

With the firing tax, the firm's employment decision is no longer static, and therefore the characterization is not as straightforward as in the case without the firing tax. We can derive a partial characterization, however, that greatly eases the computational burden of the numerical solution method. The main idea is to formulate the model in terms of the deviations from the frictionless outcome. The details of the derivation are in Appendix B.

First, define the *frictionless level of employment with $\alpha = 1$* as

$$\ell^*(\hat{q}; \hat{w}, \hat{Y}) \equiv \arg \max_{\ell'} ([\hat{q}]^\psi \ell'^{-\psi} \hat{Y}^\psi - \hat{w}) \ell' = [(1 - \psi)/\hat{w}]^{\frac{1}{\psi}} \hat{q} \hat{Y}.$$

Let us denote by $\tilde{\ell} \equiv \ell/\ell^*(\hat{q}; \hat{w}, \hat{Y})$ the deviation of *past* employment from the *current* frictionless level and by $\tilde{\ell}' \equiv \ell'/\ell^*(\hat{q}; \hat{w}, \hat{Y})$ the deviation of *current* employment from the *current* frictionless level.

We can show that the period profit in (9) is linear in \hat{q} and can be written as $\hat{q}\tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I)$, where

$$\tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) \equiv \left(\left[\frac{\alpha}{\Omega(\hat{w}, \hat{Y})} \right]^\psi \tilde{\ell}'^{-\psi} \hat{Y}^\psi - \hat{w} \right) \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' - \theta_I x_I^\gamma - \tau \Omega(\hat{w}, \hat{Y}) \hat{w} \max\langle 0, \tilde{\ell} - \tilde{\ell}' \rangle,$$

with $\Omega(\hat{w}, \hat{Y}) \equiv \ell^*(\hat{q}; \hat{w}, \hat{Y})/\hat{q}$.

All the value functions are also linear in \hat{q} . We use the tilde notation to denote the value functions normalized by \hat{q} . For example $\tilde{Z}(\alpha, \tilde{\ell})$ is defined from $\hat{Z}(\hat{q}, \alpha, \ell) = \hat{q}\tilde{Z}(\alpha, \tilde{\ell})$, and equation (7) can be rewritten as

$$\tilde{Z}(\alpha, \tilde{\ell}) = (1 - \delta)\tilde{V}^s(\alpha, \tilde{\ell}) + \delta\tilde{V}^o(\tilde{\ell}),$$

where

$$\tilde{V}^o(\tilde{\ell}) = -\tau\hat{w}\Omega(\hat{w}, \hat{Y})\tilde{\ell}$$

and $\tilde{V}^s(\alpha, \tilde{\ell})$ is

$$\tilde{V}^s(\alpha, \tilde{\ell}) = \max_{\tilde{\ell}' \geq 0, x_I} \left\{ \tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) + \beta \left((1 - \mu) \frac{\tilde{S}(x_I, \tilde{\ell}')}{1 + g_q} - \mu\tau\hat{w}\Omega(\hat{w}, \hat{Y})\tilde{\ell}' \right) \right\}. \quad (11)$$

The linearity of the value functions implies that

$$\frac{\tilde{S}(x_I, \tilde{\ell}')}{1 + g_q} = (1 - x_I)E_{\alpha'} \left[\tilde{Z} \left(\alpha', (1 + g_q)\tilde{\ell}' \right) \right] \frac{1}{1 + g_q} + x_I E_{\alpha'} \left[\tilde{Z} \left(\alpha', \frac{(1 + g_q)\tilde{\ell}'}{1 + \lambda_I} \right) \right] \frac{1 + \lambda_I}{1 + g_q}$$

also holds.

There are two choice variables in the optimization problem in (11), $\tilde{\ell}'$ and x_I . The first-order condition for x_I is

$$\gamma\theta_I x_I^{\gamma-1} = \Gamma_I$$

and thus x_I can be computed from

$$x_I = \left(\frac{\Gamma_I}{\gamma\theta_I} \right)^{1/(\gamma-1)},$$

where $\Gamma_I \equiv \beta(1 - \mu)E_{\alpha'} \left[\tilde{Z}(\alpha', (1 + g_q)\tilde{\ell}'/(1 + \lambda_I))(1 + \lambda_I) - \tilde{Z}(\alpha', (1 + g_q)\tilde{\ell}') \right] / (1 + g_q)$. From here, it is easy to see that x_I is uniquely determined once we know $\tilde{\ell}'$. Let the decision rule for $\tilde{\ell}'$ in the right-hand side of (11) be $\mathcal{L}'(\alpha, \tilde{\ell})$. Then the optimal x_I can be expressed as $x_I = \mathcal{X}_I(\alpha, \tilde{\ell})$. This implies that x_I is independent of \hat{q} .

4 Quantitative analysis

In this section, we conduct the main experiment of the paper. We calibrate the model without firing taxes to the US economy, and we analyze the effects of firing taxes on job flows, employment and output levels, and productivity growth.

4.1 Computation and calibration

The details of the computational methods are described in Appendix C. Our method involves similar steps to solving the standard general-equilibrium firm dynamics model. As in Hopenhayn and Rogerson (1993) and Lee and Mukoyama (2008), we first make a guess on the relevant aggregate variables (in our case \hat{w} , μ , g , and \hat{Y}), solve the optimization problems given these variables, and then update the guess using the equilibrium conditions. This procedure is also similar to how the Bewley-Huggett-Aiyagari models of heterogeneous consumers are typically computed (see, for example, Huggett (1993) and Aiyagari (1994)). This approach separates our work from recent models of innovation and growth, such as Klette and Kortum (2004), Acemoglu et al. (2013), and Akcigit and Kerr (2015), as these models heavily rely on analytical characterizations in a continuous-time setting. Being able to use a standardized numerical method to compute the equilibrium is particularly useful in our experiment because the firing tax introduces a kink in the firm's objective function, which makes it difficult to obtain analytical characterizations to the maximization problem.

Following a strategy similar to Hopenhayn and Rogerson (1993), we calibrate the parameters of the model under the assumption that firing costs are equal to zero, and we use US data to compute our targets. In addition to the standard targets that are widely used in the macroeconomic literature, we use establishment-level labor market data to pin down the parameters that relate to the establishment dynamics.²⁷

The first set of targets is relatively standard. The model period is one year. The discount factor β is set to 0.947 in line with Cooley and Prescott (1995). Similarly to

²⁷Our model does not distinguish between firms and establishments. As 95 percent of US firms are single-establishment firms, the results would be similar if we had instead calibrated the model on firm-level labor market data.

Hopenhayn and Rogerson (1993), we set the value of the disutility of labor ξ so that the employment to population ratio is equal to its average value in the US. The value of ψ is set to 0.2, which implies an elasticity of substitution across goods of 5. This value is in the range of Broda and Weinstein’s (2006) estimates. Our value of 0.2 implies a markup of 25%. We set the curvature of the innovation cost γ to 2. As noted by Acemoglu et al. (2013), $1/\gamma$ can be related to the elasticity of patents to R&D spending, which has been found to be between 0.3 and 0.6.²⁸ These estimates indicate that γ is between 1.66 and 3.33.

Next, we turn to the size of the innovations by entrants and incumbents, λ_E and λ_I . As underlined by Acemoglu and Cao (2015), various studies suggest that the innovations developed by entrants are more radical than those developed by incumbents, that is $\lambda_E > \lambda_I$.²⁹ We set $\lambda_E = 1.5$ and $\lambda_I = 0.25$, based on the recent estimates of Bena et al. (2015). These numbers are also similar to the ones used by Acemoglu and Cao (2015). The implied innovation advantage of entrants, $(1 + \lambda_E)/(1 + \lambda_I)$ is equal to 2, which is also in line with estimates suggested by patent data when we interpret the number of citations of a patent as indicative of the size of the innovation embedded in the patent.³⁰ To set the innovation costs parameters, we assume that the innovation cost is proportional to its size, that is $\theta_E/\theta_I = \lambda_E/\lambda_I$, and thus radical innovations are more costly than incremental ones. We then set the level of θ_I to match the average growth rate of output per worker. When θ_I is smaller, the probability to innovate is higher, and thus the output growth rate is higher. Finally, we set ϕ to match the average job creation rate by entrants in the data. When ϕ is small, there is more entry, and therefore the job creation rate by entrants is larger. We assume that the transitory shock α is uniformly distributed, and can take three values $\{1 - \varepsilon, 1, 1 + \varepsilon\}$, with probability 1/3 for each value. The value of ε is set to replicate the aggregate job creation rate. The job flows are larger when ε is larger. The overall job creation rate and the job creation rate by entrants, used as targets for ϕ and ε , are computed from the Business Dynamics Statistics (BDS) published by the Census Bureau.³¹ The data on the employment-to-population ratio and the growth rate of output per worker are computed from the Bureau of Labor Statistics (BLS) and the Bureau of Economic Analysis (BEA) data. All averages are computed

²⁸See for example Griliches (1990).

²⁹One recent example is Akcigit and Kerr (2015).

³⁰To approximate the innovation advantage of entrants, we look at the relative number of patent citations for entrants and incumbents. Using data on patents of Compustat firms, Balasubramanian and Lee (2008) compute the number of patent citations by firm age and find that the mean patent citation is equal to 15.7 at age 1 and equal to 8.2 at age 25, which implies a ratio of the citations at age 1 over the citations at age 25 equal to 1.9. We thank the authors for making these data available to us.

³¹The job creation rates data are publicly available at <http://www.census.gov/ces/dataproducts/bds/>.

Table 1: Calibration

	Parameter	Calibrated values
Discount rate	β	0.947
Disutility of labor	ξ	1.4747
Demand elasticity	ψ	0.2
Innovation step: entrants	λ_E	1.50
Innovation step: incumbents	λ_I	0.25
Innovation cost curvature	γ	2.0
Innovation cost level: entrants	θ_E	7.992
Innovation cost level: incumbents	θ_I	1.332
Entry cost	ϕ	0.1644
Exogenous exit (depreciation) rate	δ	0.001
Transitory shock	ε	0.267
Avg productivity from inactive lines	h mean	0.976
Firing tax	τ	0.0

over 1977-2012.

When an entrant innovates on an inactive product line, the entrant draws the (normalized) productivity upon which it innovates from a uniform distribution over $[0, 2\bar{q}^h]$. We set the mean $\bar{q}^h = 1$, so that the inclusion of new product lines does not alter the value of average \hat{q} .³² The exogenous exit (depreciation) probability δ is set so that the tail index κ of the productivity distribution matches the value of 1.06 estimated by Axtell (2001) on the US Census data.³³ A large δ implies a larger tail index, which indicates a thinner tail.³⁴ The parameter values are summarized in Table 1.

Table 2 compares the baseline outcome and the targets. We also report the R&D expenditures as a share of aggregate output although we do not use it as a target in the calibration. The R&D ratio, which is about 12%, is larger than what we typically see from conventional measures of R&D spending. Because our model intends to capture innovation in a broad sense, which includes productivity improvements that come from non-R&D activities, such as improvements in the production process or from learning by doing, it is more appropriate to compare the model R&D spending to a broader statistic than the conventional R&D measure. Here, the output share of R&D spending is in line with Corrado et al.'s (2009) estimate of the 1990s US intangible investments.

The baseline model can also be used to assess the contribution of incumbents and

³²Note that the approximation over discrete states creates a slight deviation from the target value of 1.

³³Axtell reports a value of 1.059. He also reports values ranging from 0.994 to 1.098 depending on the dataset used. Luttmer (2011) reports the value of 1.05 for the US firms. Ramsden and Kiss-Haypál (2000) reports the US estimate of 1.25, along with estimates from other countries.

³⁴See Section 3.1 for the expression of the tail index.

Table 2: Comparison between the US data and the model outcome

	Data	Model
Growth rate of output g (%)	1.48	1.48
Employment L	0.613	0.613
Tail index κ	1.06	1.06
Job creation rate (%)	17.0	17.0
Job creation rate from entry (%)	6.4	6.4
Job destruction rate (%)	15.0	17.0
Job destruction rate from exit (%)	5.3	2.8
R&D spending ratio (R/Y) (%)		11.5

Note: The growth rate and employment targets are computed using BEA and BLS data; The tail index is from Axtell's (2001) estimate; the job flows data are computed from the Census Bureau BDS dataset. The job destruction rate, job destruction rate from exit and R&D spending are not targeted in the calibration.

entrants to aggregate productivity growth.³⁵ Using Proposition 3, we can decompose the growth rate of output into the contribution of the incumbents' innovation and that of the entrants. The contribution of incumbents is computed as $[(1 - \delta)\lambda_I x_I (1 - \mu)]/g_q$ and that of entrants is $[(1 - \delta)\lambda_E \mu + \delta((1 + \lambda_E)\bar{q}^h - 1)]/g_q$. In the baseline calibration, we find that incumbents account for 33% of the growth rate of aggregate productivity.

4.2 Quantitative results

We now turn to our main experiment in which we evaluate the effects of firing costs. We study the effects of a firing tax $\tau = 0.3$: the cost of dismissal per worker amounts to 3.6 months of wages. The data from the World Bank Doing Business Dataset partly motivated the choice of this level of tax. The Doing Business dataset reports the mandatory severance payments due by firms upon firing a worker.³⁶ To ensure comparability across countries, precise assumptions are made about the firm and the worker. The worker is assumed to be a cashier in a supermarket, and the firm is assumed to have 60 workers. Figure 1 displays the distribution of severance payments across countries for this typical firm and for a typical worker with ten years of tenure. The firing tax $\tau = 0.3$ corresponds to the median severance payments indicated by the vertical line in Figure 1.³⁷ Note that

³⁵See, for example, Garcia-Macia et al. (2016) who use a similar approach to decompose the growth rate of aggregate productivity growth in the US.

³⁶The data are constructed from a questionnaire on employment regulations that is completed by local lawyers and public officials as well as from the reading of employment laws and regulations.

³⁷This is also close to the level of firing costs in France, estimated by Kramarz and Michaud (2010) to be 25 percent of a worker's annual wages. This is a somewhat milder level of firing tax compared to what has been examined in the literature. Hopenhayn and Rogerson (1993) consider $\tau = 0.5$ and $\tau = 1.0$

Table 3: The effects of firing costs

	$\tau = 0.0$	Experiment			Fixed entry
		$\tau = 0.3$	$\tau = 1$	$\tau = 2$	$\tau = 0.3$
Growth rate of output g (%)	1.48	1.39	1.28	1.16	1.48
Average innovation probability by incumbents x_I	0.084	0.091	0.108	0.133	0.086
Innovation probability by entrants x_E	0.143	0.143	0.143	0.143	0.143
Creative destruction rate μ (%)	2.65	2.30	1.76	1.11	2.65
Employment L	100	98.8	98.1	99.5	100.0
Normalized output \hat{Y}	100	98.1	96.5	96.6	99.4
Normalized average productivity \hat{Y}/L	100	99.3	98.3	97.0	99.4
Number of active products N	0.964	0.958	0.946	0.917	0.964
Job creation rate (%)	17.0	4.7	3.1	1.8	5.3
Job creation rate from entry (%)	6.4	4.3	2.8	1.5	4.9
Job destruction rate (%)	17.0	4.7	3.1	1.8	5.3
Job destruction rate from exit (%)	2.8	2.4	1.9	1.2	2.8
R&D ratio R/Y (%)	11.5	10.6	9.2	7.4	11.6

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

this is a conservative estimate of the median firing costs since firing costs include not only severance payments but also the cost related to the length and the complexity of the dismissal procedure.³⁸ We also report the results for higher values of the firing tax and consider a tax that amounts to 1 year and 2 years of wages.

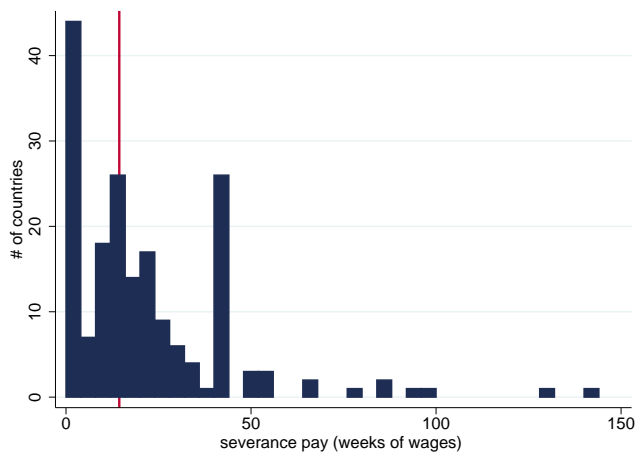
The columns of Table 3 compare the baseline outcome with the outcome when firms are subject to a firing tax. To facilitate the comparison, the variables L , \hat{Y} , and \hat{Y}/L are normalized to 100 in the baseline. Similarly to Hopenhayn and Rogerson (1993), employment L declines when the firing tax is imposed. The firing tax has two effects on employment. On the one hand, it reduces the firm's incentive to contract when a bad shock arrives. On the other hand, knowing this, the firm also becomes more reluctant to hire when there is a good shock. Here, as in Hopenhayn and Rogerson (1993) and Moscoso Boedo and Mukoyama (2012), the latter effect dominates.³⁹

(one period in their model lasts five years, therefore a firing tax of 10% in their model is equal to 50% of the annual wage). Moscoso Boedo and Mukoyama (2012) consider numbers ranging between $\tau = 0.7$ (average of high income countries) and $\tau = 1.2$ (average of low income countries). Moscoso Boedo and Mukoyama (2012) also use the Doing Business Data, but they consider a broader concept of firing tax than only severance payments.

³⁸Lazear (1990) argues that mandatory severance payments can potentially be undone by contractual arrangements between a firm and a worker. However, his empirical analysis shows that severance pay requirements do have real effects. Our notion of firing costs is also broader and can contain many elements other than mandatory severance payments.

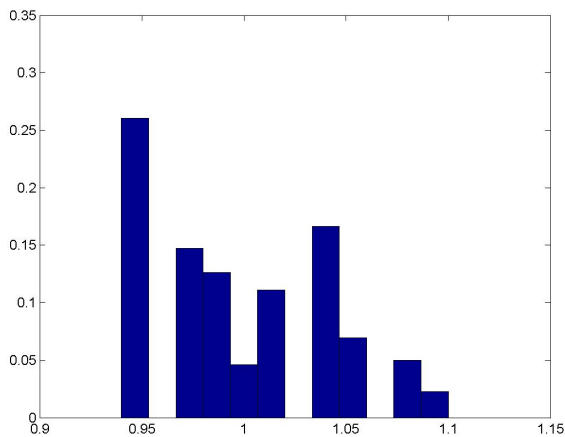
³⁹In a recent empirical study, Autor et al. (2006) document that, during the 1970s and 1980s, many US states have adopted common-law restrictions (wrongful-discharge laws) that limits firms' ability to fire. They show that these restrictions resulted in a reduction in state employment.

Figure 1: Severance payments across the world



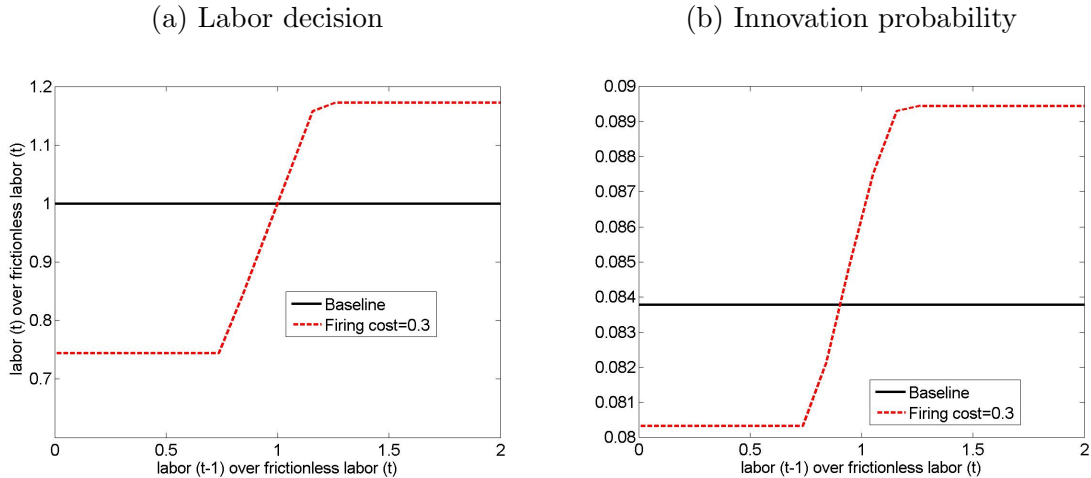
Notes: This figure shows the distribution of severance payments for a worker with ten years of tenure in the retail industry. The vertical line indicates the median. Source: Doing Business dataset (2015), World Bank.

Figure 2: Misallocation of labor



Notes: This figure shows the distribution of the marginal productivity of labor in the model for the baseline experiment where the firing tax is equal to 0.3. The marginal productivity is normalized by the wage rate \hat{w} . Without the firing tax, the marginal productivity of labor would be equalized across establishments and the normalized marginal productivity would be equal to 1.

Figure 3: Labor and innovation decision function, constant entry rate



Notes: This figure displays the firm’s labor decision in deviation from the current frictionless level $\tilde{\ell}'$ as a function of the previous labor level $\tilde{\ell}$ when μ is kept constant to its baseline value. The transitory shock is set to one.

The output level \hat{Y} declines more than employment does, mainly because of *misallocation*: the allocation of labor across firms is not aligned with the firms’ productivity when the firms face firing costs because firms do not adjust their labor as much as they would in the frictionless economy. This outcome can most vividly be seen by the large decline in job flows. The reduction in labor reallocation is consistent with the recent empirical evidence by Micco and Pagés (2007) and Haltiwanger et al. (2014). While the marginal product of labor is equalized across firms in the frictionless equilibrium, there is, by contrast, a notable dispersion in the marginal product of labor in the economy with a firing tax as shown in Figure 2. The marginal product of labor deviates by more than 5 percent from the equilibrium wage for about 35 percent of firms. Entry also decreases with the firing tax. As shown in the Table, this reduces the number of active intermediate products N , which further reduces the aggregate productivity level. Overall however, the effect of the firing tax is modest. We find that average productivity is reduced by 0.7 percent when $\tau = 0.3$.⁴⁰

In addition to these *level effects* that have already been studied in the literature, our model features *growth effects*. First, firing costs reduce the entrants’ incentives to innovate. The total innovation rate by entrants, represented by μ , falls by about 0.3

⁴⁰The magnitude is in line with the results found in previous studies. When $\tau = 1$, we find a reduction in average productivity of 1.7 percent, which is in the same order of magnitude as Hopenhayn and Rogerson (1993) who find a reduction in average productivity of 2.1 percents.

percentage points.⁴¹ Two factors reduce the entrants' incentive to innovate. First, the firing tax reduces expected profits because it raises the cost of operating a firm (*direct effect*). Second, firing costs prevent firms from reaching their optimal scale, and this misallocation reduces the entrants' expected profits (*misallocation effect*).

By contrast, the incumbents' innovation probability *increases* by about 0.7 percentage point as a result of the firing tax. The consequences of the firing tax on the incumbents' incentive to innovate are theoretically ambiguous. On the one hand, the firing tax makes it more costly to operate the firm, which reduces the profits from innovation. In addition, the misallocation of labor is costly because the firm will not operate at its optimal size after innovating. As for the entrants, both the direct effect and misallocation effect tend to reduce the incumbents' innovation. On the other hand, the firms that are larger than their optimal size, either because of a negative transitory shock or because they have been unsuccessful at innovating, now have stronger incentives to invest in R&D. A successful innovation allows these firms to avoid paying the firing tax because they no longer need to reduce their employment (*tax-escaping effect*).⁴² In addition, the incumbents' incentives to innovate further depend on the entry rate (*creative-destruction effect*). A lower entry rate reduces the risk that incumbents be taken over by an entrant, which, in turn, raises the return of the firm's R&D investment. In effect, a lower creative destruction rate raises the incumbents' planning horizon.

To assess the importance of the creative-destruction effect, we conduct an additional experiment. Here, we hold the value of the creative destruction rate μ fixed to the value in the baseline economy by not imposing the free-entry condition (5). The experiment also allows us to illustrate the ambiguous effect of the firing tax on the incumbents' innovation. Figure 3 shows the labor decision and the innovation probability of firms when entry is held constant. As is already well known, the firing tax creates an inaction zone in the labor decision of the firm. We find that the shape of the innovation decision follows closely that of the labor decision. More importantly, the figure shows that the firing tax leads firms that are below their optimal size to reduce their innovation probability. As explained above, this negative effect comes both from the direct tax effect and the misallocation effect. For firms that are larger than their optimal size, on the contrary, the tax-escaping effect leads to a higher innovation probability since innovating provides the added benefit of avoiding paying the firing tax.⁴³ Overall, the results displayed in the

⁴¹Note that the equilibrium value of x_E is not affected by the tax (see equation (6)), and thus the change in μ is all due to the change in the number of potential entrants, m .

⁴²Koeniger (2005) makes a related point, in the context of firm exit. In his model, one firm hires only one worker, and thus it cannot analyze the dependence on size that we emphasize.

⁴³These opposite effects tend to reduce the static misallocation. Since firms that are larger than their

last column of Table 3 indicate that those two effects on incumbents largely offset each other. When the entry rate is held constant, the incumbents' innovation increases by only 0.2 percentage point vs 0.7 percentage point in the baseline. Hence, the decline in entry accounts for two-thirds of the increase in the incumbents' innovation. This result suggests that the decline in the entry rate is the key to understanding the increase in the incumbents' innovation.

With the increase in the incumbents' innovation and the decline in the entrants' innovation, the aggregate growth rate could, in principle, increase or decrease after the introduction of the firing tax. In our baseline experiment, the negative effect on entrants dominates, and results in the reduction of the growth rate. The growth rate of output is 1.39% in the economy with the firing tax $\tau = 0.3$ and 1.48% without the firing tax. To gauge the growth effect relative to the well-studied level effect, we conduct a back-of-the-envelope calculation and compute the consumption-equivalent welfare change induced by the growth effect of the firing tax. The details of the calculation are provided in Appendix B.5. We find that the growth effect of firing costs in our baseline experiment is equivalent to a permanent drop in consumption by 1.6%, which is larger than the level effect (equivalent to a 0.9% drop in consumption).

5 Extensions and discussions

5.1 Extensions

Our baseline model is intentionally kept simple to deliver sharp insights. While this simplicity allows us to characterize the model analytically in the absence of firing costs, it limits the ability of the model to fit the data. In this section, we relax some of the simplifying assumptions to improve the fit of the model and we show that these extensions do not alter the paper's main results. In particular, we find that our main results are robust to allowing for a persistent transitory shock and for an alternative innovation process that delivers a better fit for firm dynamics statistics.

5.1.1 Extension 1: persistent exogenous shocks

In the baseline model, the exogenous productivity shocks are assumed to be purely transitory. This simplifying assumption may affect the quantitative evaluation of the effects

optimal size tend to have a lower than average marginal productivity, a higher innovation probability for those firms contributes to reducing the dispersion in marginal productivity and thus this can reduce the level of misallocation.

of firing taxes on aggregate productivity. Because the persistence of the shocks affects by how much firms adjust their employment in response to the shocks, the persistence may matter for the cost of operating a firm, and hence for the innovation decision of entrants and incumbents, as well as for the level of misallocation. In this section, we introduce persistence in the exogenous productivity shock and study the implications for the effects of the firing tax on the level and growth rate of aggregate productivity. We find that the negative effects of the firing tax are reinforced when the persistence of the exogenous productivity shock is accounted for. The persistence of the exogenous productivity shocks turns out to be more important for the level effect than for the growth effect of the firing tax.

To incorporate the persistence of the exogenous productivity shock α_t , we now assume that α_t follows a Markov chain, with transition probabilities given by

$$\Pr(\alpha_{t+1} = \underline{\alpha}_j | \alpha_t = \underline{\alpha}_i) = \begin{cases} \rho & \text{if } i = j \\ (1 - \rho)/2 & \text{if } i \neq j, \end{cases}$$

where ρ governs the persistence of the process. As in the baseline case, α_t can take three values $\underline{\alpha}_1 = 1 - \varepsilon$, $\underline{\alpha}_2 = 1$, and $\underline{\alpha}_3 = 1 + \varepsilon$. To identify ρ and ε , we use the variance and the autocovariance of establishment-level employment growth. As shown in Appendix D, the variance of employment growth is determined by the variance of changes in the endogenous productivity \hat{q} and that of changes in the exogenous productivity α while the autocovariance of employment growth is a function of the variance of α and the persistence parameter. Given the parameters of the endogenous productivity process, we can then infer the size of the shock ε and the persistence parameter ρ from these two statistics.

We estimate the variance and the autocovariance of establishment-level employment growth in the US using census microdata from the Longitudinal Business Database (LBD).⁴⁴ More details on the data are given in Appendix D. We estimate the variance and autocovariance to be equal to 0.24 and -0.05 , which leads us to set ρ at 0.718 and ε at 0.564. Note that this calibration implies not only more persistent shocks but also larger shocks than in the baseline. The other parameters β , ψ , λ_I , λ_E , γ , and δ are set to the same values as in the baseline case, while the parameters ϕ , ξ and θ_I are re-calibrated to match the job creation rate by entrants, the average employment rate, and the average growth rate of output per worker in the US. The parameters values are

⁴⁴We used the Synthetic LBD (U.S. Census Bureau, 2011) which is accessible through the virtual RDC. The results were then validated with the Census Bureau.

Table 4: Persistent exogenous shocks

	Baseline		Persistent α	
	$\tau = 0.0$	$\tau = 0.3$	$\tau = 0.0$	$\tau = 0.3$
Growth rate of output g (%)	1.48	1.39	1.48	1.37
Innovation probability: incumbents \bar{x}_I	0.084	0.091	0.084	0.089
Innovation probability: entrants x_E	0.143	0.143	0.143	0.143
Creative destruction rate μ (%)	2.65	2.30	2.66	2.25
Employment L	100	98.8	100	98.0
Normalized output \hat{Y}	100	98.1	100	96.9
Normalized average productivity \hat{Y}/L	100	99.3	100	98.9
Number of active products N	0.964	0.958	0.964	0.957
Job creation rate (%)	17.0	4.7	17.0	7.5
Job creation rate from entry (%)	6.4	4.3	6.4	4.3
Job destruction rate (%)	17.0	4.7	17.0	7.5
Job destruction rate from exit (%)	2.8	2.4	2.8	2.3
R&D ratio R/Y (%)	11.5	10.6	11.5	10.4

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

reported in Table 6, and the targets are reported in Table 7, both in Appendix D.⁴⁵

We report the results of the model with persistent exogenous shocks in Table 4. We find that when persistence is introduced, the firing tax leads to a larger decline both in the level and the growth rate of productivity.

The larger decline in average productivity may be surprising as firms would adjust more of their labor in response to persistent shocks, and hence the level of misallocation should be lower when shocks are persistent. In fact, the stronger effect of the firing tax is not due to the increase in the persistence in itself. As explained above, in the new calibration, the exogenous shocks are not only persistent but also more dispersed, which tends to increase misallocation. The effects of larger shocks dominate that of higher persistence, resulting in a lower average productivity than in the baseline.

The fact that the calibration leads to larger transitory shocks also matters for the growth effect. Here again, the fact that the exogenous shocks are larger dominate over the effects of the higher persistence. In particular, in the new calibration, the firms more frequently face situations where a large downsizing is necessary. Overall, the negative growth effect is only slightly stronger with this specification compared to the baseline.

⁴⁵Though the overall job creation rate is not a target in this calibration, the job creation rate is equal to 17.02 which is virtually identical to the value in the baseline.

5.1.2 Extension 2: small entrants and heterogeneous growth

A shortcoming of the baseline model is that it generates entrants that are larger compared to incumbents, which is not consistent with US data. Also, existing empirical evidence suggests that Gibrat’s law does not hold for small firms; small firms grow faster than large firms (see, for example, Evans (1987) and Hall (1987)). In this section, we modify the assumptions on the entry process and on the innovation of incumbents while maintaining the assumption that α is i.i.d. as in the baseline case. We find that the main results of the paper are robust to these modifications that improve the fit of the model along these dimensions.

To improve the fit of the model, we first assume that entrants are more likely to innovate over lower-quality products. This is likely to be more reasonable than the assumption of random innovation, considering that innovations tend to be cumulative (see, for example, Aghion et al. (2001) and Mukoyama (2003)), and it is difficult to improve upon a very advanced product. Second, we also assume that the firms with lower (relative) quality have a lower innovation cost. Previous literature on R&D and innovation emphasizes positive spillovers across firms, and a lower-quality product is more likely to benefit from these spillovers. These assumptions help the model match several empirical regularities that the baseline model is not able to match. First, since entrants tend to innovate over low-quality products, entrants tend to be less productive and therefore smaller compared to the baseline case. Second, since lower-quality firms, who are small, innovate more frequently, small (and young) firms tend to grow faster. This allows the model to deviate from Gibrat’s law.

More specifically, we first make the probability that an incumbent is taken over by an entrant dependent on the product’s relative quality. Let $u(\hat{q})$ be the probability that an incumbent with adjusted-quality \hat{q} is taken over by an entrant. We assume that $u(\hat{q})$ takes the form

$$u(\hat{q}) \equiv \frac{\omega(\hat{q})}{\bar{\omega}} \mu,$$

where $\mu = mx_E$ is the aggregate creative destruction rate and $\omega(\hat{q})$ is the weight function that determines the displacement probability of product \hat{q} .

We also assume that $\omega'(\hat{q}) \leq 0$. Given the density function of \hat{q} , $\bar{f}(\hat{q})/N$, the average weight $\bar{\omega}$ is defined as $\bar{\omega} \equiv \int \omega(\hat{q})\bar{f}(\hat{q})/Nd\hat{q}$. Note that $u'(\hat{q}) < 0$ holds. One interpretation of this specification is that a more advanced technology is more difficult to be imitated. This embeds the idea of cumulative innovation (or “step-by-step innovation”) of Aghion et al. (2001) and Mukoyama (2003) into our model in a parsimonious manner. The aggregate probability that an active production line is taken over is $\int u(\hat{q})\bar{f}(\hat{q})d\hat{q} = N\mu$,

which is the same as the baseline model. The rest of the entrants' innovation, $(1 - N)\mu$, is on the inactive production lines.

From the viewpoint of the entrants, once they successfully innovate, the probability that they innovate upon an active line is N , and the probability that they innovate upon an inactive line is $(1 - N)$. Conditional on innovating upon an active line, the density function of \hat{q} that they improve upon is denoted $p(\hat{q})$, where

$$p(\hat{q}) \equiv \frac{\omega(\hat{q}) \bar{f}(\hat{q})}{\bar{\omega} N} = \frac{u(\hat{q}) \bar{f}(\hat{q})}{\mu N}.$$

Conditional on innovating upon an inactive line, the density function of \hat{q} is assumed to be $h(\hat{q})$, which is the same as the baseline model. Note that when $\omega(\hat{q})$ is constant across \hat{q} , the specification becomes identical to the baseline model and $u(\hat{q}) = \mu$ for all \hat{q} and $p(\hat{q}) = \bar{f}(\hat{q})/N$.

The second modification is that we allow the incumbents' innovation cost to depend on the firm's relative quality. We keep the same notation for the innovation cost θ_I , but instead of being a parameter, θ_I is now a function of \hat{q} , denoted $\theta_I(\hat{q})$.

The model structure is the same as the baseline model, except for $u(\hat{q})$, $p(\hat{q})$, and $\theta_I(\hat{q})$. The description of the rest of the model is relegated to Appendix D. The computation of this version of the model is more complex than the baseline model because the value functions are not linear in \hat{q} , even after the transformation on ℓ . Nevertheless, we can, once again, simplify the computation of the model by rewriting the choice of labor relative to the frictionless level.⁴⁶

To compute the model, we must specify both the weight function and the innovation cost function. We assume that the weight function takes the form

$$\omega(\hat{q}) = 1 + \chi_1 e^{-\chi_2 \hat{q}}, \tag{12}$$

where $\chi_1 \geq 0$ and $\chi_2 \geq 0$. The parameter χ_1 controls the relative displacement probability of high- and low-productivity firms, whereas χ_2 controls the slope of the decline in the displacement probability.⁴⁷

The innovation cost is assumed to take the form

$$\theta_I(\hat{q}) = \bar{\theta}_I (1 - (1 - \chi_3) e^{-\chi_4 \hat{q}}), \tag{13}$$

where $\bar{\theta}_I > 0$, $\chi_3 \in [0, 1]$, and $\chi_4 > 0$. The parameter χ_3 represents the relative

⁴⁶The details of the computation method are described in Appendix D.

⁴⁷Note that $\lim_{\hat{q} \rightarrow \infty} u(\hat{q})/u(0) = \lim_{\hat{q} \rightarrow \infty} \omega(\hat{q})/\omega(0) = 1/(1 + \chi_1)$.

Table 5: Smaller entrants and the deviation from Gibrat’s law

	Baseline		Extension	
	$\tau = 0.0$	$\tau = 0.3$	$\tau = 0.0$	$\tau = 0.3$
Growth rate of output g (%)	1.48	1.39	1.48	1.39
Innovation probability: incumbents \bar{x}_I	0.084	0.091	0.213	0.237
Innovation probability: entrants x_E	0.143	0.143	0.079	0.079
Creative destruction rate μ (%)	2.65	2.30	5.17	4.25
Employment L	100	98.8	100	98.4
Normalized output \hat{Y}	100	98.1	100	96.6
Normalized average productivity \hat{Y}/L	100	99.3	100	98.2
Number of active products N	0.964	0.958	0.773	0.735
Job creation rate (%)	17.0	4.7	17.5	5.3
Job creation rate from entry (%)	6.4	4.3	6.7	4.3
Job destruction rate (%)	17.0	4.7	17.5	5.3
Job destruction rate from exit (%)	2.8	2.4	4.3	3.7
Entry rate(%)			6.6	5.7
R&D ratio R/Y (%)	11.5	10.6	11.9	10.9

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

ease of innovation for low-productivity firms.⁴⁸ The value of χ_4 influences how fast the cost increases with \hat{q} . The details of the calibration, including the values of the new parameters (χ_1 and χ_2 in equation (12) and χ_3 and χ_4 in equation (13)), are presented in Appendix D. As shown in Appendix D, this model better fits the data in terms of the firm size distribution.

As in Section 4, we consider an experiment of setting $\tau = 0.3$. Table 5 shows the results.⁴⁹ The baseline results are also presented for the purpose of comparison. The qualitative results are identical to those obtained with the baseline model. In particular, the firing tax leads to opposite response of the incumbents’ and the entrants’ innovation. The incumbents’ innovation increases, and the entrants’ innovation decreases; the overall effect on growth is negative.

In this extended model, there is an additional incentive for incumbents to innovate. Because a firm with a larger \hat{q} faces a lower probability of being replaced, an incumbent firm can avoid paying the firing tax that accompanies exit when \hat{q} is large. This encourages innovation when the firing tax is imposed; the mechanism is similar to the tax-escaping effect in the previous section, but works through the incentive to avoid exit

⁴⁸Note that $\lim_{\hat{q} \rightarrow \infty} \theta_I(\hat{q}) = \bar{\theta}_I$ and $\theta_I(0) = \chi_3 \bar{\theta}_I$.

⁴⁹It is still the case that the job creation rate by entry is larger than the entry rate in the extended model, indicating that the size of entrants is still larger than the size of incumbents. However, in comparison to the baseline case, the relative size of entrants is substantially smaller. With our functional forms, this turns out to be the lower bound of the entrants’ size in the parameterizations that we can compute.

instead of expansion. Another effect is through the productivity composition of firms. Because the incumbents' innovation cost varies with \hat{q} , a change in the stationary composition of \hat{q} has an effect on overall innovation by incumbents. The overall impact of these new additional effects on the final outcome turns out to be quantitatively small. The results of the previous section are robust to the modifications that bring the model outcome closer to the data.

5.2 Discussion

Here we discuss our baseline quantitative results and, in particular, the robustness to other model specifications. We also discuss the results of the empirical effects of firing costs on growth. A large part of the analysis is delegated to the Appendix.

5.2.1 The sources of innovation

We find that firing costs can affect the innovation of entrants and incumbents in opposite directions. The overall effect on the aggregate economy therefore depends on the details of the innovation process of entrants and incumbents. We investigate below to which extent different specifications and calibration strategies can generate results that differ from our baseline results (presented in Section 4).

Innovation size. We first analyze the role of the innovation size of incumbents and entrants. We consider two alternative calibration strategies, one in which entrants have a smaller innovative advantage, and the other one in which the innovation size is smaller for both entrants and incumbents. The results are reported in Appendix D.3. As in the baseline calibration, the firing tax leads to an increase in the innovation of incumbents and to a reduction in the innovation of entrants. We also find that the overall effect on growth can depend on this particular part of the calibration. The effect of firing costs on growth is smaller for those two variants and becomes positive when the innovation size of entrants and incumbents is identical. In the two alternative calibrations considered, entry accounts for a smaller share of innovation than in our baseline calibration. The results thus suggest that firing costs are less detrimental to growth when entry accounts for a smaller share of innovation. A recent study by Garcia-Macia et al. (2016) follows a different strategy of mapping the model to the data, and finds a larger contribution of incumbents' innovation in overall growth than in our analysis.⁵⁰ Thus firing costs would

⁵⁰Garcia-Macia et al. (2016) assume that the innovation sizes of incumbents and entrants are the same. There are other differences in the model; for example, they allow both incumbents and entrants to come up with new varieties.

most likely have a growth-enhancing effect if we follow their calibration strategy.

Only entrants innovate. We also consider the extreme case where only entrants innovate. The results are reported in Appendix D.3. In line with previous results, we find that the firing tax leads to a larger decline in innovation and aggregate productivity growth. Furthermore, since the positive impact on the incumbents' innovation is absent, the firing tax unambiguously reduces the growth rate. These results illustrate the importance of including the incumbents' innovation in the analysis. Ignoring the innovation by incumbents would have therefore led us to overestimate the consequences of the firing tax on growth.

Other margins of innovation. In our model, innovation leads to improvements in product quality. Another possible margin of innovation to consider is the creation of new varieties. To complement the analysis, we build a separate model with expanding varieties à la Romer (1990) and compute the effect of firing taxes in that setting. Appendix D.5 contains a full description of the model and the results. The firing tax reduces profitability and hence lowers the creation of new products. We show that the growth rate decreases with the firing tax. The mechanism is similar to the one behind the lower entry rate in our baseline model.

Another type of innovation that we do not consider in this paper is the creative destruction by incumbents, whereby incumbent firms can come up with a better-quality version of products that are already produced by other firms. The drivers of the incumbents' creative destruction are very similar to that of the entrants' innovation in our model. In fact, if such innovations result in the creation of new establishments, our analysis already includes this type of innovation in the entry component since the labor market statistics used to calibrate the model are computed at the establishment level. One caveat of this interpretation is that we do not account for the possibility that firms can move workers from one establishment to another. In fact, the tax-escaping effect can work *across* establishments—firms may want to open another establishment to avoid firing taxes. In this case, the effect of the firing tax on incumbents' creative-destruction innovation may qualitatively differ from the baseline model because the tax-escaping effect encourages the incumbents' creative-destruction. Similarly, the creation of new (-to-the-market) varieties by incumbents can be captured in the model by the creation of new varieties by entrants discussed above. Here again, the same caveat applies.

5.2.2 Comparison with labor taxes and R&D subsidies

The effects of tax and transfer policies in the labor market have been extensively studied. As the firing tax and the standard labor tax both generate distortions in the labor market, comparing the two policies will give us additional insights on the specific effects of the firing tax. In addition, comparing the effect of a labor tax in our model to the results of the literature allows us to check the validity of our model.

We introduce a tax rate $\eta \in [0, 1]$ on labor to the baseline model. The budget constraint for the consumer changes to

$$A_{t+1} + C_t = (1 + r_t)A_t + (1 - \eta)w_tL_t + T_t.$$

To facilitate the comparison with the literature, we consider a more general form of period utility function

$$\log(C_t) - \xi \frac{L_t^{1+\nu}}{1+\nu},$$

where $\nu \geq 0$. Note that $\nu = 0$ corresponds to our baseline specification.⁵¹ We report the outcomes of a 5% labor tax (that is, $\eta = 0.05$) for different values of ν in Appendix D.4. With the baseline specification ($\nu = 0$), we find that a 5% labor tax reduces the growth rate to 1.38% while the firing tax reduces the growth rate to 1.39%.⁵² The growth effect of the 30% firing tax is therefore of the same magnitude as that of a 5% labor tax.

Another way of assessing the quantitative impact of the firing tax is to compute the R&D subsidy that would be needed to offset the negative impact of the firing tax on growth. The R&D subsidy changes the innovation cost for incumbents to

$$\mathbf{r}_{Ijt} = (1 - s)\theta_I Q_t \frac{q_{jt}}{q_t} x_{Ijt}^\gamma$$

and the innovation cost for entrants to

$$\mathbf{r}_{Ejt} = (1 - s)\theta_E Q_t x_{Ejt}^\gamma.$$

We find that, in order to cancel out the growth effect of the 30% firing tax, we need to introduce a 7.3% innovation subsidy ($s = 0.073$).

⁵¹Ohanian et al. (2008) use a similar utility function and show that the model can fit the patterns of labor supply in postwar OECD countries once the changes in taxes are taken into account. Rogerson and Wallenius (2009) consider this form of utility function in a richer model of life cycle labor supply. See Appendix D.4 for details.

⁵²We have also made comparisons to Rogerson and Wallenius (2009) results by recalibrating the model to $\eta = 0.30$ and running an experiment of changing the tax to $\eta = 0.50$. See Appendix D.4 for details.

5.2.3 Empirical analysis of the effect of firing costs on growth

Our baseline results suggest that firing costs reduce the growth rate of the economy. As explained in Section 4.2, however, the overall effect on growth is the result of two opposing effects; firing costs may raise the incumbents' innovation whereas they reduce the entrants' innovation. In principle, the overall effect can be positive or negative depending on which of these two effects dominate. To gain further insights on this question, we analyze empirically the effects of firing costs on innovation spending. Several studies have investigated the consequences of firing costs for job reallocation (Micco and Pagés, 2007; Haltiwanger et al., 2014; Davis and Haltiwanger, 2014) but only a few studies focus on aggregate productivity (Autor et al., 2007; Bassanini et al., 2009; Acharya et al., 2013; Ueda and Claessens, 2016). To complement these existing studies, we first evaluate whether there is a relationship between industry-level R&D spending and the strictness of employment protection regulation. Then, we go beyond the variation across countries and over time, and we exploit the variation across industries as well. Similarly to Bassanini et al. (2009), we test whether stricter employment protection regulations tend to reduce R&D spending more in industries where dismissal regulations are more likely to be binding. We use the industry layoff rate in the US as a measure of how binding the employment regulation is in each industry. The empirical results are reported in Appendix E. We find that countries with stricter employer protection tend to have lower R&D spending. Employment protection regulation, however, does not have a systematically larger effect in industries with a higher layoff rate. From the viewpoint of our theoretical model, while the overall effect on growth is negative in our baseline calibration, it is plausible that the positive and negative effects of employment protection on R&D can offset each other to produce mixed results.

6 Conclusion

In this paper, we construct a general equilibrium model of firm dynamics with endogenous innovation. In contrast to standard firm dynamics models, firms decide not only on entry, production, and employment, but also on investments that enhance their productivity. We use this framework to show that a policy that modifies the reallocation of inputs across firms influences not only the *level* but also the *growth* rate of aggregate productivity. The model that we propose is flexible and can easily accommodate various extensions. We believe that our model will be useful for future studies of how other barriers to reallocation affect aggregate productivity growth.

We examine a particular type of barriers: firing costs. We find that firing costs can have opposite effects on entrants' innovation and incumbents' innovation. Firing costs reduces entrants' innovation while they may enhance incumbents' innovation. As a result, firing costs change the composition of innovation, and to the extent that the effect on incumbents and the effect on entrants do not offset each other, they also affect aggregate innovation. Our quantitative results show that the overall effect on growth is negative although some specifications can result in more nuanced outcomes due to these opposing forces. Our results also suggest that the welfare effect coming from the growth channel can be significant.

While the focus of this paper is theoretical, the opposing effects on entrants and incumbents call for an empirical analysis of the consequences of firing costs on growth. The existing literature on the topic has obtained mixed results. To complement existing work, we have run several regressions using cross-country data on industry-level R&D spending. We find that although stricter employment protection regulation is associated with lower industry-level R&D spending, R&D spending is not systematically lower for industries that are more likely to be constrained by the regulation. While the mixed results are not surprising in light of the opposing forces unveiled by our model, we believe that further empirical investigations on the effects of firing costs on innovation are needed.

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Appendix (not for publication)

A Stationary distribution

The stationary measure is the fixed point of the mapping $f \rightarrow \mathbf{T}f$, where $\mathbf{T}f$ gives the probability for the next period state given that the current state is drawn according to the probability measure f . The mass of firms in the set $[0, q'] \times [0, \alpha'] \times [0, \ell']$ next period is given by

$$\int_0^{\alpha'} \int_0^{\ell'} \int_0^{q'} \mathbf{T}f(\hat{q}, \alpha, \ell) d\hat{q} d\alpha d\ell = (1 - \delta)[(1 - \mu)M_s(\hat{q}', \alpha', \ell') + M_e(\hat{q}', \alpha', \ell')].$$

The first term M_s is the mass of non-displaced firms.

$$\begin{aligned} M_s(\hat{q}', \alpha', \ell') = & \int_0^{\alpha'} \int_{\alpha} \int_{\hat{q}/(1+g_q) \leq \hat{q}'} \int_{\mathcal{L}'(\hat{q}, \alpha, \ell) \leq \ell'} g_{\alpha}(\alpha') (1 - \mathcal{X}_I(\hat{q}, \alpha, \ell)) f(\hat{q}, \alpha, \ell) d\hat{q} d\ell d\alpha d\alpha' \\ & + \int_0^{\alpha'} \int_{\alpha} \int_{(1+\lambda_I)\hat{q}/(1+g_q)} \int_{\mathcal{L}'(\hat{q}, \alpha, \ell) \leq \ell'} g_{\alpha}(\alpha') \mathcal{X}_I(\hat{q}, \alpha, \ell) f(\hat{q}, \alpha, \ell) d\hat{q} d\ell d\alpha d\alpha'. \end{aligned}$$

The second term M_e is the mass of entering firms, which includes firms entering on inactive products and firms entering on existing products:

$$\begin{aligned} M_e(\hat{q}', \alpha', \ell') = & \mu(1 - N) \int_0^{\alpha'} \int_{\alpha} \int_{(1+\lambda_E)\hat{q}/(1+g_q) \leq \hat{q}'} h(\hat{q}) \bar{g}(\alpha') d\hat{q} d\alpha' \\ & + \mu \int_0^{\alpha'} \int_{\alpha} \int_{(1+\lambda_E)\hat{q}/(1+g_q) \leq \hat{q}'} \int g_{\alpha}(\alpha') f(\hat{q}, \alpha, \ell) d\hat{q} d\ell d\alpha d\alpha', \end{aligned}$$

where $\bar{g}(\alpha')$ is the invariant distribution of the transitory shock.

The expression of the stationary distribution is simpler when the model is rewritten in deviation to the frictionless values (see Section 3.2) and when the transitory shock α is i.i.d. as assumed in the baseline calibration. In that case, the stationary distribution can then be rewritten as a function of the deviation of labor from its frictionless value $\tilde{\ell}$ instead of ℓ , and the next period transitory shock draw becomes independent of next period productivity and labor states.

With these two modifications, M_s becomes

$$M_s(\hat{q}', \alpha', \tilde{\ell}') = G(\alpha') \left[\int_{\alpha} \int_{\hat{q}/(1+g_q) \leq \hat{q}'} \int_{\mathcal{L}'(\alpha, \tilde{\ell}) \leq \tilde{\ell}'} (1 - \mathcal{X}_I(\alpha, \tilde{\ell})) f(\hat{q}, \alpha, \tilde{\ell}) d\hat{q} d\alpha d\tilde{\ell} \right. \\ \left. + \int_{\alpha} \int_{(1+\lambda_I)\hat{q}/(1+g_q) \leq \hat{q}'} \int_{\mathcal{L}'(\alpha, \tilde{\ell}) \leq \tilde{\ell}'} \mathcal{X}_I(\alpha, \tilde{\ell}) f(\hat{q}, \alpha, \tilde{\ell}) d\hat{q} d\alpha d\tilde{\ell} \right].$$

The mass of entrants M_e can be rewritten as

$$M_e(\hat{q}', \alpha', \tilde{\ell}') = G(\alpha') \left[\mu(1 - N) \int_{\alpha} \int_{(1+\lambda_E)\hat{q}/(1+g_q) \leq \hat{q}'} h(\hat{q}) d\hat{q} \right. \\ \left. + \mu \int_{\alpha} \int_{(1+\lambda_E)\hat{q}/(1+g_q) \leq \hat{q}'} \int \int f(\hat{q}, \alpha, \tilde{\ell}) d\hat{q} d\tilde{\ell} d\alpha \right].$$

B Analytical characterizations

This section characterizes the model without the firing tax and boils it down to a system of nonlinear equations. The derivations also serve as proofs for the Propositions.

B.1 Model solution

Note first that for a given μ , the number of actively produced product, N , is calculated by (10). Recall that μ is an endogenous variable and is determined by the entrants' innovation:

$$\mu = m x_E^*.$$

As we have seen, x_E^* is given by

$$x_E^* = \left(\frac{\phi}{\theta_E(\gamma - 1)} \right)^{\frac{1}{\gamma}},$$

and thus μ (and also N) is a function of m . In particular, note that N is an increasing function of m .

Because there are no firing taxes, the previous period employment, ℓ , is no longer a state variable. The measure of individual states can be written as $f(\hat{q}, \alpha)$, and because \hat{q} and α are independent, we can write $f(\hat{q}, \alpha) = \hat{z}(\hat{q})g(\alpha)$. In particular, note that $\int \hat{q} \hat{z}(\hat{q}) d\hat{q} = N$, because \hat{q} is the value of q_t normalized by its average. We also assume that $g(\alpha)$ is such that $\int \alpha g(\alpha) d\alpha = 1$.

Without firing costs, labor can be adjusted freely. Thus the intermediate-good firm's

decision for ℓ' is static:

$$\max_{\ell'} \hat{\pi} \equiv ([\alpha \hat{q}]^\psi \ell'^{-\psi} \hat{Y}^\psi - \hat{w}) \ell'. \quad (14)$$

From the first-order condition,

$$\ell' = \left(\frac{1 - \psi}{\hat{w}} \right)^{\frac{1}{\psi}} \alpha \hat{q} \hat{Y} \quad (15)$$

holds. Because $y = \ell'$, we can plug this into the definition of \hat{Y} :

$$\hat{Y} = \left(\int \int [\alpha \hat{q}]^\psi y^{1-\psi} \hat{z}(\hat{q}) \omega(\alpha) d\hat{q} d\alpha \right)^{\frac{1}{1-\psi}}.$$

This yields

$$\hat{Y} = \hat{Y} \left(\frac{1 - \psi}{\hat{w}} \right)^{\frac{1}{\psi}} N^{\frac{1}{1-\psi}}$$

and therefore

$$\hat{w} = (1 - \psi) N^{\frac{\psi}{1-\psi}}. \quad (16)$$

Recall that N is a function of the endogenous variable m . Thus \hat{w} is also a function of m .

Combining the equations (15) and (16), we get

$$\ell' = \alpha \hat{q} \hat{Y} N^{-\frac{1}{1-\psi}}. \quad (17)$$

Integrating this across all active firms yields

$$L = N^{-\frac{\psi}{1-\psi}} \hat{Y}.$$

One way of looking at this equation is that \hat{Y} can be pinned down once we know L and N (and thus L and m). Plugging (16) and (17) into (14) yields

$$\hat{\pi} = \psi \alpha \hat{q} \frac{\hat{Y}}{N}.$$

Now, let us characterize the innovation decision of an intermediate-good firm. Recall that the value functions are

$$\hat{Z}(\hat{q}, \alpha) = (1 - \delta) \hat{V}^s(\hat{q}, \alpha),$$

where

$$\hat{V}^s(\hat{q}, \alpha) = \max_{x_I} \psi \alpha \hat{q} \frac{\hat{Y}}{N} - \theta_I \hat{q} x_I^\gamma + \beta(1 - \mu) \hat{S}(x_I, \hat{q}/(1 + g_q)) \quad (18)$$

and

$$\hat{S}(x_I, \hat{q}/(1 + g_q)) = (1 - x_I) \int \hat{Z}(\hat{q}/(1 + g_q), \alpha') \omega(\alpha') d\alpha' + x_I \int \hat{Z}((1 + \lambda_I) \hat{q}/(1 + g_q), \alpha') \omega(\alpha') d\alpha'.$$

We start from making a guess that $\hat{Z}(\hat{q}, \alpha)$ takes the form

$$\hat{Z}(\hat{q}, \alpha) = \mathcal{A} \alpha \hat{q} + \mathcal{B} \hat{q},$$

where \mathcal{A} and \mathcal{B} are constants. With this guess, the first-order condition in (18) for x_I is

$$\gamma \theta_I \hat{q} x_I^{\gamma-1} = \frac{\beta(1 - \mu) \lambda_I (\mathcal{A} + \mathcal{B}) \hat{q}}{1 + g_q}.$$

Thus

$$x_I = \left(\frac{\beta(1 - \mu) \lambda_I (\mathcal{A} + \mathcal{B})}{(1 + g_q) \gamma \theta_I} \right)^{\frac{1}{\gamma-1}} \quad (19)$$

and x_I is constant across \hat{q} and α . Substituting for x_I , the value function can be written

$$\hat{Z}(\hat{q}, \alpha) = (1 - \delta) \left(\psi \alpha \hat{q} \frac{\hat{Y}}{N} - \theta_I \hat{q} x_I^\gamma + \beta(1 - \mu) \frac{1 + x_I \lambda_I}{1 + g_q} (\mathcal{A} + \mathcal{B}) \hat{q} \right).$$

Thus, the guess is verified with

$$\mathcal{A} = (1 - \delta) \psi \frac{\hat{Y}}{N}$$

and \mathcal{B} solves

$$\mathcal{B} = (1 - \delta) \left(-\theta_I x_I^\gamma + \beta(1 - \mu) \frac{1 + x_I \lambda_I}{1 + g_q} (\mathcal{A} + \mathcal{B}) \right) = (1 - \delta) \beta(1 - \mu) \left(1 + \frac{\gamma - 1}{\gamma} \lambda_I x_I \right) \frac{\mathcal{A} + \mathcal{B}}{1 + g_q},$$

where x_I is given by (19). Therefore, we found that x_I (and the coefficients of the function $\hat{Z}(\hat{q}, \alpha)$) is a function of the endogenous aggregate variables μ , g_q , \hat{Y} , and N . We have already seen that we can pin down μ and N if we know m , and \hat{Y} can be pinned down if we know m and L .

We now turn to the growth rate of productivity g_q . As we have seen above, the transitory shock α does not affect the innovation decision and can therefore be ignored when calculating the transition function of q_t . Consider the measure of productivity (without the normalization) q_t for active products, $z(q_t)$. A fraction $(1 - \mu)x_I(1 - \delta)$ of

active lines are products that have been innovated upon by incumbents and the fraction $(1 - \mu - (1 - \mu)x_I)(1 - \delta)$ is owned by the incumbents but the innovation was unsuccessful. The fraction $\mu(1 - \delta)$ of active products is innovated upon by entrants. The fraction $\mu(1 - \delta)$ of inactive products is innovated upon by entrants. The productivity distribution of inactive product lines is $h(q_t/\bar{q}_t)$ rather than $z(q_t)/N$. Thus g_q can be calculated from

$$1 + g_q = (1 - \delta) \left[(1 + \lambda_I x_I)(1 - \mu) + (1 + \lambda_E)\mu + (1 + \lambda_E)\mu \frac{1 - N \bar{q}^h}{N \bar{q}^z} \right],$$

where \bar{q}^h and \bar{q}^z are averages of q_t with respect to the distributions h and z . Thus $\bar{q}^h/\bar{q}^z = \int q_t h(q_t/\bar{q}_t) dq_t / \int q_t [z(q_t)/N] dq_t = \int \hat{q} h(\hat{q}) d\hat{q} / \int \hat{q} [\hat{z}(\hat{q})/N] d\hat{q}$. The first term is the productivity increase of the surviving incumbents, the second term is the entry into active products, and the last is the entry into inactive products. Using the expression for N in (10) and the fact that $\bar{q}^z = 1$,

$$g_q = (1 - \delta)[(1 + \lambda_I x_I)(1 - \mu) + (1 + \lambda_E)\mu] + \delta(1 + \lambda_E)\bar{q}^h - 1.$$

Thus, g_q can be written as a function of μ and x_I , and therefore m and L .

Hence, we can determine all endogenous variables in the economy once we pin down m and L . The values of m and L can be pinned down by two additional conditions: the labor-market equilibrium condition and the free-entry condition. To see this, let us first be explicit about each variable's (and each coefficient's) dependence on m and L : $\hat{w}(m)$, $N(m)$, $\hat{Y}(m, L)$, $x_I(m, L)$, $g_q(m, L)$, $\mathcal{A}(m, L)$, and $\mathcal{B}(m, L)$. Also note that the total R&D, \hat{R} , can be written as

$$\hat{R} = \int \theta_I \hat{q} x_I(m, L)^\gamma \hat{z}(\hat{q}) d\hat{q} + m(\phi + \theta_E x_E^\gamma) = \theta_I N(m) x_I(m, L)^\gamma + m(\phi + \theta_E x_E^\gamma) \quad (20)$$

and therefore we can write $\hat{R}(m, L)$.

The labor-market equilibrium condition is

$$\frac{\hat{w}(m)}{\hat{Y}(m, L) - \hat{R}(m, L)} = \xi$$

and the free-entry condition is

$$\frac{\gamma \theta_E x_E^{\gamma-1}}{\beta} = \hat{V}_E, \quad (21)$$

where

$$\begin{aligned}
\hat{V}_E &= \int \left[\int \hat{Z}((1 + \lambda_E)\hat{q}/(1 + g_q), \alpha)(\hat{z}(\hat{q}) + (1 - N)h(\hat{q}))d\hat{q} \right] \omega(\alpha)d\alpha \\
&= \int \frac{\mathcal{A}(m, L) + \mathcal{B}(m, L)}{1 + g_q(m, L)}(1 + \lambda_E)\hat{q}(\hat{z}(\hat{q}) + (1 - N)h(\hat{q}))d\hat{q} \\
&= \frac{\mathcal{A}(m, L) + \mathcal{B}(m, L)}{1 + g_q(m, L)}(1 + \lambda_E)[N(m) + (1 - N(m))\bar{q}^h].
\end{aligned}$$

B.2 Productivity distribution

The invariant distribution $\hat{z}(\hat{q})$ can be easily computed. The next-period mass at relative quality \hat{q} is the sum of four components: (i) the incumbents' innovation, $(1 - \delta)(1 - \mu)x_I\hat{z}((1 + g_q)\hat{q}/(1 + \lambda_I))d\hat{q}$; (ii) the entrants' innovation, $(1 - \delta)\mu\hat{z}((1 + g_q)\hat{q}/(1 + \lambda_E))d\hat{q}$; (iii) the downgrade from products that were not innovated upon, $((1 - \delta)(1 - \mu - (1 - \mu)x_I)\hat{z}((1 + g_q)\hat{q}))d\hat{q}$; and (iv) the entry from inactive products, $(1 - \delta)\mu(1 - N)h(\hat{q}/(1 + \lambda_E))d\hat{q}$. The sum of these four components must be equal to $\hat{z}(\hat{q})d\hat{q}$ along the stationary growth path.

We can characterize the right tail of the distribution analytically, when the distribution $h(\hat{q})$ is bounded. Let the density function of the stationary distribution be $s(\hat{q}) \equiv \hat{z}(\hat{q})/N$. Because $h(\hat{q})$ is bounded, there is no direct inflow from the inactive product lines at the right tail.

Consider the point \hat{q} and the interval Δ around that point. The outflow from that interval is $s(\hat{q})\Delta$ because all the firms will either move up, move down, or exit.

The inflow comes from two sources. The first source is the mass of firms who innovated. Innovation is either done by incumbents or entrants. Let $\gamma_i \equiv (1 + \lambda_I)/(1 + g_q) > 1$ be the (adjusted) improvement of \hat{q} after innovation by an incumbent. The probability of innovation by an incumbent is $(1 - \delta)(1 - \mu)x_I$ and the corresponding mass of this inflow is $(1 - \delta)(1 - \mu)x_I s(\hat{q}/\gamma_i)\Delta/\gamma_i$. Similarly, letting $\gamma_e \equiv (1 + \lambda_E)/(1 + g_q) > 1$ be the improvement of \hat{q} after innovation by an entrant, the mass of the inflow due to the entrants' innovation is $(1 - \delta)\mu s(\hat{q}/\gamma_e)\Delta/\gamma_e$. The second source of inflow is the surviving firms that did not innovate. With probability $(1 - \delta)(1 - \mu)(1 - x_I)$, incumbents firms are not successful at innovating. Let $\gamma_n \equiv 1/(1 + g_q) < 1$ be the (adjusted) quality ratio when there is no innovation. The corresponding mass of this inflow is $(1 - \delta)(1 - \mu)(1 - x_I)s(\hat{q}/\gamma_n)\Delta/\gamma_n$.

In the stationary distribution, the inflows are equal to the outflows, and therefore

$$s(\hat{q})\Delta = (1 - \delta) \left[(1 - \mu)x_I s\left(\frac{\hat{q}}{\gamma_i}\right) \frac{\Delta}{\gamma_i} + \mu s\left(\frac{\hat{q}}{\gamma_e}\right) \frac{\Delta}{\gamma_e} + (1 - \mu - (1 - \mu)x_I) s\left(\frac{\hat{q}}{\gamma_n}\right) \frac{\Delta}{\gamma_n} \right],$$

or

$$s(\hat{q}) = (1 - \delta) \left[(1 - \mu)x_I s \left(\frac{\hat{q}}{\gamma_i} \right) \frac{1}{\gamma_i} + \mu s \left(\frac{\hat{q}}{\gamma_e} \right) \frac{1}{\gamma_e} + (1 - \mu - (1 - \mu)x_I) s \left(\frac{\hat{q}}{\gamma_n} \right) \frac{1}{\gamma_n} \right],$$

Guess that the right-tail of the density function is Pareto and has the form $s(x) = Fx^{-(\kappa+1)}$. The parameter $\kappa >$ is the shape parameter, and the expected value of x exists only if $\kappa > 1$. Plugging this guess into the expression above yields

$$\begin{aligned} F\hat{q}^{-(\kappa+1)} = \\ (1 - \delta) \left[(1 - \mu)x_I F \left(\frac{\hat{q}}{\gamma_i} \right)^{-(\kappa+1)} \frac{1}{\gamma_i} + \mu F \left(\frac{\hat{q}}{\gamma_e} \right)^{-(\kappa+1)} \frac{1}{\gamma_e} \right. \\ \left. + (1 - \mu - (1 - \mu)x_I) F \left(\frac{\hat{q}}{\gamma_n} \right)^{-(\kappa+1)} \frac{1}{\gamma_n} \right], \end{aligned}$$

or

$$1 = (1 - \delta) [(1 - \mu)x_I \gamma_i^\kappa + \mu \gamma_e^\kappa + (1 - \mu - (1 - \mu)x_I) \gamma_n^\kappa].$$

The parameter κ is the solution of this equation.

B.3 Growth rate

The growth rate of aggregate productivity is given by

$$g_q = (1 - \delta)[(1 + \lambda_I x_I)(1 - \mu) + (1 + \lambda_E)\mu] + \delta(1 + \lambda_E)\bar{q}^h - 1,$$

where \bar{q}_h is the average relative productivity of inactive product lines. This can be shown by a simple accounting relation. Let the measure of q_t (without normalization) for active products be $z(q_t)$.⁵³ Innovation by incumbents occurs on a fraction $(1 - \mu)x_I(1 - \delta)$ of active product lines, no innovation occurs on a fraction $(1 - \mu - (1 - \mu)x_I)(1 - \delta)$ of active lines. There is innovation by entrants on a fraction $\mu(1 - \delta)$ of active products. Among the inactive products, the fraction $\mu(1 - \delta)$ becomes active from the innovation by entrants, but it is an upgrade from the distribution $h(q_t/\bar{q}_t)$ rather than $z(q_t)/N$. Thus g_q can be calculated from

$$1 + g_q = (1 - \delta) \left[(1 + \lambda_I x_I)(1 - \mu) + (1 + \lambda_E)\mu + (1 + \lambda_E)\mu \frac{1 - N \bar{q}^h}{N \bar{q}^z} \right].$$

⁵³In relation to the general model, $z(q_t)$ corresponds to $\bar{f}(q_t/\bar{q}_t)$ in terms of \bar{f} in Section 2.5.2. The normalized version $\hat{z}(\hat{q})$ exactly corresponds to $\bar{f}(\hat{q})$.

Here, \bar{q}^h and \bar{q}^z are averages of q_t with respect to the distributions h and z . Thus $\bar{q}^h/\bar{q}^z = \int q_t h(q_t/\bar{q}_t) dq_t / \int q_t [z(q_t)/N] dq_t = \int \hat{q} h(\hat{q}) d\hat{q} / \int \hat{q} [z(\hat{q})/N] d\hat{q}$. Combining this with the expression for N in (10) and the fact that $\bar{q}^z = 1$ yields the above result.

B.4 Details of Section 3.2

Under the notations of Section 3.2, the period profit (9) can be rewritten as

$$\hat{\Pi}(\hat{q}, \alpha, \ell, \ell', x_I) = \left(\left[\frac{\alpha}{\Omega(\hat{w}, \hat{Y})} \right]^\psi \tilde{\ell}'^{-\psi} \hat{Y}^\psi - \hat{w} \right) \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' - \theta_I \hat{q} x_I^\gamma - \tau \hat{w} \max\langle 0, \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' - \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' \rangle.$$

Thus this is linear in \hat{q} , and can be rewritten as $\hat{q} \tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I)$, where

$$\tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) \equiv \left(\left[\frac{\alpha}{\Omega(\hat{w}, \hat{Y})} \right]^\psi \tilde{\ell}'^{-\psi} \hat{Y}^\psi - \hat{w} \right) \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' - \theta_I x_I^\gamma - \tau \Omega(\hat{w}, \hat{Y}) \hat{w} \max\langle 0, \tilde{\ell}' - \tilde{\ell}' \rangle.$$

Because the period return function is linear in \hat{q} , it is straightforward to show that all value functions are linear in \hat{q} . Defining $\tilde{Z}(\alpha, \tilde{\ell})$ from $\tilde{Z}(\hat{q}, \alpha, \ell) = \hat{q} \tilde{Z}(\alpha, \tilde{\ell})$, (7) can be rewritten as

$$\tilde{Z}(\alpha, \tilde{\ell}) = (1 - \delta) \tilde{V}^s(\alpha, \tilde{\ell}) + \delta \tilde{V}^o(\tilde{\ell}),$$

where $\tilde{V}^o(\tilde{\ell})$ is from $\hat{V}^o(\ell) = \hat{q} \tilde{V}^o(\tilde{\ell})$ and thus

$$\tilde{V}^o(\tilde{\ell}) = -\tau \hat{w} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}$$

and $\tilde{V}^s(\alpha, \tilde{\ell})$ is from $\hat{V}^s(\hat{q}, \alpha, \ell) = \hat{q} \tilde{V}^s(\alpha, \tilde{\ell})$ with

$$\tilde{V}^s(\alpha, \tilde{\ell}) = \max_{\tilde{\ell}' \geq 0, x_I} \left\{ \tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) + \beta \left((1 - \mu) \frac{\tilde{S}(x_I, \tilde{\ell}')}{1 + g_q} - \mu \tau \hat{w} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' \right) \right\}$$

Here, the expression $\tilde{S}(x_I, \tilde{\ell}')/(1+g_q)$ comes from $\hat{S}(x_I, \hat{q}/(1+g_q), \ell') = \hat{q} \tilde{S}(x_I, \tilde{\ell}')/(1+g_q)$. The linearity of the value functions implies that

$$\frac{\tilde{S}(x_I, \tilde{\ell}')}{1 + g_q} = (1 - x_I) E_{\alpha'} \left[\tilde{Z} \left(\alpha', (1 + g_q) \tilde{\ell}' \right) \right] \frac{1}{1 + g_q} + x_I E_{\alpha'} \left[\tilde{Z} \left(\alpha', \frac{(1 + g_q) \tilde{\ell}'}{1 + \lambda_I} \right) \right] \frac{1 + \lambda_I}{1 + g_q}$$

also holds. Here we used that

$$\hat{Z}(\hat{q}', \alpha', \ell') = \hat{q}' \tilde{Z} \left(\alpha', \frac{\ell'}{\ell^*(\hat{q}'; \hat{w}', \hat{Y}')} \right) = \hat{q}' \tilde{Z} \left(\alpha', \frac{\ell^*(\hat{q}; \hat{w}, \hat{Y})}{\ell^*(\hat{q}'; \hat{w}', \hat{Y}')} \frac{\ell'}{\ell^*(\hat{q}; \hat{w}, \hat{Y})} \right)$$

with $\hat{w}' = \hat{w}$, $\hat{Y}' = \hat{Y}$; and that $\ell^*(\hat{q}; \hat{w}, \hat{Y})/\ell^*(\hat{q}'; \hat{w}', \hat{Y}') = \hat{q}/\hat{q}'$ yields

$$\hat{Z}(\hat{q}', \alpha', \ell') = \hat{q}' \tilde{Z} \left(\alpha', \frac{\hat{q}}{\hat{q}'} \tilde{\ell}' \right)$$

for $\hat{q}' = \hat{q}/(1 + g_q)$ and $\tilde{\ell}' = (1 + \lambda_I)\hat{q}/(1 + g_q)$.

B.5 Measuring the welfare loss of the growth decline

To evaluate the size of the growth effect of the firing tax and to better compare our results to the literature, we conduct a back-of-the-envelope calculation of the consumption-equivalent welfare change. We compare the balanced-growth welfare of consumers in the economies with and without the firing tax.⁵⁴

The consumer's utility under balanced growth is

$$\sum_{t=0}^{\infty} \beta^t [\log(\hat{C}(1 + g)^t) - \xi L],$$

which can be separated into two components:

$$\sum_{t=0}^{\infty} \beta^t [\log(\hat{C}(1 + g)^t) - \xi L] = \sum_{t=0}^{\infty} \beta^t [\log(\hat{C}(1 + g)^t)] - \sum_{t=0}^{\infty} \beta^t \xi L.$$

The first component is the effect of consumption, and the second component is the effect of labor. In our baseline outcome, g , \hat{C} , and L all decline with the firing tax. Note that the decline in L leads to a welfare *gain*. We focus here on the welfare losses of the firing tax, and therefore abstract from the effects of the firing tax on labor.⁵⁵

The consumption component of welfare can itself be separated in two parts

$$\sum_{t=0}^{\infty} \beta^t [\log(\hat{C}(1 + g)^t)] = \sum_{t=0}^{\infty} \beta^t \log(\hat{C}) + \sum_{t=0}^{\infty} \beta^t \log((1 + g)^t)$$

⁵⁴This is not a complete analysis of consumer welfare, as the comparison below does not take the transition dynamics into account. Our analysis here is meant to be illustrative.

⁵⁵This approach (ignoring the change in disutility of labor and focusing on consumption) is similar to the approach that Lucas (1987) used in his welfare cost calculation for the business cycles.

We call the first term the *level effect* and the second term the *growth effect* on consumer welfare.

Let the consumption level and the growth rate be \hat{C}_0 and g_0 in the economy without the firing tax, and \hat{C}_1 and g_1 in the economy with the firing tax. We compute the welfare loss from the level effect and the growth effect of the firing tax as the permanent drop in consumption that would make the representative consumer in the economy without the firing tax indifferent between the two economies.

For the level effect, the permanent drop in consumption φ_L is such that

$$\sum_{t=0}^{\infty} \beta^t \log((1 - \varphi_L)\hat{C}_0) = \sum_{t=0}^{\infty} \beta^t \log(\hat{C}_1).$$

Thus

$$\varphi_L = 1 - \exp(\log(\hat{C}_1) - \log(\hat{C}_0)) = 1 - \frac{\hat{C}_1}{\hat{C}_0}.$$

For the growth effect, the permanent drop in consumption φ_G is computed as

$$\sum_{t=0}^{\infty} \beta^t \log((1 - \varphi_G)(1 + g_0)^t) = \sum_{t=0}^{\infty} \beta^t \log((1 + g_1)^t).$$

This equation can be solved to obtain

$$\varphi_G = 1 - \left(\frac{1 + g_1}{1 + g_0} \right)^{\frac{\beta}{1-\beta}}.$$

In our baseline experiment ($\tau = 0.3$), we find that $\varphi_L = 1 - \hat{w}_1/\hat{w}_0 = 0.9\%$ and $\varphi_G = 1 - \left(\frac{1.0139}{1.0148} \right)^{\frac{0.947}{1-0.947}} = 1.6\%$. The growth effect is larger than the level effect.

C Details of computation

The computation solution consists of first guessing the values of the relevant aggregate variables, solving for the value function and the stationary distribution of firms, and then updating the guess. The procedure is as follows.

1. Construct a grid for productivity \hat{q} and labor $\tilde{\ell}$. We use a log grid for \hat{q} with 100 points between 0 and 10^9 . For $\tilde{\ell}$, we use a linear grid with 30 points between 0 and 4.
2. Compute the innovation from entrants and the value from entry consistent with

the free entry condition

$$x_E^* = \left(\frac{\phi}{\theta_E(\gamma - 1)} \right)^{\frac{1}{\gamma}},$$

$$\hat{V}_E = \frac{\gamma\theta_E}{\beta} x_E^{\gamma-1}.$$

3. Guess \hat{Y} , \hat{w} , m , and g . Given m , we can calculate the value of μ by $\mu = X_E = mx_E^*$.
4. Solve for the value function by iterating on the value function and using linear interpolation between grid points.
5. Using the optimal decision rules, solve for the stationary distribution $f(\hat{q}, \alpha, \tilde{\ell})$ by iterating over the density.
6. Then check if the equilibrium conditions are verified. The four conditions are the following

(a) Aggregate output

$$\hat{Y} = \left(\int \int \alpha^\psi [\Omega(\hat{w}, \hat{Y}) \mathcal{L}'(\alpha, \tilde{\ell})]^{1-\psi} \int \hat{q} f(\hat{q}, \alpha, \tilde{\ell}) d\hat{q} d\alpha d\tilde{\ell} \right)^{\frac{1}{1-\psi}}$$

(b) Resource constraint

$$\hat{Y} = \hat{C} + \hat{R},$$

with $\hat{C} = \hat{w}/\xi$ and $\hat{R} = \theta_I \int \int \mathcal{X}_I(\alpha, \tilde{\ell})^\gamma \int \hat{q} f(\hat{q}, \alpha, \tilde{\ell}) d\hat{q} d\alpha d\tilde{\ell} + m(\phi + \theta_E x_E^\gamma)$

(c) Consistency condition for productivity

$$\frac{1}{N} \int \int \int \hat{q} f(\hat{q}, \alpha, \tilde{\ell}) d\alpha d\tilde{\ell} d\hat{q} = 1$$

(d) Free-entry condition

$$\hat{V}_E = \frac{\gamma\theta_E}{\beta} x_E^{\gamma-1}$$

where⁵⁶

$$\hat{V}_E = \int \tilde{Z}(\alpha, 0) \omega(\alpha) d\alpha \left[N + (1 - N) \int h(\hat{q}) d\hat{q} \right] (1 + \lambda_E) / (1 + g_q).$$

We use condition (a) to update the value for \hat{w} . When \hat{w} is too high, aggregate output implied by the firms decision is too low. We use condition (b) to update the value for \hat{Y} . If \hat{Y} is too high then the resource constraint is not satisfied. We update g_q using condition (c). Intuitively, when g_q is too small, the stationary density $f(\hat{q}, \alpha, \tilde{\ell})$ implies the values of \hat{q} that are too large. To update the value of m we use condition (d). Because a large m implies a large μ , which in turn lowers \tilde{Z} . Thus the value of m affects the computed value of \hat{V}_E , through \tilde{Z} .

7. Go back to Step 3, until convergence.

D Details of the extensions and robustness checks of Section 5

D.1 Extension 1: persistent exogenous shocks

This section complements section 5.1.1 by giving more details on the calibration of the extension with persistent transitory shocks.

D.1.1 Calibration

We use the variance and autocovariance of establishment-level employment growth to identify the size of the shock ε and the persistence parameter ρ . To give the intuition behind this strategy, let us assume that instead of following a discrete-valued Markov process, the exogenous productivity α follows an AR(1) process (in logs), that is $\ln \alpha_t = \varphi \ln \alpha_{t-1} + u_t$ where u_t is i.i.d. with mean zero and variance σ_u^2 . This assumption simplifies the expression of the variance and covariance of log employment changes. In the absence of firing costs, the employment of the firm is given by $\ell = \left(\frac{1-\psi}{\hat{w}}\right)^{\frac{1}{\psi}} \alpha \hat{q} \hat{Y}$, the variance of

⁵⁶Computed from

$$\begin{aligned} \hat{V}_E &= \int \left[\int \hat{Z}((1 + \lambda_E)\hat{q}/(1 + g_q), \alpha, 0) (\bar{f}(\hat{q}) + (1 - N)h(\hat{q})) d\hat{q} \right] \omega(\alpha) d\alpha \\ &= \int \left[\int \hat{q} \tilde{Z}(\alpha, 0) (1 + \lambda_E)/(1 + g_q) (\bar{f}(\hat{q}) + (1 - N)h(\hat{q})) d\hat{q} \right] \omega(\alpha) d\alpha \end{aligned}$$

Table 6: Calibration of the two extensions

	Parameter	Extension 1	Extension 2
Discount rate	β	0.947	0.947
Disutility of labor	ξ	1.475	1.480
Demand elasticity	ψ	0.2	0.2
Innovation step: entrants	λ_E	1.50	1.50
Innovation step: incumbents	λ_I	0.25	0.25
Innovation cost curvature	γ	2.0	2.0
Innovation cost: entrants	θ_E	7.995	6.807
Innovation cost: incumbents	θ_I	1.333	1.134
Entry cost	ϕ	0.1642	0.0420
Exogenous exit rate	δ	0.001	0.015
Transitory shock: size	ε	0.564	0.258
Transitory shock: persistence	ρ	0.718	-
Avg productivity from inactive lines	h mean	0.976	0.050
Firing tax	τ	0.000	0.000
Weight function parameter	χ_1	-	35.0
Weight function parameter	χ_2	-	26.5
θ_I function parameter	χ_3	-	0.4
θ_I function parameter	χ_4	-	0.8

log employment changes is then $V(\ln \ell_t - \ln \ell_{t-1}) = V(\ln \alpha_t - \ln \alpha_{t-1} + \ln \hat{q}_t - \ln \hat{q}_{t-1})$. Abstracting from the correlation between x_{It-1} and α_{t-1} , we can write the variance of log employment changes as a function of the variance of the changes in the endogenous productivity \hat{q}_t and that of changes in the exogenous productivity α_t . Using the AR(1) assumption, we get

$$V(\ln \ell_t - \ln \ell_{t-1}) = \frac{2(1-\varphi)}{1-\varphi^2} \sigma_u^2 + V(\ln \hat{q}_t - \ln \hat{q}_{t-1}).$$

The covariance of log employment changes can be written as a function of the variance of α and the persistence parameter:

$$Cov(\ln \ell_t - \ln \ell_{t-1}, \ln \ell_{t-1} - \ln \ell_{t-2}) = -\frac{(1-\varphi)^2}{1-\varphi^2} \sigma_u^2.$$

Given the variance of endogenous productivity $V(\ln \hat{q}_t - \ln \hat{q}_{t-1})$, we can infer the variance of the innovation σ_u^2 and the persistence parameter φ from these two statistics. Similarly, when α follows a Markov chain, the variance and the covariance of log employment changes can be used to infer the size of the shock ε and the persistence parameter ρ . The full calibration is reported in Table 6 and the comparison with the models targets are given in Table 7.

Table 7: Comparison between model outcome and the targets for the two extensions

	Data	Model	
		Extension 1	Extension 2
Growth rate of output g (%)	1.48	1.48	1.48
Employment L	0.613	0.613	0.613
Tail index κ	1.06	1.06	-
Job creation rate (%)	17.0	-	17.5
Job creation rate from entry (%)	6.4	6.4	6.7
Entry rate (%)	12.6	-	6.6
Variance of employment growth	0.24	0.24	-
Auto-cov. of employment growth	-0.05	-0.05	-

Note: The growth rate and employment targets are computed using BEA and BLS data; for the tail index, we use Axtell (2001)'s estimate; the job flows data are computed from the Census Bureau BDS dataset and the variance and auto-covariance of employment growth are measured from LBD micro data.

D.1.2 Data

We estimate the variance and covariance of annual log employment changes using US census microdata from the Longitudinal Business Database (LBD). The LBD is an exhaustive establishment-level dataset that covers nearly all the non-farm private economy. The dataset provides longitudinally linked data on employment and payroll data for 21 million establishments over 1976-2000. The dataset is constructed using information from the business register, economic censuses and surveys.⁵⁷ We used the Synthetic LBD (U.S. Census Bureau, 2011), which is accessible through the virtual RDC. The results were then validated with the Census Bureau. We compute the variance and covariance of annual log employment change over the period 1976-2000 after excluding the three-digit SIC sectors 100 and 800 to 999. The estimated variance is 0.24 and the covariance is -0.05 .

D.2 Extension 2: smaller entrants and the deviation from Gibrat's law

This section provides the details on the analysis of Section 5.1.2.

D.2.1 Model setup

The (normalized) value of a firm at the beginning of period is

$$\hat{Z}(\hat{q}, \alpha, \ell) = (1 - \delta)\hat{V}^s(\hat{q}, \alpha, \ell) + \delta\hat{V}^o(\ell),$$

⁵⁷For a detailed description of the dataset, see <https://www.census.gov/ces/dataproducts/datasets/lbd.html>.

where

$$\hat{V}^o(\ell) = -\tau\hat{w}\ell$$

is the value of exit. The value of survival is

$$\hat{V}^s(\hat{q}, \alpha, \ell) = \max_{\ell' \geq 0, x_I} \left\{ \hat{\Pi}(\hat{q}, \alpha, \ell, \ell', x_I) + \beta \left((1 - u(\hat{q}))\hat{S} \left(x_I, \frac{\hat{q}}{1 + g_q}, \ell' \right) - u(\hat{q})\tau\hat{w}\ell' \right) \right\},$$

where

$$\hat{S} \left(x_I, \frac{\hat{q}}{1 + g_q}, \ell' \right) = (1 - x_I)E_{\alpha'} \left[\hat{Z} \left(\frac{\hat{q}}{1 + g_q}, \alpha', \ell' \right) \right] + x_I E_{\alpha'} \left[\hat{Z} \left(\frac{(1 + \lambda_I)\hat{q}}{1 + g_q}, \alpha', \ell' \right) \right].$$

The period profit is

$$\hat{\Pi}(q, \alpha, \ell, \ell', x_I) = ([\alpha\hat{q}]^\psi \ell'^{-\psi} \hat{Y}^\psi - \hat{w})\ell' - \theta_I(\hat{q})\hat{q}x_I^\gamma - \tau\hat{w} \max(0, \ell - \ell').$$

For the entrants, the free entry condition is

$$\max_{x_E} \left\{ -\theta_E x_E^\gamma - \phi + \beta x_E \hat{V}_E \right\} = 0,$$

where x_E satisfies the optimality condition

$$\beta \hat{V}_E = \gamma \theta_E x_E^{\gamma-1}.$$

The expected benefit of entry, \hat{V}_E , is now calculated from

$$\hat{V}_E = \int \left[\int \hat{Z} \left(\frac{(1 + \lambda_E)\hat{q}}{1 + g_q}, \alpha, 0 \right) (Np(\hat{q}) + (1 - N)h(\hat{q})) d\hat{q} \right] \omega(\alpha) d\alpha.$$

D.2.2 Transformed model and computation

Define the frictionless level of employment without temporary shock as

$$\ell^*(\hat{q}; \hat{w}, \hat{Y}) \equiv \arg \max_{\ell'} ([\alpha\hat{q}]^\psi \ell'^{-\psi} \hat{Y}^\psi - \hat{w})\ell'$$

with $\alpha = 1$; that is,

$$\ell^*(\hat{q}; \hat{w}, \hat{Y}) = \left(\frac{1 - \psi}{\hat{w}} \right)^{\frac{1}{\psi}} \hat{q}\hat{Y}.$$

Also define $\Omega(\hat{w}, \hat{Y})$ by

$$\Omega(\hat{w}, \hat{Y}) \equiv \frac{\ell^*(\hat{q}; \hat{w}, \hat{Y})}{\hat{q}}.$$

In addition, define the deviation of employment from the frictionless level by

$$\tilde{\ell} \equiv \frac{\ell}{\ell^*(\hat{q}; \hat{w}, \hat{Y})}.$$

Similarly, let

$$\tilde{\ell}' \equiv \frac{\ell'}{\ell^*(\hat{q}; \hat{w}, \hat{Y})}.$$

Then, the period profit can be rewritten as

$$\hat{\Pi}(\hat{q}, \alpha, \ell, \ell', x_I) = \left(\left[\frac{\alpha}{\Omega(\hat{w}, \hat{Y})} \right]^\psi \tilde{\ell}'^{-\psi} \hat{Y}^\psi - \hat{w} \right) \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' - \theta_I(\hat{q}) \hat{q} x_I^\gamma - \tau \hat{w} \max\langle 0, \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell} - \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' \rangle.$$

Thus this is linear in \hat{q} , and can be rewritten as $\hat{q} \tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I)$, where

$$\tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) \equiv \left(\left[\frac{\alpha}{\Omega(\hat{w}, \hat{Y})} \right]^\psi \tilde{\ell}'^{-\psi} \hat{Y}^\psi - \hat{w} \right) \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' - \theta_I(\hat{q}) x_I^\gamma - \tau \Omega(\hat{w}, \hat{Y}) \hat{w} \max\langle 0, \tilde{\ell} - \tilde{\ell}' \rangle.$$

Although the value function is not linear in \hat{q} , we still utilize the transformation on ℓ by defining the new value functions (abusing the $\tilde{\cdot}$ notation on the value functions) as

$$\tilde{Z}(\hat{q}, \alpha, \tilde{\ell}) = (1 - \delta) \tilde{V}^s(\hat{q}, \alpha, \tilde{\ell}) + \delta \tilde{V}^o(\hat{q}, \tilde{\ell}),$$

where

$$\tilde{V}^o(\hat{q}, \tilde{\ell}) = -\tau \hat{w} \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}.$$

$$\tilde{V}^s(\hat{q}, \alpha, \tilde{\ell}) = \max_{\tilde{\ell}' \geq 0, x_I} \left\{ \hat{q} \tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) + \beta \left((1 - u(\hat{q})) \tilde{S} \left(x_I, \frac{\hat{q}}{1 + g_q}, \tilde{\ell}' \right) - u(\hat{q}) \tau \hat{w} \hat{q} \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' \right) \right\},$$

where

$$\tilde{S} \left(x_I, \frac{\hat{q}}{1 + g_q}, \tilde{\ell}' \right) = (1 - x_I) E_{\alpha'} \left[\tilde{Z} \left(\frac{\hat{q}}{1 + g_q}, \alpha', (1 + g_q) \tilde{\ell}' \right) \right] + x_I E_{\alpha'} \left[\tilde{Z} \left(\frac{(1 + \lambda_I) \hat{q}}{1 + g_q}, \alpha', \frac{(1 + g_q) \tilde{\ell}'}{1 + \lambda_I} \right) \right],$$

where the transformation of $\tilde{\ell}'$ is similar to the baseline case.

For a given $\tilde{\ell}'$, x_I can be solved from the first-order condition

$$\gamma \theta_I(\hat{q}) \hat{q} x_I^{\gamma-1} = \Gamma_I,$$

where

$$\Gamma_I \equiv \beta(1-u(\hat{q})) \left\{ E_{\alpha'} \left[\tilde{Z} \left(\frac{(1+\lambda_I)\hat{q}}{1+g_q}, \alpha', \frac{(1+g_q)\tilde{\ell}'}{1+\lambda_I} \right) \right] - E_{\alpha'} \left[\tilde{Z} \left(\frac{\hat{q}}{1+g_q}, \alpha', (1+g_q)\tilde{\ell}' \right) \right] \right\}.$$

The expected benefit of entry, \hat{V}_E , is calculated with the same formula as above

$$\hat{V}_E = \int \left[\int \tilde{Z} \left(\frac{(1+\lambda_E)\hat{q}}{1+g_q}, \alpha, 0 \right) (Np(\hat{q}) + (1-N)h(\hat{q})) d\hat{q} \right] \omega(\alpha) d\alpha,$$

because $\tilde{\ell} = 0$ is equivalent to $\ell = 0$.

The computational steps are similar to the baseline model. The only difference is that we need to guess $\bar{f}(\hat{q})$ before performing the optimization. We update the guess at the same time as we update the aggregate variables. (It can also be done within the aggregate variables loop.) The following are the steps:

1. First, several variables can be computed from parameters. First, calculate x_E^* from

$$x_E^* = \left(\frac{\phi}{\theta_E(\gamma-1)} \right)^{\frac{1}{\gamma}}.$$

2. Then \hat{V}_E can be computed from

$$\hat{V}_E = \frac{\gamma\theta_E}{\beta} x_E^{\gamma-1}.$$

3. Start the iteration. Guess \hat{Y} , \hat{w} , m , and g . Guess $\bar{f}(\hat{q})$.

Given m , we can calculate the value of μ by $\mu = X_E = mx_E^*$. From $\bar{f}(\hat{q})$ and μ , we can obtain $u(\hat{q})$ and $p(\hat{q})$. (The value of N can still be calculated by the same formula as in the baseline case.)

Now we are ready to solve the Bellman equation for the incumbents. We have two choice variables, $\tilde{\ell}'$ and x_I . The first-order condition for x_I is

$$\gamma\theta_I(\hat{q})\hat{q}x_I^{\gamma-1} = \Gamma_I,$$

and thus x_I can be computed from

$$x_I = \left(\frac{\Gamma_I}{\gamma\theta_I(\hat{q})\hat{q}} \right)^{1/(\gamma-1)},$$

where

$$\Gamma_I \equiv \beta(1-u(\hat{q})) \left\{ E_{\alpha'} \left[\tilde{Z} \left(\frac{(1+\lambda_I)\hat{q}}{1+g_q}, \alpha', \frac{(1+g_q)\tilde{\ell}'}{1+\lambda_I} \right) \right] - E_{\alpha'} \left[\tilde{Z} \left(\frac{\hat{q}}{1+g_q}, \alpha', (1+g_q)\tilde{\ell}' \right) \right] \right\}.$$

We can see that x_I is uniquely determined once we know $\tilde{\ell}'$. Let the decision rule for $\tilde{\ell}'$ be $\mathcal{L}'(\hat{q}, \alpha, \tilde{\ell}')$. Then $x_I = \mathcal{X}_I(\hat{q}, \alpha, \tilde{\ell}')$.

4. Once all decision rules are computed, we can find $f(\hat{q}, \alpha, \tilde{\ell}')$ by iterating over the density.
5. Now, we check if the first guesses are consistent with the solution from the optimization. First, $\bar{f}(\hat{q})$ can be calculated from $f(\hat{q}, \alpha, \tilde{\ell}')$.

The value \hat{w} is checked from

$$\hat{Y} = \left(\int \int \int [\alpha \hat{q}]^\psi [\ell^*(\hat{q}; \hat{w}, \hat{Y}) \mathcal{L}'(\hat{q}, \alpha, \tilde{\ell}')]^{1-\psi} f(\hat{q}, \alpha, \tilde{\ell}') d\hat{q} d\alpha d\tilde{\ell}' \right)^{\frac{1}{1-\psi}}.$$

The value of \hat{Y} is checked from

$$\frac{\hat{w}}{\hat{Y} - \hat{R}} = \xi,$$

where

$$\hat{R} = \int \int \int \theta_I \hat{q} \mathcal{X}_I(\hat{q}, \alpha, \tilde{\ell}')^\gamma f(\hat{q}, \alpha, \tilde{\ell}') d\hat{q} d\alpha d\tilde{\ell}' + m(\phi + \theta_E x_E^\gamma).$$

To check the value of g_q , the condition $\frac{1}{N} \int \int \int \hat{q} f(\hat{q}, \alpha, \tilde{\ell}') d\alpha d\tilde{\ell}' d\hat{q} = 1$ is used. Intuitively, when g_q is too small, the stationary density $f(\hat{q}, \alpha, \tilde{\ell}')$ implies the values of \hat{q} that are too large.

To set m , we look at the free-entry condition. Because a large m implies a large μ , which in turn lowers \tilde{Z} . Thus the value of m affects the computed value of \hat{V}_E , through \tilde{Z} . Recall that

$$\hat{V}_E = \frac{\gamma \theta_E}{\beta} x_E^{\gamma-1}$$

has to be satisfied, and this has to be equal to

$$\hat{V}_E = \int \left[\int \tilde{Z} \left(\frac{(1+\lambda_E)\hat{q}}{1+g_q}, \alpha, 0 \right) (Np(\hat{q}) + (1-N)h(\hat{q})) d\hat{q} \right] \omega(\alpha) d\alpha.$$

6. Go back to Step 3, until convergence.

Table 8: Size distribution, Comparison between the US data and the model outcome

	Data	Baseline	Extension
0-4	0.495	0.917	0.491
5-9	0.223	0.017	0.253
10-19	0.138	0.020	0.135
20-49	0.089	0.025	0.087
50-99	0.030	0.009	0.017
100-249	0.017	0.006	0.011
250-499	0.004	0.002	0.003
500-999	0.002	0.001	0.001
1000+	0.001	0.001	0.001

Note: The establishment size distribution is computed from the US Census BDS dataset (average over 1976-2012).

D.2.3 Calibration

The overall calibration follows similar steps as the baseline case. The values of β , ψ , λ_I , γ , and δ are the same as the baseline model. For ξ , we target $L = 0.61$ as in the baseline case. The values ϕ and ε are set so that the model generates the amount of overall job creation rate and the job creation rate by entrants close to the data. We assume that $\lambda_E = 1.50$. As in the baseline model, the level parameter of incumbent innovation cost, now represented by $\bar{\theta}_I$ in equation (13), is set so that the overall growth rate of output, g , is 1.48%. We set θ_E so that $\theta_E/\bar{\theta}_I = \lambda_E/\lambda_I$.

The new parameters of this extended model are χ_1 and χ_2 in equation (12) and χ_3 and χ_4 in equation (13). The value of χ_1 is set as a large number so that the size of entrants becomes closer to the data. Given the job creation rate from entrants, the size of entrants is reflected in the entry rate. A large value of χ_1 makes the size of entrants small and thus increases the entry rate for a given job creation rate by entrants. The value of χ_3 relates to the speed of growth by a small firm and thus is reflected in the size distribution of firms for small firms. The other two parameters, χ_2 and χ_4 , also have effects on the size distribution of firms. Thus, these parameters are picked so that the size distribution of firms is close to the data. The parameter values are summarized in Table 6.

Table 8 compares the size distribution of firms in the data, the baseline model, and the extended model. The extended model is very close to the data.

Table 7 describes the outcomes of the models for $\tau = 0$ in the baseline model and the extension. The discrepancy in the entry rate between the model and the data is substantially smaller in the extended model. While it is not perfect, this seems to be

Table 9: Age distribution, Comparison between the US data and the model outcome

	Data	Baseline	Extension
0	0.095	0.028	0.066
1	0.076	0.027	0.059
2	0.067	0.026	0.054
3	0.060	0.025	0.050
4	0.055	0.025	0.046
5	0.050	0.024	0.043
6-10	0.187	0.110	0.177
11-15	0.123	0.096	0.129
16-20	0.090	0.083	0.095
21-25	0.066	0.072	0.071
26+	0.130	0.484	0.211

Note: The age distribution is computed from the US Census BDS dataset (average over 2003-2012). - age not available for all age classes before 2003. may not sum to one due to rounding.

Table 10: Exit rate by age, Comparison between the US data and the model outcome

	Data	Baseline	Extension
1	0.227	0.028	0.109
2	0.160	0.028	0.086
3	0.138	0.028	0.076
4	0.123	0.028	0.071
5	0.115	0.028	0.068
6-10	0.093	0.028	0.064
11-15	0.073	0.028	0.060
16-20	0.063	0.028	0.058
21-25	0.057	0.028	0.057
26+	0.048	0.028	0.057

Note: The exit rate by age is computed from the US Census BDS dataset (average over 2003-2012).- age not available for all age classes before 2003. May not sum to one due to rounding.

Table 11: Exit rate by size, Comparison between the US data and the model outcome

	Data	Baseline	Extension
0-4	0.182	0.028	0.092
5-9	0.047	0.028	0.041
10-19	0.034	0.028	0.041
20-49	0.028	0.028	0.041
50-99	0.023	0.028	0.041
100-249	0.018	0.028	0.041
250-499	0.011	0.028	0.041
500-999	0.009	0.028	0.041
1000+	0.007	0.028	0.041

Note: The exit rate by size is computed from the US Census BDS dataset (average over 1976-2012).

the closest we can achieve given the functional forms. What is important here is that the results, and their intuitions, remain the same with these modifications that make the model outcome closer to the data.

D.3 Robustness checks to the innovation size

D.3.1 Smaller innovation advantage of entrants

We assess the robustness of the results to a smaller innovative advantage of entrants. In the baseline, we set $(1 + \lambda_E)/(1 + \lambda_I) = 2$ in line with the ratio of the number of patent citations of entrants over that of incumbents. Given the value of λ_I , the baseline calibration implies $\lambda_E/\lambda_I = 6$. An alternative interpretation of the relative citation rate of entrants is to set $\lambda_E/\lambda_I = 2$. We therefore solve the model for $\lambda_E = 2\lambda_I$, where λ_I is kept at the same value as in the baseline calibration. As an additional robustness check, we also report the results for the case where the entrants and the incumbents have the same innovative step, that is $\lambda_E = \lambda_I$. The other parameters, reported in Table 12, are set using the same strategy as in the baseline (see Section 4.1). We reset θ_I , ϕ , ε and δ to match the same targets as the baseline and keep the other parameters at the same value as in the baseline.

The results when entrants have a lower innovative advantage are reported in Table 14. We find that the consequences of the firing tax for entrants and incumbents are qualitatively robust to these changes in the calibration. As in the baseline calibration, the firing tax leads to higher innovation rates for incumbents and lower innovation rates for entrants. The overall effect of the firing tax on growth is however sensitive to the exact calibration. The overall negative effect of firing costs on the growth rate is reduced

when entrants have a lower innovative advantage. The growth rate of output declines only by 0.01 percentage point when $\lambda_E = 2\lambda_I$, and it even rises slightly relative to the frictionless benchmark when $\lambda_E = \lambda_I$. When entrants have a lower innovative advantage, they also account for a smaller share of aggregate productivity growth, which dampens the consequences of the decline in entry on growth. This calibration shows that the contribution of entrants to aggregate productivity growth is key for the consequences of the firing tax on aggregate productivity growth.

D.3.2 Smaller innovation steps

In the benchmark calibration, the size of the incumbents' innovation step λ_I is set at 0.25, following estimates by Bena et al. (2015). In this section we adopt an alternative strategy and use data on the establishment-level employment dynamics to calibrate this parameter. We set λ_I to match the relative proportion of establishments creating jobs and destroying jobs. We measure the relative proportion of establishments creating and destroying jobs from the BLS annual Business Employment Dynamics Data and find a ratio of 1.05.⁵⁸ The incumbents' innovation step is closely related to this statistic. For a given growth rate g_q , a smaller λ_I implies a higher innovation probability x_I and hence a larger proportion of establishment creating jobs. In fact, the same growth rate can be reached either with a high λ_I and low x_I , or with a low λ_I and high x_I . To match the ratio of the relative proportion of establishments creating jobs, we set λ_I at 0.0832, which is lower than the baseline value. We continue to assume $\lambda_E = 6\lambda_I$; hence λ_E is also lower than in the baseline. The rest of the parameters are set following the same strategy as in the baseline. The parameters and the targeted statistics are in Tables 12 and 13.

We report the results of this calibration in Table 15. As expected, with the lower innovation step λ_I , the incumbents' probability to innovate is higher than in the baseline. On average, 48% of incumbents innovate in a given year compared with 8.4% in the baseline calibration. Overall the results are qualitatively robust to this alternative calibration strategy. The firing tax leads to a decline in average productivity, an increase in the innovation of incumbents and to a reduction in the innovation of entrants. Quantitatively, the effects of the firing tax on the growth rate, however, differ from the baseline. We find that the growth rate of aggregate productivity is virtually unaffected by the firing tax. This smaller negative effect of the firing tax on the growth rate comes

⁵⁸We compute the average share of expanding establishments over the average share of contracting establishments over the available period (March 1994-March 2015). The data are publicly available at <https://www.bls.gov/bdm/bdmann.htm>.

Table 12: Calibration: alternative innovation sizes

	Parameter	$\lambda_E = 2\lambda_I$	$\lambda_E = \lambda_I$	Small λ_I	$\lambda_I = 0$
Discount rate	β	0.947	0.947	0.947	0.947
Disutility of labor	ξ	1.483	1.487	1.482	1.496
Demand elasticity	ψ	0.2	0.2	0.2	0.2
Innovation step: entrants	λ_E	0.50	0.25	0.50	1.50
Innovation step: incumbents	λ_I	0.25	0.25	0.08	0.00
Innovation cost curvature	γ	2.0	2.0	2.0	2.0
Innovation cost: entrants	θ_E	1.256	0.483	0.417	5.750
Innovation cost: incumbents	θ_I	0.628	0.483	0.070	0.958
Entry cost	ϕ	0.3243	0.5477	0.8502	0.1644
Exogenous exit rate	δ	0.00090	0.00097	0.00056	0.00127
Transitory shock: size	ε	0.245	0.234	0.260	0.223
Avg productivity from inactive lines	h mean	0.976	0.976	0.976	0.976
Firing tax	τ	0.0	0.0	0.0	0.0

here again from the smaller contribution of entrants to the growth rate. Despite the higher innovation advantage of entrants ($\lambda_E = 6\lambda_I$), the contribution of entrants to the growth rate is lower than in the baseline. The decline in the entry rate has therefore less impact on aggregate productivity growth. The results of this calibration are very similar to the case where $\lambda_E = 2\lambda_I$. Note that the two calibrations have very similar values for λ_E . In the end, these calibrations suggest that the key parameter for the overall effect of the firing tax on productivity growth is λ_E rather than λ_E/λ_I .

D.3.3 When only entrants innovate

To evaluate the importance of including the incumbents' innovation in the model, we consider the case when only entrants innovate. We set $\lambda_I = 0$ and re-calibrate the parameters θ_E , ξ , ε , and δ to match the growth rate of output per worker, the employment rate, the job creation rate, and the tail index of the firm size distribution. The other parameters (including ϕ) are kept identical to the baseline calibration. Since entrants are the only innovators, we can no longer match the job creation rate by entrants because θ_E needs to be set at a value that is consistent with the growth rate. The results, reported in Table 14, show that ignoring the incumbents' innovation would lead to overestimating the decline in the growth rate. When only entrants innovate, the positive impact on the incumbents' innovation is absent and the firing tax therefore leads to a larger decline in innovation and aggregate productivity growth. The effect is quantitatively substantial. We find that the growth rate in the economy with firing costs is 1.33% vs 1.39% in our baseline.

Table 13: Comparison between model outcome and the targets for the alternative calibrations

	Data	$\lambda_E = 2\lambda_I$	$\lambda_E = \lambda_I$	Small λ_I	$\lambda_I = 0$
Growth rate of output g (%)	1.48	1.48	1.48	1.48	1.48
Employment L	0.613	0.613	0.613	0.613	0.613
Tail index κ	1.06	1.06	1.06	1.06	1.06
Job creation rate (%)	17.0	17.0	17.0	17.0	17.0
Job creation rate from entry (%)	6.4	6.4	6.4	6.4	9.6
Positive employment growth	1.05	-	-	1.04	-

Note: The growth rate and employment targets are computed using BEA and BLS data; for the tail index, we use Axtell (2001)'s estimate; the job flows data are computed from the Census Bureau BDS dataset and the variance and autocovariance of employment growth are measured from LBD micro data. "Positive employment growth" refers to the ratio of expanding private sector establishments over contracting establishments computed from the BLS BED dataset. Missing points indicate that the statistics is not used as a target in the calibration.

Table 14: Robustness: innovative advantage of entrants

	Baseline		$\lambda_E = 2\lambda_I$		$\lambda_E = \lambda_I$	
	$\tau = 0.0$	$\tau = 0.3$	$\tau = 0.0$	$\tau = 0.3$	$\tau = 0.0$	$\tau = 0.3$
Growth rate of output g (%)	1.48	1.39	1.48	1.48	1.48	1.51
Innovation probability: incumbents \bar{x}_I	0.084	0.091	0.162	0.172	0.202	0.216
Innovation probability: entrants x_E	0.143	0.143	0.508	0.508	1.0	1.0
Creative destruction rate μ (%)	2.65	2.30	4.46	4.01	5.35	4.58
Employment L	100	98.8	100	98.4	100	98.4
Normalized output \hat{Y}	100	98.1	100	97.8	100	97.8
Normalized average productivity \hat{Y}/L	100	99.3	100	99.4	100	99.3
Number of active products N	0.964	0.958	0.980	0.978	0.982	0.979
Job creation rate (%)	17.0	4.7	17.0	5.6	17.0	5.7
Job creation rate from entry (%)	6.4	4.3	6.4	4.6	6.4	4.4
Job destruction rate (%)	17.0	4.7	17.0	5.6	17.0	5.7
Job destruction rate from exit (%)	2.8	2.4	4.6	4.1	5.5	4.7
R&D ratio R/Y (%)	11.5	10.6	12.0	11.6	12.2	11.6

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

Table 15: Robustness: smaller innovation steps

	Baseline		Small λ_I		$\lambda_I = 0$	
	$\tau = 0.0$	$\tau = 0.3$	$\tau = 0.0$	$\tau = 0.3$	$\tau = 0.0$	$\tau = 0.3$
Growth rate of output g (%)	1.48	1.39	1.480	1.490	1.48	1.33
Innovation probability: incumbents \bar{x}_I	0.084	0.091	0.483	0.528	0.000	0.000
Innovation probability: entrants x_E	0.143	0.143	1.000	1.000	0.169	0.169
Creative destruction rate μ (%)	2.65	2.30	4.50	3.83	4.00	3.57
Employment L	100	98.8	100	98.8	100	98.2
Normalized output \hat{Y}	100	98.1	100	98.0	100	97.7
Normalized average productivity \hat{Y}/L	100	99.3	100	99.4	100	99.5
Number of active products N	0.964	0.958	0.988	0.986	0.969	0.966
Job creation rate (%)	17.0	4.7	17.0	5.03	17.0	6.8
Job creation rate from entry (%)	6.4	4.3	6.4	4.42	9.6	6.6
Job destruction rate (%)	17.0	4.7	17.0	5.02	17.0	6.8
Job destruction rate from exit (%)	2.8	2.4	4.9	3.88	4.1	3.7
R&D ratio R/Y (%)	11.5	10.6	12.0	11.27	12.8	11.7

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

D.4 The effect of labor taxes and innovation subsidies

Our model can easily be extended to analyze the effects of taxes and subsidies. Here, we consider a labor tax of the rate $\eta \in [0, 1]$ and R&D subsidies at the rate $s \in [0, 1]$ to both incumbents and entrants. The budget constraint for consumer changes to

$$A_{t+1} + C_t = (1 + r_t)A_t + (1 - \eta)w_tL_t + T_t.$$

This changes the first-order condition for the consumer to

$$\frac{w_t}{C_t} = \frac{\xi}{1 - \eta}. \quad (22)$$

The other equilibrium conditions are unchanged. Because ξ is endogenously targeted in the calibration, this implies that the baseline model is identical even when the baseline value of η is nonzero. When the value of ξ is ξ_0 when $\eta = 0$, the equilibrium is identical when ξ is set at $\xi_0(1 - \eta)$ in the economy with $\eta > 0$. Moreover, the experiment starting from $\eta = \eta_0$ to $\eta = \eta_1$ is equivalent to the experiment starting from $\eta = 0$ to $\eta = 1 - (1 - \eta_1)/(1 - \eta_0)$. For example, an experiment of changing $\eta = 0.3$ to $\eta = 0.5$ is the same as starting from $\eta = 0$ to $\eta = 1 - (1 - 0.5)/(1 - 0.3) = 0.286$. Changing $\eta = 0.3$ to $\eta = 0.35$ is the same as starting from $\eta = 0$ to $\eta = 1 - (1 - 0.35)/(1 - 0.3) = 0.071$.

To facilitate the comparison to the literature, we also consider a more general form of preferences. Following Rogerson and Wallenius (2009), we specify the period utility

as

$$\log(C_t) - \xi \frac{L_t^{1+\nu}}{1+\nu},$$

where $\nu \geq 0$. Our baseline model is the special case of $\nu = 0$.⁵⁹ For this utility function, only change that is necessary for the equilibrium conditions is the optimality condition for the labor-leisure choice. Instead of (2), the first-order condition is

$$\frac{w_t}{C_t} = \frac{\xi L_t^\nu}{1-\eta}. \quad (23)$$

The analytical characterization of the model (Appendix B) is almost identical, except that the labor-market equilibrium condition must be modified to

$$\frac{\hat{w}(m)}{\hat{Y}(m, L) - \hat{R}(m, L)} = \frac{\xi L^\nu}{1-\eta}.$$

The computation of the model (Appendix C) is also similar, except that the step 6 (b) uses the consumption value of $\hat{C} = L^{-\nu} \hat{w} / \xi$.

First, we compare the effects of the firing tax to that of a 5% labor tax ($\eta = 0.05$). The baseline case is $\eta = 0$. We report the results for different values of the Frisch elasticity parameter ν in Table 16. In our baseline calibration ($\nu = 0$), we find that the labor tax reduces the growth rate to 1.38% while the firing tax reduces the growth rate to 1.39%. This shows that the effect of the baseline firing tax is of the same magnitude as a 5% labor tax.

Next, we make a comparison to the previous study by Rogerson and Wallenius (2009). To be consistent with their study (they consider the 30% case as the US and the 50% case as the continental Europe), we recalibrate the baseline case with $\eta = 0.30$. Then we compare the outcome with the case of $\eta = 0.50$.

We report the results in Table 17. We find that the employment rate is reduced by 31% for our baseline calibration ($\nu = 0$). We find that when we set ν to the same value as Rogerson and Wallenius (2009) ($\nu = 0.5$), labor declines by 21.3%, which is similar to

⁵⁹Ohanian et al. (2008) consider a similar utility function of the form (omitting the subsistence consumption and government consumption)

$$\alpha \log(C_t) + (1-\alpha) \frac{(\bar{L} - L_t)^{1-\gamma} - 1}{1-\gamma},$$

where $\alpha \in (0, 1)$, $\gamma \geq 0$, and $\bar{L} > 0$, in our notation. Note that our baseline specification corresponds to the case with $\gamma = 0$. Their overall conclusion is that a neoclassical model with this form of utility (with subsistence consumption) with changes in taxes explain the post-war change in hours across OECD countries. They note that the results are robust to the values of $\gamma \in [0, 2]$.

Table 16: Labor tax (5%) and the labor supply elasticity

	$\eta = 0.0$	$\eta = 0.05$			
		$\nu = 0.0$	$\nu = 0.5$	$\nu = 2$	$\nu = 10$
Growth rate of output g (%)	1.48	1.38	1.42	1.45	1.47
Innovation probability: incumbents \bar{x}_I	0.084	0.084	0.084	0.084	0.084
Innovation probability: entrants x_E	0.143	0.143	0.143	0.143	0.143
Creative destruction rate μ (%)	2.65	2.37	2.47	2.56	2.63
Employment L	100	94.5	96.4	98.3	99.5
Normalized output \hat{Y}	100	94.4	96.4	98.2	99.5
Normalized average productivity \hat{Y}/L	100	99.9	99.9	100	100.0
Number of active products N	0.964	0.959	0.961	0.962	0.963
Job creation rate (%)	17.0	16.6	16.7	16.9	17.0
Job creation rate from entry (%)	6.4	5.8	6.0	6.2	6.4
Job destruction rate (%)	17.0	16.6	16.7	16.9	17.0
Job destruction rate from exit (%)	2.8	2.5	2.6	2.7	2.7
R&D ratio R/Y	11.5	11.1	11.2	11.4	11.5

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

Table 17: Effect of a large increase in the labor tax (0.30 to 0.50)

	$\eta = 0.30$	$\eta = 0.50$			
		$\nu = 0$	$\nu = 0.5$	$\nu = 2$	$\nu = 10$
Growth rate of output g (%)	1.48	0.95	1.11	1.29	1.43
Innovation probability: incumbents \bar{x}_I	0.084	0.085	0.085	0.084	0.084
Innovation probability: entrants x_E	0.143	0.143	0.143	0.143	0.143
Creative destruction rate μ (%)	2.65	1.09	1.56	2.09	2.50
Employment L	100	68.8	78.7	89.1	97.0
Normalized output \hat{Y}	100	68.0	78.2	88.9	96.9
Normalized average productivity \hat{Y}/L	100	98.7	99.4	99.7	99.9
Number of active products N	0.964	0.916	0.940	0.954	0.961
Job creation rate (%)	17.0	14.5	15.3	16.1	16.8
Job creation rate from entry (%)	6.4	2.8	3.9	5.1	6.1
Job destruction rate (%)	17.0	14.5	15.3	16.1	16.8
Job destruction rate from exit (%)	2.8	1.2	1.7	2.2	2.6
R&D ratio R/Y	11.5	8.2	9.44	10.6	11.3

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

Table 18: Firing tax and the labor supply elasticity

	$\tau = 0.0$	$\tau = 0.3$			
		$\nu = 0.0$	$\nu = 0.5$	$\nu = 2$	$\nu = 10$
Growth rate of output g (%)	1.48	1.39	1.40	1.40	1.41
Innovation probability: incumbents \bar{x}_I	0.084	0.091	0.091	0.091	0.091
Innovation probability: entrants x_E	0.143	0.143	0.143	0.143	0.143
Creative destruction rate μ (%)	2.65	2.30	2.31	2.33	2.34
Employment L	100	98.8	99.2	99.6	99.9
Normalized output \hat{Y}	100	98.1	98.5	98.9	99.2
Normalized average productivity \hat{Y}/L	100	99.3	99.3	99.3	99.3
Number of active products N	0.964	0.958	0.959	0.959	0.959
Job creation rate (%)	17.0	4.7	4.8	4.8	4.8
Job creation rate from entry (%)	6.4	4.3	4.3	4.3	4.4
Job destruction rate (%)	17.0	4.7	4.8	4.8	4.8
Job destruction rate from exit (%)	2.8	2.4	2.4	2.4	2.4
R&D ratio R/Y	11.5	10.6	10.6	10.6	10.6

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

Rogerson and Wallenius’ 22% decline. This is not trivial since Rogerson and Wallenius (2009) consider a substantially richer labor supply model (with life-cycle heterogeneity, extensive and intensive margin of labor supply).

We find that the large increase in the labor tax reduces the growth rate from 1.48% to 0.95% for $\nu = 0$, and to 1.11% for $\nu = 0.5$. As we discussed in the main text, for the firing tax, the decline in the growth rate is a result of two opposite effects for incumbents and entrants. In the case of the labor tax, entry is reduced because the labor tax reduces profitability, similarly to the firing tax case. The incumbents’ innovation is higher here, too, but note that the mechanism is not the same as in the case of the firing tax. The tax-escaping effect that we highlight in the case of firing tax is absent in the case of labor tax because innovating would not affect the tax rate. The incumbent innovation increases because entrants’ innovation decreases (what we called the “creative destruction effect” in the main text).

For completeness, we repeat the effects of the firing tax for different values of ν . The results are reported in Table 18. We find that the effect of the firing tax when $\nu = 0.5$ is virtually identical to the baseline case. When ν is higher, a high firing tax leads to a smaller decline in the growth rate compared to a low ν because the entry rate decreases less. The overall growth effects, however, are very similar across different values of ν .

Now we consider the R&D subsidy. The R&D subsidy changes the cost for innovation

for incumbents and entrants. In particular, the innovation cost for the incumbents is

$$\mathbf{r}_{Ijt} = (1 - s)\theta_I Q_t \frac{q_{jt}}{\hat{q}_t} x_{Ijt}^\gamma$$

and the innovation cost for the entrant is

$$\mathbf{r}_{Ejt} = (1 - s)\theta_E Q_t x_{Ejt}^\gamma.$$

In the analytical characterization (Appendix B), this does not alter the calculation of \hat{R} in (20) is the same, as the resource cost does not change with subsidies. The equilibrium conditions change in the incumbent's innovation choice and the entrants' choices. The value functions for incumbents are now

$$\hat{Z}(\hat{q}, \alpha) = (1 - \delta)\hat{V}^s(\hat{q}, \alpha),$$

where

$$\hat{V}^s(\hat{q}, \alpha) = \max_{x_I} \psi \alpha \hat{q} \frac{\hat{Y}}{N} - (1 - s)\theta_I \hat{q} x_I^\gamma + \beta(1 - \mu)\hat{S}(x_I, \hat{q}/(1 + g_q))$$

and

$$\hat{S}(x_I, \hat{q}/(1 + g_q)) = (1 - x_I) \int \hat{Z}(\hat{q}/(1 + g_q), \alpha') \omega(\alpha') d\alpha' + x_I \int \hat{Z}((1 + \lambda_I)\hat{q}/(1 + g_q), \alpha') \omega(\alpha') d\alpha'.$$

Similar to the baseline, the value function is linear:

$$\hat{Z}(\hat{q}, \alpha) = \mathcal{A}\alpha\hat{q} + \mathcal{B}\hat{q},$$

The optimal x_I is

$$x_I = \left(\frac{\beta(1 - \mu)\lambda_I(\mathcal{A} + \mathcal{B})}{(1 + g_q)\gamma(1 - s)\theta_I} \right)^{\frac{1}{\gamma-1}}$$

and the constants are

$$\mathcal{A} = (1 - \delta)\psi \frac{\hat{Y}}{N}$$

and \mathcal{B} solves

$$\mathcal{B} = (1 - \delta)\beta(1 - \mu) \left(1 + \frac{\gamma - 1}{\gamma} \lambda_I x_I \right) \frac{\mathcal{A} + \mathcal{B}}{1 + g_q}.$$

The change from the baseline is therefore the expression for x_I only. For entrants, the

optimal innovation rate for the potential entrant is now

$$x_E^* = \left(\frac{\phi}{(1-s)\theta_E(\gamma-1)} \right)^{\frac{1}{\gamma}},$$

and the free entry condition (21) must be changed to

$$\frac{\gamma(1-s)\theta_E x_E^{\gamma-1}}{\beta} = \hat{V}_E,$$

The computation of the model (Appendix C) would change in a few places. First, in step 2, the entrants' innovation equations are

$$x_E^* = \left(\frac{\phi}{(1-s)\theta_E(\gamma-1)} \right)^{\frac{1}{\gamma}},$$

$$\hat{V}_E = \frac{(1-s)\gamma\theta_E}{\beta} x_E^{\gamma-1}.$$

Second, in step 4, because now the flow profit is

$$\tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) \equiv \left(\left[\frac{\alpha}{\Omega(\hat{w}, \hat{Y})} \right]^{\psi} \tilde{\ell}'^{-\psi} \hat{Y}^{\psi} - \hat{w} \right) \Omega(\hat{w}, \hat{Y}) \tilde{\ell}' - (1-s)\theta_I x_I^{\gamma} - \tau \Omega(\hat{w}, \hat{Y}) \hat{w} \max\langle 0, \tilde{\ell} - \tilde{\ell}' \rangle,$$

with $\Omega(\hat{w}, \hat{Y}) \equiv \ell^*(\hat{q}; \hat{w}, \hat{Y})/\hat{q}$, the first-order condition for x_I is

$$\gamma(1-s)\theta_I x_I^{\gamma-1} = \Gamma_I$$

and thus x_I can be computed from

$$x_I = \left(\frac{\Gamma_I}{\gamma(1-s)\theta_I} \right)^{1/(\gamma-1)},$$

where $\Gamma_I \equiv \beta(1-\mu)E_{\alpha'} \left[\tilde{Z}(\alpha', (1+g_q)\tilde{\ell}'/(1+\lambda_I))(1+\lambda_I) - \tilde{Z}(\alpha', (1+g_q)\tilde{\ell}') \right] / (1+g_q)$.

Third, in step 6(b), the free-entry condition is once again

$$\hat{V}_E = \frac{\gamma(1-s)\theta_E}{\beta} x_E^{\gamma-1}.$$

The results are in Table 19. We find that we would need a subsidy equal to 7.3% to offset the effect of the firing tax on the growth rate.

Table 19: Innovation subsidies

	Baseline, $\tau = 0$ $s = 0.0$	$\tau = 0.3$ $s = 0.073$
Growth rate of output g (%)	1.48	1.48
Innovation probability: incumbents \bar{x}_I	0.084	0.094
Innovation probability: entrants x_E	0.143	0.149
Creative destruction rate μ (%)	2.65	2.50
Employment L	100	99.7
Normalized output \hat{Y}	100	99.0
Normalized average productivity \hat{Y}/L	100	99.3
Number of active products N	0.964	0.961
Job creation rate (%)	17.0	5.1
Job creation rate from entry (%)	6.4	4.6
Job destruction rate (%)	17.0	5.1
Job destruction rate from exit (%)	2.8	2.6
R&D ratio R/Y	11.5	11.4

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

D.5 Expanding variety model

Our baseline model is a quality-ladder model. There are other popular formulations of innovation, and it is of interest to see how different formulations of innovation can affect the outcome of the model. Here, we consider the expanding variety model of Romer (1990). While there is some overlap in the notation with the main text, we intend this section to be self-contained.

As in the standard expanding variety model, we assume that innovation is conducted only by the entrants. For simplicity, we assume that there is no exogenous exit of firms. Furthermore, we assume that there are no exogenous productivity shocks, which implies that the model has a limited ability to match observed job flows. In the model, there is no job creation by incumbents, and job destruction is uniform across incumbents, and job flows will therefore be lower than in the data. In contrast, with innovation conducted by the incumbents, the quality ladder model naturally generates both job creation and job destruction by incumbents even without exogenous shocks. While we could easily add exogenous shocks to the model to better fit the job flows data, we choose here to consider the simplest version of the model to study the main mechanism through which firing taxes affect growth.

The production structure is similar to our baseline model. The final goods, which are used both for consumption and R&D, are produced using only intermediate goods, and the differentiated intermediate goods are produced by monopolists using labor. Let the final goods produced at time t be Y_t . The quantity of intermediate good j used at

time t is denoted y_{jt} . The final goods production function is

$$Y_t = \left(\int_0^{N_t} y_{jt}^{1-\psi} dj \right)^{\frac{1}{1-\psi}}, \quad (24)$$

where $1/\psi$ is the elasticity of substitution across goods, N_t is the number of goods produced at time t . The final goods market is perfectly competitive. After the cost minimization by the final goods producers, the inverse demand function for intermediate good j is

$$p_{jt} = y_{jt}^{-\psi} Y_t^\psi,$$

where p_{jt} is the price of good j at time t .

Each entrant needs to pay $\eta^{-1} N_t^{\frac{\psi}{1-\psi}-1}$ units of final goods to come up with a new variety and enter. The free-entry condition for innovation equates the value of innovation, \mathbf{V}_t , to the cost. Thus

$$\mathbf{V}_t = \eta^{-1} N_t^{\frac{\psi}{1-\psi}-1}. \quad (25)$$

The growth in the number of variety depends on aggregate spending by entrants on developing new varieties R_t .

$$N_{t+1} - N_t = \eta N_t^{1-\frac{\psi}{1-\psi}} R_t. \quad (26)$$

On the consumer side, the preferences are assumed to be the same as in the baseline model:

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t [\log(C_t) - \xi L_t],$$

where C_t is the final goods consumption and L_t is labor supply.

Let us focus on the balanced-growth path. Let the growth rate of N_t be g , that is $(N_{t+1} - N_t)/N_t = g$. Along the balanced-growth path, Y_t , C_t , R_t , and w_t grow at the rate g_Y , where

$$1 + g_Y = (1 + g)^{\frac{\psi}{1-\psi}}. \quad (27)$$

From the Euler equation of the consumer, the firm discounts future profits at rate $\beta(C_t/C_{t+1}) = \beta/(1 + g_Y)$. Thus, the Bellman equation for the intermediate good producer is

$$V_t(\ell_{t-1}) = \max_{\ell_t} \ell_t^{1-\psi} Y_t^\psi - w_t \ell_t - \tau w_t \max\langle 0, \ell_{t-1} - \ell_t \rangle + \frac{\beta}{1 + g_Y} V_{t+1}(\ell_t).$$

As in the quality ladder model, we can make the Bellman equation stationary. Here,

we normalize by dividing Y_t , C_t , R_t , and w_t by $N_t^{\frac{\psi}{1-\psi}}$ and dividing ℓ_t by N_t^{-1} . The value function $V_t(\ell_{t-1})$ is divided by $N_t^{\frac{\psi}{1-\psi}-1}$.

$$\hat{V}_t(\hat{\ell}_{t-1}) = \max_{\hat{\ell}_t} \hat{\ell}_t^{1-\psi} \hat{Y}_t^\psi - \hat{w}_t \hat{\ell}_t - \tau \hat{w}_t \max\langle 0, (1+g)\hat{\ell}_{t-1} - \hat{\ell}_t \rangle + \frac{\beta}{1+g} \hat{V}_{t+1}(\hat{\ell}_t).$$

Note that we used $\ell_{t-1}/N_t^{-1} = (1+g)\ell_{t-1}/N_{t-1}^{-1} = (1+g)\hat{\ell}_{t-1}$ and $V_{t+1}(\hat{\ell}_t)/N_t^{\frac{\psi}{1-\psi}-1} = (1+g)^{\frac{\psi}{1-\psi}-1} V_{t+1}(\hat{\ell}_t)/N_{t+1}^{\frac{\psi}{1-\psi}-1} = (1+g_Y)(1+g)^{-1} \hat{V}_{t+1}(\hat{\ell}_t)$. This can be written without the time subscript as

$$\hat{V}(\hat{\ell}) = \max_{\hat{\ell}'} \hat{\ell}'^{1-\psi} \hat{Y}^\psi - \hat{w} \hat{\ell}' - \tau \hat{w} \max\langle 0, (1+g)\hat{\ell} - \hat{\ell}' \rangle + \frac{\beta}{1+g} \hat{V}(\hat{\ell}'). \quad (28)$$

Note that because all the entrants are identical, and the employment decision only depends on past employment $\hat{\ell}$, the employment process is deterministic and identical across firms: firms with the same age have the same employment level.

D.5.1 No firing tax case

With no firing tax, the intermediate good firm's problem becomes static. The maximization problem yields the optimal ℓ_t as

$$\ell_t^* = \left(\frac{1-\psi}{w_t} \right)^{\frac{1}{\psi}} Y_t = \left(\frac{1-\psi}{\hat{w}} \right)^{\frac{1}{\psi}} \hat{Y} N_t^{-1}.$$

In the context of the normalized Bellman, equation (28), $\hat{\ell}'$ is thus $((1-\psi)/\hat{w})^{\frac{1}{\psi}} \hat{Y}$, and therefore the value function satisfies ($\hat{\ell}$ is now no longer a state variable)

$$\hat{V} = \psi(1-\psi)^{\frac{1}{\psi}-1} \hat{w}^{1-\frac{1}{\psi}} \hat{Y} + \frac{\beta}{1+g} \hat{V}.$$

Thus

$$\hat{V} = \frac{\psi}{1-\beta/(1+g)} (1-\psi)^{\frac{1}{\psi}-1} \hat{w}^{1-\frac{1}{\psi}} \hat{Y}.$$

Note that a constant value of $\hat{\ell}'$ implies that $\ell_t = \ell_{t-1}/(1+g)$. Thus, $\ell_{t-1} - \ell_t = g\ell_t$ and the firm fires a proportion g of its employees every period.

From the free-entry condition,

$$\eta^{-1} = \frac{\psi}{1-\beta/(1+g)} (1-\psi)^{\frac{1}{\psi}-1} \hat{w}^{1-\frac{1}{\psi}} \hat{Y} \quad (29)$$

holds, because \hat{V} here corresponds to $\mathbf{V}_t/N_t^{\frac{\psi}{1-\psi}-1}$ in (25).

In the case with no tax, the employment ℓ_t is the same across different goods. From the production function,

$$Y_t = N_t^{\frac{1}{1-\psi}} \ell_t, \quad (30)$$

and thus

$$\hat{Y} = \left(\frac{1-\psi}{\hat{w}} \right)^{\frac{1}{\psi}} \hat{Y},$$

which implies

$$\hat{w} = 1 - \psi.$$

Therefore, the free-entry condition (29) can be rewritten as

$$\hat{Y} = \frac{1 - \beta/(1+g)}{\psi\eta}.$$

Note that this equation represents a positive relationship between \hat{Y} and g because the profit for each intermediate good producer contracts over time due to increase in N_t . When N_t is large, there are many intermediate-good firms who compete for limited production resources (labor in the current model). This implies that the wage increases as N_t becomes larger. When N_t grows faster, the future profit shrinks faster due to the wage increase. One can interpret this as a form of *creative destruction effect*, because the creation of new varieties forces existing firms to contract. Thus when g is large, it is necessary to have a large \hat{Y} (which supports a larger flow profit) in order to satisfy the free entry condition.

Because firms are symmetric, the aggregate labor can be written as

$$L_t = \ell_t N_t.$$

Because $\hat{\ell} = \ell_t/N_t^{-1} = \ell_t N_t$, this implies that $L_t = \hat{\ell}$. From (30) together with $\hat{Y} = Y_t/N_t^{\frac{\psi}{1-\psi}}$, we then have

$$\hat{Y} = \hat{\ell} = L_t.$$

The growth rate is determined by the R&D input in (26). To determine the R&D input, we use the condition $\hat{R} = \hat{Y} - \hat{C}$ and the consumer's static first-order condition

$$\hat{C} = \frac{\hat{w}}{\xi},$$

yielding

$$\hat{R} = \frac{1 - \beta/(1 + g)}{\psi\eta} - \frac{1 - \psi}{\xi}.$$

Thus, because $g = \eta\hat{R}$ from (26),

$$g = \frac{1 - \beta/(1 + g)}{\psi} - \frac{\eta(1 - \psi)}{\xi}. \quad (31)$$

D.5.2 Computation of the model with firing taxes

At each point in time, the normalized number of firms at age s , f_s , is

$$f_s \equiv \frac{gN_{t-s-1}}{N_t} = \frac{g}{(1 + g)^{1+s}}. \quad (32)$$

The computation proceeds as follows.

1. Guess g and \hat{w} . Because $g = \eta\hat{R} = \eta(\hat{Y} - \hat{C})$ and $\hat{C} = \hat{w}/\xi$,

$$\hat{Y} = \frac{g}{\eta} + \frac{\hat{w}}{\xi}.$$

2. Solve the Bellman equation (28). Let the policy function be $\hat{\ell}' = \mathcal{L}(\hat{\ell})$. Then we can calculate the normalized employment for firms of age s , $\hat{\ell}_s$, as

$$\hat{\ell}_0 = \mathcal{L}(0)$$

and

$$\hat{\ell}_s = \mathcal{L}(\hat{\ell}_{s-1})$$

for $s = 1, 2, \dots$

3. Check whether the guess is correct by looking at the two equilibrium conditions. First, from the production function, normalized output has to be

$$\hat{Y} = \left(\sum_{s=0}^{\infty} \hat{\ell}_s^{1-\psi} f_s \right)^{\frac{1}{1-\psi}},$$

where f_s is calculated by (32). The free-entry condition is

$$\hat{V}(0) = \eta^{-1}$$

because $\hat{V}(0)$ corresponds to $\mathbf{V}_t/N_t^{\frac{\psi}{1-\psi}-1}$ in the notation of (25). Adjust g and \hat{w} until these two conditions are satisfied.

D.5.3 Calibration

Similar to the benchmark model, we calibrate the frictionless economy to match the U.S growth rate and employment rate. The efficiency of innovation η is set so that $g_Y = 0.0148$. The disutility labor ξ is set so that $L_t = 0.613$. We use the same values of β and ψ as for the benchmark model: $\beta = 0.947$ and $\psi = 0.2$.

From

$$1 + g = (1 + g_Y)^{\frac{1-\psi}{\psi}}$$

from 27, we can obtain $g = 0.0605$. Then because

$$L_t = \hat{Y} = \frac{1 - \beta/(1 + g)}{\psi\eta},$$

with the target $L_t = 0.613$, we can obtain $\eta = 0.873$. We can solve (31) for ξ and obtain $\xi = 1.471$.

D.5.4 Results

We consider two experiments: $\tau = 0.3$ and $\tau = 1.0$. Table 20 summarizes the results. As in our model in the main text, firing taxes have both level effects and growth effects. The growth rate falls, and the quantitative impact is larger than in the main text, largely because innovation occurs here only through entry, which is affected negatively by τ . The incentive to enter is lower because the total benefit of entry is reduced by the tax. In the model in the main text, incumbents' innovation increases and counteracts this effect, while here we assume that incumbents do not innovate on their own variety.

Table 20: Results: expanding variety model

	$\tau = 0.0$	$\tau = 0.3$	$\tau = 1.0$
Growth rate of output g_Y (%)	1.48	1.31	1.05
Employment L	100	99.0	98.7
Normalized output \hat{Y}	100	98.9	98.1
Normalized average productivity \hat{Y}/L	100	99.9	99.4

Note: L , \hat{Y} , and \hat{Y}/L are set at 100 in the baseline simulation.

The level effect on employment and normalized output are also negative. As for the baseline model, this outcome is not trivial. Entrants are smaller than in the frictionless

case, but after entering, firms do not fire workers for some time, and eventually they will become larger compared to the frictionless case (recall that, in frictionless case, the firm size becomes smaller over time). After a certain point, they start firing workers at a constant rate (at the rate g , as in the frictionless case). Depending on the size of entrants, the size at which firms start firing workers, and the change in the overall growth rate, the aggregate labor demand can be higher or lower. Labor supply can also change because of the wealth effect.

The average productivity \hat{Y}/L falls because of misallocation, as in the main text. The production function (24) implies that the maximum production given L is achieved when $\hat{\ell}$ is constant across firms. Firing taxes generate dispersion in $\hat{\ell}$ (and thus the marginal product of labor) across firms and thus reduces aggregate productivity.

Overall, the expanding variety model produces similar results to the quality ladder model in our baseline model. The largest difference is that here we assume that the incumbents do not innovate. As a consequence, the effects that are intrinsic to incumbents, such as the tax-escaping effect, are not present in the current model.

E Empirical analysis

The quantitative results with the baseline calibration suggest that firing costs reduce the growth rate of the economy. As explained in Section 4.2, however, the overall effect on growth is the result of two opposing effects. Firing costs may increase the incumbents' innovation while discouraging the innovation by entrants. The overall effect could be positive or negative depending on which of these two effects dominate. To gain further insight on this question, we conduct in this section an empirical analysis of the effect of firing costs on innovation. Several studies have shown the effects of firing costs on job reallocation (Micco and Pagés, 2007; Haltiwanger et al., 2014; Davis and Haltiwanger, 2014), but only a few studies have investigated the consequences of firing costs for aggregate productivity. Using differences across the US states in the adoption of more stringent labor laws, Autor et al. (2007) find evidence suggesting that firing costs reduce total factor productivity. More closely related to our objective, Bassanini et al. (2009) investigate the effects of firing costs on total factor productivity growth. They find that more stringent dismissal regulations tend to reduce total factor productivity growth in industries where dismissal regulations are more likely to be binding.

In this section, we complement the existing studies by focusing on innovation spending. We analyze two different empirical models. First, we exploit the variation in employment protection regulations across countries and over time to evaluate how industry-

level R&D spending is correlated with these regulations. Second, we exploit the variation across industries as well and conduct an analysis similar to Bassanini et al. (2009).

E.1 Data

R&D spending (R&D): We compute R&D as R&D business expenditures, divided by the gross output of the industry. We use data on R&D business expenditures by industry and by country from the OECD ANBERD database (Analytical Business Enterprise Research and Development). The data are available at the two-digit ISIC Rev.3 level and are classified in industries according to the main activity of the enterprise carrying out the R&D. We remove the financial intermediation sector from the dataset. The ANBERD dataset includes statistical estimates, which leads to fewer missing values and more extensive time series than the raw data. The ANBERD dataset covers 32 OECD countries and 6 non-member countries between 1987 and 2011, with gaps and breaks in some of the series. The gross output data, obtained from the OECD STAN database, is also available at the two-digit ISIC Rev.3 level.

Employment protection indicator (EPL): We use two indicators of the strictness of employment protection constructed by the OECD. The indicator **EPL1** measures the strictness of dismissal regulation for individual dismissal, and the indicator **EPL2** includes measures of the strictness of the regulation on collective dismissal as well.⁶⁰ The indicators are constructed from the reading of statutory laws, collective bargaining agreements and case law combined with advice from officials from OECD member countries and country experts. The indicators are compiled from scores between 0 and 6 on the notification procedure, the severance pay and the difficulty of dismissal. The indicator **EPL1** is available between 1985 and 2013, and **EPL2** is available between 1998 and 2013. The dataset covers 34 OECD countries and 38 non OECD countries (for most non OECD countries the series is not available before 2008). The Employment protection indicators are publicly available at <http://stats.oecd.org/> and a comprehensive description of the method used to construct the indicator can be found at <http://www.oecd.org/els/emp/oecdindicatorsofemploymentprotection.htm>.

Layoff rate (layoff): To measure the sensitivity of each industry to firing costs, we use the layoff rate by industry in the US. Dismissal regulation in the US is less strict than in the rest of the countries considered. The US layoff rate can therefore be used as a proxy for the propensity of each industry to lay off workers. Following Bassanini et al. (2009), we

⁶⁰The OECD codes for EPL1 and EPL2 are EPRC_V1 and EPRC_V2.

estimate the US layoff rate by industry using data from the 2004 “Displaced workers, Employee, Tenure and Occupational Mobility” supplement of the Current Population Survey (CPS). We measure the layoff rate as the total number of displaced workers in the three years preceding the survey (2001, 2002 and 2003) divided by total employment in the industry in January 2004. A displaced worker is a worker who has lost his job because of the following reasons: “plant closing,” “insufficient work,” “position abolished,” “seasonal job ended,” or “self-operated business failed.” We use the Uniform Extract of CPS made available by the Center for Economic and Policy Research (<http://ceprdata.org/cps-uniform-data-extracts/cps-displaced-worker-survey/cps-dws-data/>). The data are organized according to the 2002 census industry classification. To be consistent with the R&D data, we convert the layoff data into the two-digit ISIC Rev. 3 classification. The correspondence between the two classification is reported in Table 21. Though the exact procedure used to estimate the US layoff rate differs from Bassanini et al. (2009), the two measures are strongly correlated (correlation coefficient of 0.71).

The merged dataset contains data on 27 OECD countries and 19 industries between 1987 and 2009, with breaks and gaps in the series. The 27 countries are: Austria, Belgium, Canada, Czech Republic, Estonia, Finland, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Switzerland, and United States. We excluded the primary sectors, the financial intermediation industry, as well as public and personal services (education, health, etc). The 19 industries used are listed in Table 21.

E.2 Empirical specifications

First, we utilize the variation in employment protection regulations across countries and over time to evaluate how R&D spending is correlated to these regulations. We estimate the following equation at the industry level

$$\log(\text{R\&D}_{jct}) = \beta_0 + \beta_1 \text{EPL}_{ct} + \gamma_j + \varepsilon_{jct}, \quad (33)$$

where γ_j is the industry fixed effect. R\&D_{jct} is the R&D spending of industry j in country c and year t , computed as the share of the industry’s output and EPL_{ct} is the indicator of employment protection. A high value of EPL_{ct} indicates that dismissal regulation is strict, and it is thus more costly to fire workers. The parameter of interest is β_1 , which indicates how R&D spending is related to the strictness of employment protection regulation.

Table 21: CPS-OECD industry classification correspondence

code	CPS label	code	OECD label
4	Construction	F	Construction
5	Nonmetallic mineral product manufacturing	26	Non-metallic mineral products
6	Primary metals and fabricated metal products	27-28	Basic metals and fabricated metal
7	Machinery manufacturing	29	Machinery n.e.c.
8	Computer and electronic product manufacturing	30-33	Electrical and optical equipment
9	Electrical equipment, appliance manufacturing	30-33	Electrical and optical equipment
10	Transportation equipment manufacturing	34-35	Transport equipment
11	Wood products	20	Wood and wood products
12	Furniture and fixtures manufacturing	36-37	Manufacturing, n.e.c.; recycling
13	Miscellaneous and not specified manufacturing	36-37	Manufacturing, n.e.c.; recycling
14	Food manufacturing	15-16	Food and beverages
15	Beverage and tobacco products	15-16	Food and beverages
16	Textile, apparel, and leather manufacturing	17-19	Textiles, wearing app. and leather
17	Paper and printing	21-22	Paper, printing and publ
18	Petroleum and coal products manufacturing	23	Coke, refined petroleum, nuclear fuel
19	Chemical manufacturing	24	Chemicals and chemical products
20	Plastics and rubber products	25	Rubber and plastics
21	Wholesale trade	50-52	Trade
22	Retail trade	50-52	Trade
23	Transportation and warehousing	60-64	Transport, storage and communications
24	Utilities	E	Electricity, gas and water supply
25	Publishing industries (except internet)	60-64	Transport, storage and communications
26	Motion picture and sound recording industries	60-64	Transport, storage and communications
27	Broadcasting (except internet)	60-64	Transport, storage and communications
28	Internet publishing and broadcasting	60-64	Transport, storage and communications
29	Telecommunications	60-64	Transport, storage and communications
30	Internet service providers and data processing services	60-64	Transport, storage and communications
31	Other information services	60-64	Transport, storage and communications
34	Real estate	70-74	Real estate and business services
35	Rental and leasing services	70-74	Real estate and business services
36	Professional and technical services	70-74	Real estate and business services
37	Management of companies and enterprises	70-74	Real estate and business services
38	Administrative and support services	70-74	Real estate and business services
45	Accommodation	H	Hotels and Restaurants
46	Food services and drinking places	H	Hotels and Restaurants

Notes: The CPS classification is the 2002 Census Industry Classification and the OECD classification is ISIC Rev.3.

Table 22: Regression results: log(R&D ratio)

	Individual dismissal EPL1			Individual and collective dismissal EPL2		
	[1]	[2]	[3]	[1]	[2]	[3]
EPL _{ct}	-0.461*** (0.0148)	-0.441*** (0.0197)	-0.461*** (0.0148)	-0.656*** (0.0255)	-0.754*** (0.0304)	-0.656*** (0.0255)
<i>N</i>	5755	3055	5755	3770	2233	3770
<i>R</i> ²	0.552	0.531	0.552	0.577	0.577	0.577

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The columns refer to different samples: [1] non-balanced panel [2] balanced panel [3] year=2005. The balanced panel contains data on 18 countries and 19 industries from 1995 to 2005. All regressions include industry fixed effects. Robust standard errors in parentheses. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Second, we follow the approach used by Bassanini et al. (2009). We test whether industries that have a higher propensity to lay off workers have relatively lower R&D spending in countries where firing costs are high. Cross-industry variation is used to identify the effect of the regulation, with the underlying assumption that industries with a higher layoff propensity are more sensitive to firing costs. This strategy greatly reduces the concerns about omitted variable bias as it allows us to control for both country and industry fixed effects. Hence, our results cannot be driven by other cross-country differences in regulations or policies as long as they do not affect industries with different layoff propensities differently. We estimate the following equation

$$\log(\text{R\&D}_{jct}) = \beta_0 + \beta_1 \text{EPL}_{ct} \times \log(\text{layoff}_j) + \gamma_j + \lambda_{ct} + \varepsilon_{jct}, \quad (34)$$

where λ_{ct} is the country-time fixed effect. The indicator of the industry's propensity to lay off workers layoff_j corresponds to the industry's layoff rate in the absence of any dismissal regulation. The parameter of interest here is that of the interaction between the level of employment protection and the industry's propensity to lay off workers β_1 . When $\beta_1 < 0$, countries with stricter dismissal regulation have relatively lower R&D spending in industries with a higher propensity to lay off workers. Conversely, $\beta_1 > 0$ would indicate that countries with stricter dismissal regulation have relatively higher R&D spending in industries with a higher propensity to lay off workers.

Table 23: Regression results: log(R&D ratio), with log(layoff rate) as a proxy

	Individual dismissal EPL1			Individual and collective dismissal EPL2		
	[1]	[2]	[3]	[1]	[2]	[3]
$EPL_{ct} \times \ln(\text{layoff}_j)$	0.0303 (0.0578)	0.132 (0.0861)	0.249 (0.251)	0.139 (0.102)	0.158 (0.123)	0.388 (0.334)
N	5755	3055	343	3770	2233	343
R^2	0.772	0.775	0.776	0.781	0.775	0.777

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The layoff rate is measured from the CPS displacement data. The columns refer to different samples: [1] non-balanced panel [2] balanced panel [3] year=2005. The balanced panel contains data on 18 countries and 19 industries from 1995 to 2005. The non-balanced and balanced panel regressions include industry and country-time fixed effects. The 2005 regression includes industry and country fixed effects. Robust standard errors in parentheses.

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

E.3 Empirical Results

The results of the OLS estimation of equation (33) are displayed in Table 22 for the two measures of employment protection, EPL1 and EPL2. The first column reports the results for the full sample. Because the data have missing observations for some industries and some countries, we also run the regression on the balanced panel (column [2]) and for a given year (column [3]) to ensure that the missing observations do not bias the results. All six regressions indicate that R&D spending is negatively correlated to employment protection regulation.

The OLS estimation results of equation (34) are in Table 23. The interaction term in all specifications have insignificant coefficients. Employment protection legislation does not have a systematically larger effect in industries with a higher layoff rate. From the viewpoint of our theoretical model, while the total effect is negative in our baseline calibration, it is plausible that the positive and negative effects of employment protection on R&D can offset each other to produce mixed results.

As a robustness check, we use the job destruction rate instead of the layoff rate as a proxy of the industries' sensitivity to firing costs (both computed on US data). The data, made available online by John Haltiwanger (Bartelsman et al., 2009), span the period 1989-1991 and 1994-1996. The industry classification is derived from the STAN classification, but at a higher aggregation level than the R&D data. We re-aggregate the R&D data at the level at which the job destruction rate is available. The merged dataset has 17 industries (versus 19 with our original dataset). The results, reported in Table

Table 24: Regression results: log(R&D ratio), with log(job destruction rate) as a proxy

	Individual dismissal EPL1			Individual and collective dismissal EPL2		
	[1]	[2]	[3]	[1]	[2]	[3]
$EPL_{ct} \times \ln jdrate_j$	-0.0589 (0.0507)	-0.00950 (0.0628)	0.424 (0.242)	0.0469 (0.0797)	-0.246* (0.101)	0.356 (0.329)
N	5392	2844	320	3516	2080	320
R^2	0.766	0.767	0.735	0.756	0.760	0.732

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The job destruction rate is measured using data made available by John Haltiwanger. The columns refer to different samples: [1] non-balanced panel [2] balanced panel [3] year=2005. The balanced panel contains data on 18 countries and 15 industries from 1995 to 2005. The non-balanced and balanced panel regressions include industry and country-time fixed effects. The 2005 regression includes industry and country fixed effects. Robust standard errors in parentheses. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

24, show that the coefficient of the interaction term is insignificant for most regressions, similar to what we obtained when using the layoff rate. The one significant coefficient is negative, consistently with our baseline result.

All in all, the empirical results only suggest a negative effect of firing costs on innovation spending. We find that countries with stricter dismissal regulations tend to invest less in R&D, but this effect does not hold once we control for country fixed effects and use the cross-industry variation to identify the effect of the dismissal regulation.

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