Online Appendix (not for publication) to

Industrialization and the Evolution of Enforcement Institutions

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F Details on data sources

We use the measure of contract sensitivity developed by Nunn (2007). It is based on data from the input-output tables of the U.S. in 1997. Nunn (2007) assigns a contract-sensitivity score to 386 out of 486 categories in the 1997 IO classification. We focus the analysis on manufacturing, partly because some categories are missing in the service sector, whereas the data are complete for manufacturing. Another reason we use the manufacturing sector is to stay away from the issue of structural transformation, which can be driven by other forces, such as the change in demand composition. We use data on production from the World KLEMS initiative (`http://www.worldklems.net/`). KLEMS uses ISIC 3.1. industry classifications at the two-digit level (60 industries). We employ the following procedure to assign a contract-sensitivity measure on each of these industries. The BEA provides a concordance between 1997 IO and the 1997 version of the North American Industry Classification System (NAICS). The Census provides a concordance between the 2002 version of NAICS and ISIC 3.1. Finally, we use the concordance between the 1997 and 2002 versions of NAICS, provided by the BEA.

Unfortunately, this procedure does not yield a one-to-one mapping: Many IO codes are mapped onto multiple ISIC 3.1 categories. For category i at the two-digit ISIC 3.1 classification, we assign the following contract-sensitivity measure:

$$\hat{s}_i = \frac{\sum_j x_{i,j} s_j}{\sum_j x_{i,j}},$$

where j spans the 1997 IO categories, $x_{i,j}$ is the number of NAICS codes that are mapped to both the i category in ISIC and the j category in the IO classification, and s_j is the contract-sensitivity measure of Nunn (2007).

For a few countries, some of the 60 ISIC 3.1 categories are aggregated. In this case, we compute a weighted average of the scores of constituent categories and use it as a measure. We use the average share of each industry in the grouping from the UK as weights.

In Appendix G, we perform a robustness exercise and we focus on exports. We use data from Feenstra et al. (2005), which is in SITC Rev. 2 classification. The reported values are for the last year in our sample, 2000, but the results are virtually unchanged for all other years. We assign a contract sensitivity measure to each SITC category in a similar manner to Nunn (2007). We use the SITC-HS10 concordance provided by Feenstra and the HS10-IO category concordance provided by the BEA. We use the same procedure to compute the measure as for the KLEMS data.

G Additional empirical results

In Table 6, we report the estimates of regression equations (27) and (28) when the average contract sensitivity is computed using industry gross output as weights. Compared to Table 2, the magnitude and significance of the coefficients for institutions are virtually unchanged, with the exception that government effectiveness is no longer significant.

Table 7 reports the estimates of regression equation (29) when the average contract sensitivity is computed using industry gross output as weights. Compared to Table 3, the coefficients remain highly significant and their magnitude increases markedly. This observation is consistent with the findings of Jones (2011, 2013), Bartelme and Gorodnichenko (2015), and Boehm (2017) that frictions in intermediate-goods usage are significantly more severe in developing countries. Thus, contract-sensitive industries are impacted even more once we consider the amount of purchased intermediate goods. Figure 4 corresponds to Figure 1 in the main text. The two graphs look very similar to one another.

As a further robustness check, we estimate the regression equation (27) when $\mathcal{S}_c$ (average contract sensitivity) is computed using exports as weights and report the results in Table 8. A strong positive relationship exists between the quality of institutions and contract sensitivity at the country level, partly due to a large sample size. The details of the data construction can be found in Appendix F.

Table 6: Cross-sectional results; gross output weights

	[1]	[2]	[3]	[4]	[5]	[6]
	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$
Rule of law	0.049**	0.043**				
	(.023)	(.018)				
Government effectiveness			0.040	0.040*		
			(.025)	(.021)		
Control of corruption					0.019	0.023
					(.019)	(.016)
ln GDP per capita	−0.057		−0.040		−0.28	
	(.032)		(.031)		(.033)	
GDP per capita ($\times 10^{-6}$)		−2.03**		−1.73*		−1.58
		(0.96)		(0.97)		(1.05)
N	32	32	32	32	32	32

The dependent variable is average contract sensitivity weighted by gross output.
Standard errors in parenthesis.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.
Constant omitted. Institutions measures are on a -2.5: 2.5 scale.

H Derivation of the Shapley value

We follow the "heuristic derivation" of Acemoglu, Antràs, and Helpman (2007). To compute the Shapley value, we first consider all feasible permutations of all players. We then consider the marginal contribution of a particular player to the coalition ordered below her. The

Table 7: Panel regression results; gross output weights

	[1]	[2]	[3]	[4]
	$\Delta\mathcal{S}$	$\Delta\mathcal{S}$	$\Delta\mathcal{S}$	$\Delta\mathcal{S}$
Δ ln GDP per capita	0.0485***	0.0426***	0.0444***	0.0390***
	(.006)	(.006)	(.006)	(.007)
Year fixed effect	No	No	Yes	Yes
Country fixed effect	No	Yes	No	Yes
N	952	952	952	952

The dependent variable is average contract sensitivity weighted by gross output.
Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: Cross sectional results; export weights

	[1]	[2]	[3]	[4]	[5]	[6]
	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$
Law	0.0992***					
	(.021)					
Effectiveness		0.106***				
		(.021)				
Stability			0.0558***			
			(.02)			
Regulation				0.0975***		
				(.018)		
Accountability					0.0997***	
					(.017)	
Corruption						0.0796***
						(.02)
ln GDP per capita	-0.0436***	-0.0472***	-0.0148	-0.0360***	-0.0333***	-0.0307**
	(.016)	(.016)	(.015)	(.013)	(.012)	(.015)
N	154	154	152	154	154	154

The dependent variable is average contract sensitivity weighted by exports.
Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

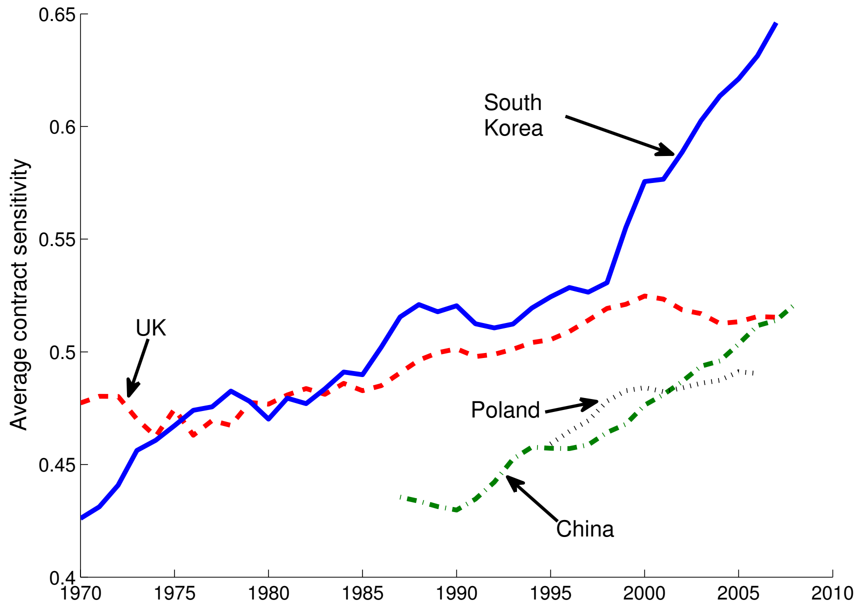

Figure 4: Average contract sensitivity, gross output weights

Shapley value is the average of these marginal contributions.

Now, consider the marginal contribution of a particular supplier. The price of intermediate good z is

$$p(z) = Y^{1-\phi} y(z)^{\phi-1},$$

and thus the revenue is

$$R(z) = Y^{1-\phi} y(z)^{\phi}.$$

Using the production function, the revenue generated by the coalition of the intermediate good firm and b suppliers ($b \in [0, 1]$) is

$$R_b = Y^{1-\phi} \left[\int_0^b \left(\exp \left(\int_0^1 \ln x(i,j) di \right) \right)^{\alpha} dj \right]^{\phi/\alpha}.$$

The marginal contribution, evaluated with $x(i,j) = x_c$ for $i \in [0, \mu]$ and $j \in [0, b]$, $x(i,j) = x_n$

for $i \in (\mu, 1]$ and $j \in [0, b)$, and $x(i,j) = x_n(j)$ for $i \in (\mu, 1]$ and $j = b$, is

$$m(j,b) \equiv \frac{\partial R_b}{\partial b} = \frac{\phi}{\alpha} Y^{1-\phi} \left[\frac{x_n(j)}{x_n} \right]^{(1-\mu)\alpha} x_c^{\phi\mu} x_n^{\phi(1-\mu)} b^{(\phi-\alpha)/\alpha}.$$

The marginal contribution of supplier j is zero if the intermediate good firm is not included in the coalition. Thus the marginal contribution is $m(j,b)$ with probability b and zero with probability $1-b$. Considering all possible orderings, the Shapley value for supplier j is

$$s_j = \int_0^1 s m(j,b) db = (1-\gamma) Y^{1-\phi} \left[\frac{x_n(j)}{x_n} \right]^{(1-\mu)\alpha} x_c^{\phi\mu} x_n^{\phi(1-\mu)}.$$

The intermediate firm receives the leftover of the revenue in the symmetric equilibrium:

$$s_i = \gamma Y^{1-\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)}.$$

I Characterization of market economy with commitment in section 3.2

This section provides characterizations of the model in section 3.2. The results will also serve as a basis for computing the equilibrium of the model.

First, one issue with the problem (SP1) is that it is not obvious whether the resource constraint is always binding. It is easy to see that for fixed λ and G, consumption is decreasing in physical capital; furthermore, λ' must increase if K increases, which may lead to lower consumption in future periods, and so on. Therefore, $v(K, G, \lambda)$ could be decreasing in physical capital, so the planner may find it optimal to destroy resources, that is, choose an allocation such that the resource constraint is not binding. The next lemma shows that without loss of generality, in the sequential problem, we may impose the additional constraint that all resources are used up.

Lemma I.1 *Suppose $\Theta(G)$ is weakly increasing in G. Let $\{c_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ be a sequence that satisfies all the constraints for* (SP1). *There exists another sequence $\{c'_t, k'_{t+1}, g'_{t+1}\}_{t=0}^{\infty}$ that also satisfies all the constraints for* (SP1), *$c'_t = c_t$ for all t, and the resource constraint is binding for all t.*

Proof. See Appendix J. ■

Thus, below we can assume the resource constraint is binding.

Second, it will be convenient to establish some properties of the admissible set $\Omega(K,G)$ in order to be able to compute it. The formal definition of $\Omega(K,G)$ is the following. Let $\Delta \equiv \{\{J_t\}_{t=0}^{\infty}, J_t \in \mathbf{R}_+^3\}$ be the space of sequences of nonnegative triplets of real numbers. Let $\Gamma : \mathbf{R}_{++} \times \mathbf{R}_+ \rightrightarrows \Delta$ be the correspondence that maps from the pair (K,G) into the set of all sequences that satisfy (17), (18), (19), and (20), where the first component of the triplet is understood as C, the second is K', and the third is G'. Define the mapping $m : \Delta \times \mathbf{R}_{++} \times \mathbf{R}_+ \to \mathbf{R}$ by

$$m((J_t), K, G) \equiv \beta(1 - \delta_K + q(G)f'(K))u'(J_0(1)),$$

where (J_t) is an element of Δ and $J_0(1)$ represents the first component of J_0 (i.e., the consumption at time 0). Then the admissible set is formally given by

$$\Omega(K,G) = \{m((J_t), K, G) : (J_t) \in \Gamma(K,G)\}. \tag{I.1}$$

Now, the following proposition shows that $\Omega(K,G)$ can be characterized by its lower bound.

Proposition I.1 *For every $(K,G) \in \mathbf{R}_{++} \times \mathbf{R}_+$, $\Omega(K,G) = [\omega(K,G), \infty)$, where ω is a continuous function that is strictly decreasing in K.*

Proof. See Appendix J. ■

Proposition I.1 implies $\Omega(K,G)$ is convex-valued and closed. The admissibility set $\Omega(K,G)$ has an obvious recursive structure: For a given admissible triplet (λ', K', G') that satisfies (24), constraints (21), (22), and (23) provide restrictions on all combinations of (λ, K, G) that makes (λ', K', G') feasible. In turn, these combinations of (λ, K, G) have to be consistent with the admissible set of (λ', K', G') postulated initially.[29]

[29] This idea was first demonstrated by Kydland and Prescott (1980) and later formalized by Abreu, Pearce

The special structure of our problem allows us to find Ω by an iterative procedure. Let $\mathbf{w} : \mathbf{R}_{++} \times \mathbf{R}_{+} \to \mathbf{R}_{+}$ be a continuous function. This is the (initial) guess for ω. Define the operator $\mathbf{T_w}$ by:

$$\mathbf{T_w}\mathbf{w}(K,G) = \min_{C>0, K'>0, G'\geq 0, \lambda'} \lambda = \beta(1-\delta_K + q(G)f'(K))u'(C)$$

subject to

$$C + K' + G' \leq \Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G,$$

$$u'(C) = \lambda',$$

and

$$\lambda' \geq \mathbf{w}(K', G').$$

This problem is well defined for a particular subset of functions, as is shown below.

Lemma I.2 *Let* $\mathcal{A} \equiv \{\mathbf{w} : \mathbf{R}_{++} \times \mathbf{R}_{+} \to \mathbf{R}_{++}$, *continuous, increasing, and* $\lim_{K\to 0} \mathbf{w}(K,G) = \infty$ *for all* $G\}$. *Then* $\mathbf{T_w}$ *is defined on* $\mathcal{A}$ *and* $\mathbf{T_w}\mathcal{A} \subseteq \mathcal{A}$.

Proof. See Appendix J. ■

Because the function ω is in the class $\mathcal{A}$, one systematic method of finding it turns out to be to start with some $\mathbf{w} \leq \omega$ and iterate on the operator $\mathbf{T}$ until convergence.

Lemma I.3 *Let* $\mathbf{w} \in \mathcal{A}$, $\mathbf{w} \leq \omega$, *and* $\mathbf{T_w}\mathbf{w} \geq \mathbf{w}$. *Then for every* (K,G), *the sequence* $(\mathbf{T_w}^n\mathbf{w})(K,G)$ *converges and* $\lim_{n\to\infty}(\mathbf{T_w}^n\mathbf{w})(K,G) = \omega(K,G)$.

Proof. See Appendix J. ■

One initial guess that satisfies the conditions of Lemma I.2 and Lemma I.3 (with Inada-like conditions on $f(K)$) is

$$\mathbf{w}(K,G) = \beta(1-\delta_K + q(G)f'(K))u'(\Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G).$$

and Stacchetti (1990).

This gives the upper bound on consumption C to the consumer in the current period, because this consumption is derived from setting $K' = 0$ and $G' = 0$ (in fact, this consumption is just unattainable because of the constraint $K' > 0$). Thus both $\mathbf{w}$ and $\mathbf{T_w w}$ cannot be larger than this $\mathbf{w}$ and therefore $\mathbf{w} \leq \omega$ and $\mathbf{w} \leq \mathbf{T_w w}$ are satisfied.

Next, we turn to solving the recursive problem (RP1) itself. Define the operator $\mathbf{T_v}$ by

$$\mathbf{T_v} v(K, G, \lambda) = \sup_{C>0, K'>0, G'\geq 0, \lambda'} u(C) + \beta v(K', G', \lambda'),$$

subject to (21), (22), (23), and (24). We aim for our model to be consistent with usual parametrization of the standard growth model, so we do not want to impose boundedness constraints on the utility function. Without a boundedness assumption, the operator $\mathbf{T_v}$ is not necessarily a contraction.

We utilize a different strategy to solve the Bellman equation. We have two tools: first, the neoclassical growth model is extensively studied, and because it is an upper bound on the planner's problem, we can use standard existence results from it. Second, consumption is bounded from above by feasibility and strict concavity of the production function. Our approach is similar to the one we use to find Ω: We start with a suitable upper bound for the value function, and iteratively apply the operator $\mathbf{T_v}$. This approach generates a decreasing sequence that converges to the true value.

Proposition I.2 *Let the real-valued function $\hat{v}(K, G, \lambda)$ be continuous, constant in G and λ, strictly increasing in K, and $\hat{v}(K, G, \lambda) \geq v^*(K, G, \lambda)$. In addition, suppose that $\mathbf{T_v}\hat{v} \leq \hat{v}$. Then for any (K, G, λ) such that $\lambda \in \Omega(K, G)$, the sequence $(\mathbf{T_v}^n \hat{v})(K, G, \lambda)$ has a limit. Denote the limit function $v(K, G, \lambda)$. Then $v(K, G, \lambda) = v^*(K, G, \lambda)$.*

Proof. See Appendix J. ■

An obvious upper bound for v^* will be the value of the competitive allocation in an economy without enforcement frictions ($\mu(z) = 1$ for all z), which implies $\Theta(G) = q(G) = 1$, and without any constraints on marginal utility in the first period.

J Additional proofs

We introduce a technical lemma, which is useful in the proofs of Lemma I.1, Proposition I.1, Lemma I.2, and Lemma I.3.

Lemma J.1 *The functions $\Theta(G)$ and $q(G)$ are continuous and $\Theta(G) > 0$, $q(G) > 0$, $\forall G$. Also $q(G)$ is strictly increasing in G.*

Proof. Let $\underline{\nu}$ be the probability measure on μ induced by $\underline{\mu}$. For any Borel set $A \subseteq [0, 1]$, define $A' = \{z \in [0, 1] : \underline{\mu}(z) \in A\}$. Then $\underline{\nu}(A)$ is equal to the Lebesgue measure of A' (which is a Borel set). Then the probability measure $\nu(\cdot, G)$ on μ in an economy with institutions G is given by:

$$\nu([0, a], G) = \underline{\nu}\left(\left[0, \max\left\{0, \frac{a - h(G)}{1 - h(G)}\right\}\right]\right).$$

Because $[0, a]$ generates all the Borel sets, we have uniquely determined $\nu(\cdot, G)$. Moreover, the CDF that $\nu(\cdot, G)$ defines is continuous in G, hence for any continuous function $g(\mu)$, the integral $\int_{[0,1]} g(\mu)\nu(d\mu, G)$ is continuous in G.

Then, given

$$q(G) = \left[\int_{[0,1]} D(\mu)^{\phi}\nu(d\mu, G)\right]^{\frac{1-\phi}{\phi}},$$

$q(G)$ is continuous because $D(\mu)$ is continuous. Furthermore, $q(G) > 0$ because for all μ, $D(\mu) > 0$. Also $q(G)$ is increasing in G because (i) $D(\mu)$ is increasing and (ii) if $G_2 > G_1$ then $\nu(\cdot, G_2)$ first-order stochastically dominates $\nu(\cdot, G_1)$.

Next,

$$\Theta(G) = \frac{\left[\int_{[0,1]} D(\mu)^{\phi}\nu(d\mu, G)\right]^{\frac{1}{\phi}}}{\int_{[0,1]} H(\mu)\nu(d\mu, G)}$$

is continuous in G because $H(\mu) > 0$, $D(\mu) > 0$ for all μ. Similarly $\Theta(G) > 0$. ■

Proof of Lemma I.1. The proof is by construction. First, set $c_t' = c_t$ for all t.

Let $M_t \equiv \Theta(g_t)f(k_t) + (1 - \delta_K)k_t + (1 - \delta_G)g_t - c_t$. If $k_{t+1} + g_{t+1} = M_t$ for all t then the original sequence satisfies all the conditions of the lemma.

Let $N(g'; M, c_a, c_b) \equiv \beta(1 - \delta_K + q(g')f'(M - g'))u'(c_a) - u'(c_b)$. Let n be the earliest period for which $M_t > k_{t+1} + g_{t+1}$. For all $t \leq n$, set $k'_t = k_t, g'_t = g_t$.

$$N(g_{n+1}; M_n, c_n, c_{n+1}) = \beta(1 - \delta_K + q(g_{n+1})f'(M_n - g_{n+1}))u'(c_{n+1}) - u'(c_n)$$

$$< \beta(1 - \delta_K + q(g_{n+1})f'(k_{n+1}))u'(c_{n+1}) - u'(c_n) = 0.$$

Similarly

$$N(M_n - k_{n+1}; M_n, c_n, c_{n+1}) = \beta(1 - \delta_k + q(M_n - k_{n+1})f'(k_{n+1}))u'(c_{n+1}) - u'(c_n)$$

$$> \beta(1 - \delta_K + q(g_{n+1})f'(k_{n+1}))u'(c_{n+1}) - u'(c_n) = 0.$$

Then, by continuity, for some $g' \in (g_{n+1}, M_n - k_{n+1})$, $N(g'; M_n, c_n, c_{n+1}) = 0$. Then set $g'_{n+1} = g'$, $k'_{n+1} = M_n - g'$. The resource constraint (17) for period n binds, and the Euler condition (20) also still holds. Because $k'_{n+1} > k_{n+1}$ and $g'_{n+1} > g_{n+1}$, the resource constraint (17) for period $n+1$ is now slack. We now modify the allocation for period $n+1$ in the same way and so on. ■

Proof of Proposition I.1. We first show that if $\lambda \in \Omega(K_0, G_0)$ and $\lambda' > \lambda$, then $\lambda' \in \Omega(K_0, G_0)$.

Let $\{C_t, K_{t+1}, G_{t+1}\}_{t=0}^{\infty} \in \Gamma(K_0, G_0)$ such that $\beta(1 - \delta_K + q(G_0)f'(K_0))u'(C_0) = \lambda$. We construct a modified sequence $\{C'_t, K'_{t+1}, G'_{t+1}\}_{t=0}^{\infty} \in \Gamma(K_0, G_0)$ such that $\beta(1 - \delta_K + q(G_0)f'(K_0))u'(C'_0) = \lambda'$.

Let $G'_t = G_t$ for all t. Let C'_0 be defined by $\beta[1 - \delta_K + q(G_0)f'(K_0)]u'(C'_0) = \lambda'$. By strict concavity and the Inada conditions of u, C'_0 exists and $0 < C'_0 < C_0$. Define K'_1 in such a way to satisfy the resource constraint:

$$K'_1 = \Theta(G_0)f(K_0) + (1 - \delta_K)K_0 + (1 - \delta_G)G_0 - G_1 - C'_0.$$

Clearly, $K'_1 > K_1$. Then, we construct the alternative sequence recursively by (for $t =$

$1, 2, 3, ...)$:

$$C'_t = u'^{-1}\left(\frac{u'(C'_{t-1})}{\beta[1-\delta_K + q(G_t)f'(K'_t)]}\right)$$

$$K'_{t+1} = \Theta(G_t)f(K'_t) + (1-\delta_k)K'_t + (1-\delta_g)G_t - G_{t+1} - C'_t.$$

By induction, $0 < C'_t < C_t$ (because $u'(C'_{t-1}) > u'(C_{t-1})$ and $f'(K'_t) < f'(K_t)$) and $K'_{t+1} > K_{t+1}$ (because $f(K'_t) > f(K_t)$ and $C'_t < C_t$), so the new sequence is feasible, and satisfies all constraints. Therefore, $\lambda' \in \Omega(K_0, G_0)$.

Then, showing that $\Omega(K_0, G_0) = [\omega(K_0, G_0), \infty)$ is equivalent to proving that the minimization problem:

$$\min\{m((x_t), K_0, G_0) : (x_t) \in \Gamma(K_0, G_0)\}$$

has a well-defined solution. We use the theorem of the maximum to prove this claim, which also demonstrates ω is continuous.

Endow Δ with the metric

$$\rho((x_t), (y_t)) = \sum_{t=0}^{\infty} 2^{-t-1}||x_t - y_t||_E,$$

where $||.||_E$ is the usual Euclidean metric. Let the t-section of a set A be the set $\{x \in \mathbf{R}^3_+ : x = x_t, (x_t) \in A\}$. Then A is closed if and only if all of its t-sections are closed. Similarly, if all t-sections of a set A are uniformly bounded in the Euclidean metric, then A is totally bounded.

Unfortunately, Γ is not compact-valued. We show that, without loss of generality, we can restrict (x_t) to some $\tilde{\Gamma}(K, G)$ such that $\tilde{\Gamma}(K, G) \subseteq \Gamma(K, G)$ and $\tilde{\Gamma}$ is compact-valued and continuous. The rest of the proofs proceeds by a sequence of claims.

Claim 1: $\Gamma(K_0, G_0)$ is totally bounded.

Let $\delta = \min\{\delta_K, \delta_G\}$. Let $\tilde{K}$ be defined as the solution of $f(\tilde{K}) = \delta\tilde{K}$. $\tilde{K}$ is an upper bound on the maximum sustainable accumulable resources: physical capital and institutional capital. Let $\bar{K}(K_0, G_0) = \max\{K_0 + G_0, \tilde{K}_0\}$ and $\bar{G}(K_0, G_0) = \max\{K_0 + G_0, \tilde{K}\}$. Define $\bar{C}(K_0, G_0) = f(\bar{K}(K_0, G_0)) + (1-\delta_K)\bar{K}(K_0, G_0) + (1-\delta_G)\bar{G}(K_0, G_0)$. Then for any feasible path, $0 \leq K_t \leq \bar{K}(K_0, G_0)$, $0 \leq G_t \leq \bar{G}(K_0, G_0)$, and $0 < C_t \leq \bar{C}(K_0, G_0)$ are satisfied. Thus $\Gamma(K_0, G_0)$ is totally bounded.

Claim 2: Define $N(K_0, G_0) \equiv \beta(1 - \delta_K + q(0)f'(\bar{K}(K_0, G_0))$. Then for any feasible K_t and G_t, we have $\beta(1 - \delta_K + q(G_t)f'(K_t)) \geq N(K_0, G_0)$.

For any feasible path, $0 < q(0) \leq q(G_t)$ and $f'(\bar{K}(K_0, G_0)) \leq f'(K_t)$ (the second inequality follows from the strict concavity of f and the fact that $K_t \leq \bar{K}(K_0, G_0)$). Then by substituting, we obtain the inequality.

Claim 3: There exists a sequence $\{\underline{c}_t\}_{t=0}^{\infty}$, $\bar{C}(K_0, G_0) \geq \underline{c}_t > 0$ such that we can impose the additional restriction $c_t \geq \underline{c}_t$ without loss of generality.

Consider a feasible policy $G_{t+1} = (1 - \delta_G)G_t$. Given this sequence of G (and T that balances the government budget), the economy has a unique equilibrium, which is continuous with respect to (K_0, G_0); let period-zero consumption of this equilibrium be denoted $\tilde{C}(K, G)$. Then, without loss of generality, we can ignore the elements $(J_t) \in \Delta$ such that $m((J_t), K, G) > \beta(1 - \delta_K + q(G_0)f'(K_0))u'(\tilde{C}(K_0, G_0))$, or $u'(c_0) > u'(\tilde{C}(K, G))$ in the minimization problem. This imposes a restriction on c_0: $u'(c_0) \leq u'(\tilde{C}(K, G))$ or equivalently

$$c_0 \geq \underline{c}_0(K, G) \equiv u'^{-1}(u'(\tilde{C}(K, G))).$$

Next, we turn to finding $\underline{c}_t$ for $t > 0$. Iterating on the Euler equation (20),

$$\begin{aligned} u'(C_t) & \\ &= \frac{u'(c_0)}{\beta(1 - \delta_K + q(G_1)f'(K_1)) \times \beta(1 - \delta_K + q(G_2)f'(K_2)) \times \cdots \times \beta(1 - \delta_K + q(G_t)f'(K_t))} \\ &\leq \frac{u'(c_0)}{N(K_0, G_0)^t} \\ &\leq \frac{u'(\underline{c}_0)}{N(K_0, G_0)^t} \end{aligned}$$

hold, where we used the restriction that $c_0 \geq \underline{c}_0$. Then, without loss of generality, we can impose the restriction on C_t, $t = 1, 2, ...$:

$$c_t \geq \underline{c}_t(K, G) \equiv u'^{-1}\left(\frac{u'(c_0)}{N(K_0, G_0)^t}\right).$$

Claim 4: There exists a sequence $\{\underline{k}_t\}_{t=0}^{\infty}$, $\bar{K}(K_0, G_0) \geq \underline{k}_t > 0$ such that we can impose the additional restriction $k_t \geq \underline{k}_t$.

Iterating on the Euler equation (20) once again,

$$\beta(1-\delta_K+q(G_t)f'(K_t)) = \frac{u'(c_0)}{u'(c_t)\beta(1-\delta_K+q(G_1)f'(K_1)) \times \cdots \times \beta(1-\delta_k+q(G_{t-1})f'(K_{t-1}))}.$$

The right-hand side is smaller than

$$\frac{u'(\underline{c}_0)}{u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}},$$

and therefore

$$q(G_t)f'(K_t) \leq \frac{u'(\underline{c}_0)}{\beta u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}} - (1-\delta_K).$$

Because $q(G_t)f'(K_t) \geq q(0)f'(K_t)$,

$$f'(K_t) \leq \frac{u'(\underline{c}_0)}{q(0)\beta u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}} - \frac{1-\delta_K}{q(0)}.$$

This inequality implies that we can define $\underline{k}_t$ as

$$\underline{k}_t = f'^{-1}\left(\frac{u'(\underline{c}_0)}{q(0)\beta u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}} - \frac{1-\delta_K}{q(0)}\right).$$

Claim 5: There exists a compact-valued and continuous correspondence $\tilde{\Gamma} : \mathbf{R}_+^2 \rightrightarrows \Delta$ such that $\tilde{\Gamma}(K,G) \subset \tilde{\Gamma}(K,G)$ for all $(K,G) \in \mathbf{R}_+^2$, and if $J \in \Gamma(K,G)$, there exists $J' \in \tilde{\Gamma}(K,G)$ such that $m(J';K,G) \leq m(J;K,G)$.

Define

$$\tilde{\Gamma}(K,G) = \{(x_t) \in \Delta : (17), (20), C_t \geq \underline{C}_t(K,G), K_t \geq \underline{K}_t(K,G)\ \forall t \text{ for given } (K,G)\}.$$

Because all the constraint functions are continuous and the inequalities are weak, all t-sections of $\tilde{\Gamma}$ are closed, and hence $\tilde{\Gamma}(K,G)$ is closed. Because $\tilde{\Gamma}(K,G) \subseteq \Gamma(K,G)$, it is also totally bounded, and hence $\tilde{\Gamma}(K,G)$ is compact-valued. Finally, because all the constraint functions are continuous, $\tilde{\Gamma}(K,G)$ is continuous.

The last part of the claim follows from the properties of $\underline{c}_t, \underline{k}_t$ established in Claims 3 and 4.

Claim 6: $\Omega(K,G) = [\omega(K,G), \infty)$, where $\omega(K,G)$ is a continuous function.

This claim follows directly from Claim 5 and the theorem of the maximum.

Claim 7: The function $\omega(K,G)$ is decreasing in K.

This claim is proven analogously to proving $\lambda \in \Omega(K,G)$ and $\lambda' > \lambda$ implies $\lambda' \in \Omega(K,G)$.

Let $\{C_t, K_{t+1}, G_{t+1}\}_{t=0}^{\infty} \in \Gamma(K,G)$ such that $\beta(1-\delta_K + q(G)f'(K))u'(C_0) = \omega(K,G)$. Let $K' > K$. We construct a modified sequence $\{C'_t, K'_{t+1}, G'_{t+1}\}_{t=0}^{\infty} \in \Gamma(K',G)$ such that $\beta(1-\delta_K+q(G)f'(K'))u'(C'_0) < \beta(1-\delta_K+q(G)f'(K'))u'(C'_0)$, and hence $\omega(K',G) < \omega(K,G)$.

Let $G'_t = G_t$ for all t. Let $C'_0 = C_0$ Define K'_1 in such a way to satisfy the resource constraint:

$$K'_1 = \Theta(G_0)f(K'_0) + (1-\delta_K)K_0 + (1-\delta_G)G_0 - G_1 - C'_0.$$

Clearly, $K'_1 > K_1$. Then, we construct the alternative sequence recursively by (for $t = 1, 2, 3, ...$):

$$C'_t = u'^{-1}\left(\frac{u'(C'_{t-1})}{\beta[1-\delta_K + q(G_t)f'(K'_t)]}\right)$$

$$K'_{t+1} = \Theta(G_t)f(K'_t) + (1-\delta_k)K'_t + (1-\delta_g)G_t - G_{t+1} - C'_t.$$

By induction, $0 < C'_t < C_t$ (because $u'(C'_{t-1}) > u'(C_{t-1})$ and $f'(K'_t) < f'(K_t)$) and $K'_{t+1} > K_{t+1}$ (because $f(K'_t) > f(K_t)$ and $C'_t < C_t$), so the new sequence is feasible, and satisfies all constraints. Moreover, $\beta(1-\delta_k + q(G)f'(K'))u'(C'_0) < \beta(1-\delta_k + q(G)f'(K))u'(C_0)$ (because $f'(K') < f'(K)$ and all the other terms are positive and the same). Therefore $\omega(K',G) < \omega(K,G)$. ■

Proof of Lemma I.2. Because K and G are fixed, the optimization problem is equivalent to maximizing consumption C subject to the constraints.

Define $\hat{G}'(K,G) \equiv \frac{1}{3}[\Theta(G)f(K)+(1-\delta_K)K+(1-\delta_G)G]$ and let $\hat{K}'(K,G)$ be implicitly defined by:

$$u'(\Theta(G)f(K)+(1-\delta_K)K+(1-\delta_G)G-\hat{G}'(K,G)-\hat{K}'(K,G)) = \mathbf{w}(\hat{K}'(K,G),\hat{G}'(K,G))).$$

Because $\lim_{c\to 0} u'(c)=\infty$ and $\lim_{K\to 0}\mathbf{w}(K,G)=\infty$, at least one solution exists. Because $u'(c)$ is strictly decreasing and $\mathbf{w}(K,G)$ is decreasing and continuous in K, the solution $\hat{K}'(K,G)$ is unique and continuous. Define $\hat{C}(K,G)$ by

$$\hat{C}(K,G)=\Theta(G)f(K)+(1-\delta_K)K+(1-\delta_G)G-\hat{G}'(K,G)-\hat{K}'(K,G).$$

Thus, the triplet $(\hat{C}(K,G),\hat{K}'(K,G),\hat{G}'(K,G))$ satisfies the constraints for $\mathbf{T_w}$ and is continuous.

Define the correspondence

$$\hat{\Gamma}(K,G)=\{(C,K',G'):(21),u'(C)\geq \mathbf{w}(K',G'),C\geq \hat{C}(K,G)\}.$$

From this definition, $\mathbf{w}(K',G')\leq u'(\hat{C}(K,G))$ holds for all $(C,K',G')\in\hat{\Gamma}(K,G)$. From this and $\lim_{K\to 0}\mathbf{w}(K,G)=\infty$, the inequality $K'>0$ is satisfied for all $(C,K',G')\in\hat{\Gamma}(K,G)$. Therefore, $\hat{\Gamma}$ respects all the constraints of $\mathbf{T_w}$, and without loss of generality, we can perform the minimization subject to $(C,K',G')\in\hat{\Gamma}(K,G)$.

Because $(C,K',G')\in[0,\Theta(G)f(K)+(1-\delta_K)K+(1-\delta_G)G]^3$, clearly $\hat{\Gamma}(G,K)$ is bounded. By continuity of the constraint functions, $\hat{\Gamma}$ is closed, and hence compact-valued. Continuity of the constraint functions also ensures continuity of the correspondence. Then, $\mathbf{T_w w}$ is continuous by the theorem of the maximum.

Finally, $\mathbf{T_w w}(K,G)\geq\beta(1-\delta_K+q(G)f'(K))u'(\Theta(G)f(K)+(1-\delta_K)K+(1-\delta_G)G)$. Therefore $\lim_{K\to 0}\mathbf{T_w w}(K,G)=\infty$ and $\mathbf{T_w w}(K,G)>0$ hold. ■

For proving Lemma I.3, we use the following results.

Lemma J.2 $\mathbf{T_w}\omega=\omega$.

Proof. The proof is standard. Clearly, $\omega\in\mathcal{A}$, so by Lemma I.2, $\mathbf{T_w}\omega$ is well-defined.

Fix K and G. Let c_0, k_1, g_1 be an arbitrary triplet satisfying the constraints in problem $\mathbf{T_w}$ for ω. Let $(c_t, k_{t+1}, g_{t+1})_{t=1}^{\infty} \in \Gamma(k_1, g_1)$ such that $u'(c_0) = \beta(1-\delta_K+q(g_1)f'(k_1))u'(c_1)$. Then $\{c_0, k_1, g_1, (c_t, k_{t+1}, g_{t+1})_{t=1}^{\infty}\} \in \Gamma(K, G)$. Therefore, $\beta(1-\delta_K + q(G)f'(K))u'(c_0) \geq \omega(K, G)$. Because the triplet was arbitrary, $\beta(1 - \delta_K + q(G)f'(K))u'(c_0) \geq \omega(K, G)$ for all c_0, k_1, g_1 that satisfy the constraints of problem $\mathbf{T_w}$, so $\mathbf{T_w}\omega(K, G) \geq \omega(K, G)$.

Next, let $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty} \in \Gamma(K, G)$ be arbitrary. Then, $(c_t, k_{t+1}, g_{t+1})_{t=1}^{\infty} \in \Gamma(k_1, g_1)$, so $\beta(1 - \delta_K + q(g_1)f'(k_1))u'(c_1) \geq \omega(k_1, g_1)$. Then, $u'(c_0) = \beta(1 - \delta_K + q(g_1)f'(k_1))u'(c_1) \geq \omega(k_1, g_1)$; also, (c_0, k_1, g_1) satisfies the resource constraint and the non-negativity constraints. Thus, (c_0, k_1, g_1) satisfies the constraints for $\mathbf{T_w}$. Then, $\beta(1 - \delta_K + q(G)f'(K))u'(c_0) \geq \mathbf{T_w}\omega(K, G)$. Because $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty} \in \Gamma(K, G)$ is arbitrary, $\omega(K, G) \geq \mathbf{T_w}\omega(K, G)$. ■

Proof of Lemma I.3. The operator $\mathbf{T_w}$ is clearly monotone, so $\mathbf{T_w w} \geq \mathbf{w}$ implies $\mathbf{T_w}^2\mathbf{w} \geq \mathbf{T_w w}$, and then by induction, $\mathbf{T_w}^{n+1}\mathbf{w} \geq \mathbf{T_w}^n\mathbf{w}$. Similarly, the assumption $\mathbf{w} \leq \omega$ implies $\mathbf{T_w}^n\mathbf{w} \leq \mathbf{T_w}^n\omega = \omega$, where we used Lemma J.2. Then, for any (K, G), $(\mathbf{T_w}^n\mathbf{w})(K, G)$ is a monotone and bounded sequence of real numbers, and hence it has a limit.

Next, we show that $\lim_{n\to\infty}(\mathbf{T_w}^n\mathbf{w})(K, G) = \omega(K, G)$. Fix K and G. Let H_n be the policy correspondence that solves problem $\mathbf{T_w}$ given $\mathbf{T_w}^{n-1}\mathbf{w}$. Let $k_{0,n} = K, g_{0,n} = G$. Then for a fixed n, define $(c_{t,n}, k_{t+1,n}, g_{t+1,n})$ recursively by $(c_{t,n}, k_{t+1,n}, g_{t+1,n}) \in H_{n-t}(k_{t,n}, g_{t,n})$ if $t \leq n - 1$ and $(c_{t,n}, k_{t+1,n}, g_{t+1,n}) = (0, 0, 0)$ if $t \geq n$.

Because $\mathbf{T_w}^n\mathbf{w} \leq \omega$, it follows that $c_{0,n} \geq \tilde{C}(K, G)$ for all n, where $\tilde{C}(K, G)$ is defined in the proof of Proposition I.1. Then, by the same reasoning as in the proof of Proposition I.1 $c_{t,n} \geq \underline{c}_t(K, G)$ if $t \leq n - 1$, where $\underline{c}_t(K, G)$ is defined in the proof of Proposition I.1.

Let $y_n = (c_{t,n}, k_{t+1,n}, g_{t+1,n})_{t=0}^{\infty}$. Define

$$\Delta_{K,G} = \{\{J_t\}_{t=0}^{\infty}, J_t \in [0, \bar{C}(K, G)] \times [0, \bar{K}(K, G)] \times [\bar{G}(K, G)]\},$$

where $\bar{C}(K, G)$, $\bar{K}(K, G)$, and $\bar{G}(K, G)$ are defined in the proof of Proposition I.1. Then, $y_n \in \Delta_{K,G}$ for all n. Because $\Delta_{K,G}$ is compact in the metric we defined above, (y_n) has a convergent subsequence (y_{n_ℓ}) with a limit in $\Delta_{K,G}$. Denote $c_t = \lim_\ell c_{t,n_\ell}, k_{t+1} = \lim_\ell k_{t+1,n_\ell}, g_{t+1} =$

$\lim_\ell g_{t+1,n_\ell}$.

By construction, $c_{n,t}, k_{n,t}, g_{n,t}, c_{n,t+1}, k_{n,t+1}, g_{n,t+1}$ satisfy (17), (18), (19), and (20) evaluated at t if $n \geq t+1$. This fact and the continuity of the constraint functions implies $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty}$ satisfy (17), (18), (19), and (20) for all t. Therefore, $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty} \in \Gamma(K,G)$, so $\beta(1-\delta_K q(G) f'(K))u'(c_0) \geq \omega(K,G)$.

Also by continuity $\beta(1-\delta_K q(G) f'(K))u'(c_0) = \lim_n \mathbf{T_w}^n \mathbf{w}(K,G) \leq \omega(K,G)$. Then, $\lim_n \mathbf{T_w}^n \mathbf{w}(K,G) = \omega(K,G)$. ■

Lemma J.3 $\mathbf{T_v} v^* = v^*$.

Proof. Standard. ■

Proof of Proposition I.2. Denote $\mathbf{T_v}^n \hat{v} \equiv v_n$ and $\hat{v} = v_0$. The operator $\mathbf{T_v}$ is monotone, thus, by induction, $v_1 = \mathbf{T_v}\hat{v} \leq \hat{v} = v_0$ implies $v_{n+1} = \mathbf{T_v}^{n+1}\hat{v} \leq \mathbf{T_v}^n \hat{v} = v_n$ for all n. Similarly, $\hat{v} \geq v^*$ implies $v_n = \mathbf{T_v}^n \hat{v} \geq \mathbf{T_v}^n v^* = v^*$, where we use Lemma J.3. This implies that for all (K,G,λ), $\lambda \in \Omega(K,G)$, $\{v_n(K,G,\lambda)\}_{n=0}^{\infty}$ is a decreasing sequence of real numbers, hence it has a limit. Denote the pointwise limit v. Then, $v \geq v^*$.

Next we show that $v \leq v^*$. Fix (K,G,λ) such that $\lambda \in \Omega(K,G)$. We will construct a sequence $\{c_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ that is feasible for (SP1) given K, G, λ, and

$$v(K,G,\lambda) \leq \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Then from the definition of v^*, the claim follows.

Let n be an arbitrary integer. Set $k_{0,n} = K, g_{0,n} = G, \lambda_{0,n} = \lambda$. We construct the n-th sequence of choice variables $\{c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n}\}_{t=0}^{\infty}$ as follows. For $t < n$, choose $(c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n})$ recursively such that

$$u(c_{t,n}) + \beta v_{n-t-1}(k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n}) = v_{n-t}(k_{t,n}, g_{t,n}, \lambda_{t,n})$$

and $(c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n})$ are feasible given $k_{t,n}, g_{t,n}, \lambda_{t,n}$.

Set $(c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n}) = (0,0,0,0)$ if $t \geq n$.

By construction,

$$v_n(K_0, G_0, \lambda_0) = \sum_{t=0}^{n-1} \beta^t u(c_{t,n}) + \beta^n v_0(k_{n,n}, g_{n,n}, \lambda_{n,n}).$$

Furthermore, by construction, the sequence is physically feasible, so for all t and n, $c_{t,n} \leq \bar{C}(K_0, G_0)$, $k_{t,n} \leq \bar{K}(K_0, G_0)$, and $g_{t,n} \leq \bar{G}(K_0, G_0)$. Because $\hat{v} = v_0$ is strictly increasing in K and constant in G and λ, it follows that $v_0(k_{n,n}, g_{n,n}, \lambda_{n,n}) \leq v_0(\bar{K}(K_0, G_0), 1, 1) \equiv A$. Therefore

$$v_n(K_0, G_0, \lambda_0) \leq \sum_{t=0}^{n-1} \beta^t u(c_{t,n}) + \beta^n A.$$

Next, we show this sequence of sequences has a converging subsequence with a limit that has the desired properties.

Let $y_n = \{c_{t,n}, k_{t+1,n}, g_{t+1,n}\}_{t=0}^{\infty}$. Define

$$\Delta_{K,G} = \{\{x_t\}_{t=0}^{\infty}, x_t \in [0, \bar{C}(K,G)] \times [0, \bar{K}(K,G)] \times [\bar{G}(K,G)]\},$$

where $\bar{C}(K,G)$, $\bar{K}(K,G)$, and $\bar{G}(K,G)$ are defined in the proof of Proposition I.1. Then, $y_n \in \Delta_{K,G}$ for all n. Because $\Delta_{K,G}$ is compact in the metric we defined above, (y_n) has a convergent subsequence (y_{n_ℓ}) with a limit in $\Delta_{K,G}$. Denote $c_t = \lim_\ell c_{t,n_\ell}$, $k_{t+1} = \lim_\ell k_{t+1,n_\ell}$, and $g_{t+1} = \lim_\ell g_{t+1,n_\ell}$. Next, we show the limit sequence satisfies all the constraints in (SP1).

By construction, $u'(c_{0,n}) = \lambda_0/[\beta(1 - \delta_q + q(G_0)f'(K_0))]$. As in the proof of Proposition I.1 (Claim 3), we can derive that if $n \geq t+1$,

$$u'(c_{t,n}) \leq \frac{u'(c_{0,n})}{N(K_0)^t} = \frac{\lambda_0}{\beta(1 - \delta_q + q(G_0)f'(K_0))N(K_0, G_0)^t}.$$

Then, if we define

$$\underline{c}_t = u'^{-1}\left(\frac{\lambda_0}{[\beta(1 - \delta_q + q(G_0)f'(K_0))]N(K_0, G_0)^t}\right),$$

$c_{t,n} \geq \underline{c}_t$ for all t, n such that $n \geq t+1$.

Because the Euler equation must hold if $n > t$,

$$\beta(1 - \delta_k + q(0)f'(k_{t,n})) \leq \beta(1 - \delta_k + q(g_{t,n})f'(k_{t,n})) = \frac{u'(c_{t-1,n})}{u'(c_{t,n})} \leq \frac{u'(\underline{c}_t)}{u'(\bar{C}(K_0, G_0))}.$$

Then,

$$f'(k_{t,n}) \leq \frac{1}{\beta q(0)} \left[\frac{u'(\underline{c}_t)}{u'(\bar{C}(K_0, G_0))} - (1 - \delta_k) \right],$$

which defines a lower bound for $k_{t,n}$. Denote it $\underline{k}_t$.

By construction, $c_{n,t}, k_{n,t}, g_{n,t}, c_{n,t+1}, k_{n,t+1}, g_{n,t+1}$ satisfy (17), (18), (19), and (20) evaluated at t if $n \geq t + 1$. This fact and the continuity of the constraint functions imply $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty}$ satisfy (17), (18), (19), and (20) for all t; moreover, by construction, $u'(c_0)\beta(1 - \delta_k + q(G_0)f'(K_0)) = \lambda_0$. Therefore, $\{c_0, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ satisfies all the constraints in the problem (SP1) given K_0, G_0, and λ_0. Also $u(c_t) \leq u(\bar{C}(K_0, G_0))$ for all t; therefore, $\sum_{t=0}^{\infty} \beta^t u(c_t)$ is well-defined (though it may be $-\infty$). Therefore,

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \leq v^*(K_0, G_0, \lambda_0).$$

Finally, using this inequality, we have that

$$\begin{aligned}
v(K_0, G_0, \lambda_0) &= \lim_n v_n(K_0, G_0, \lambda_0) \\
&= \lim_\ell v_{n_\ell}(K_0, G_0, \lambda_0) \\
&\leq \lim_\ell \left[\sum_{t=0}^{n_\ell - 1} \beta^t u(c_{t,n_\ell}) + \beta^{n_\ell} A \right] \\
&= \sum_{t=0}^{\infty} \beta^t u(c_t) \\
&\leq v^*(K_0, G_0, \lambda_0).
\end{aligned}$$

■

K Computational algorithm

K.1 Coercive government (section 3.1)

The value function v is approximated by linear interpolation on a grid of points for physical capital and institutional capital, wherever necessary.

1. Set a grid for K and G.

   We define implicitly $\underline{K}$ and $\bar{K}$ by $\beta(1 - \delta_K + q(0)f'(\underline{K})) = 1$ and $\beta(1 - \delta_K + \theta f'(\bar{K})) = 1$.

Then, we set $K_{min} = 0.5 \times \underline{K}$ and $K_{max} = 2.55 \times \bar{K}$. We construct an equally spaced grid for physical capital between K_{min} and K_{max} with 100 points. We set $G_{min} = 10^{-5}$ and $G_{max} = 2.0$ and again construct an equally spaced grid for institutional level with 80 points.

2. Initialize v.

   We set $v(K, G)$ to be the value in the competitive equilibrium without distortions.

3. For all points on the grid (K, G), we set

$$Tv(K, G) = \max_{C, K', G'} u(C) + \beta v(K', G'),$$

$$C + K' + G' \leq \Theta(G) f(K) + (1 - \delta_K)K + (1 - \delta_G)G - \tau(G, G').$$

4. If $||v - Tv||_\infty < tol$, we end. If not, we update $v = Tv$ and return to step 3.

K.2 Market economy: Commitment (section 3.2)

The value function v and the admissible set Ω are approximated by linear approximation off the grid points, whenever necessary. Proposition I.1 states that $\Omega(K, G) = [\omega(K, G), \infty)$, where ω is a continuous, real-valued function that is strictly decreasing in k.

The algorithm for solving the problem is as follows:

1. We set the grid for K and G as described in K.1.

2. Initialize the function ω.

   We set the initial guess for the function to be

$$\omega(K, G) = \beta(1 - \delta_K + q(G) f'(K)) u'(\Theta(G) f(K) + (1 - \delta_K)K + (1 - \delta_G)G)$$

   on all the points of the grid.

3. Find $T_{\mathbf{w}}\omega$.

For all points (K, G) on the grid, we set

$$\tilde{C}(K,G) = \max_{G'} \Theta(G)f(K) + (1-\delta_k)K + (1-\delta_g)G - \tau(G,G') - \tilde{K}(K,G,G'),$$

where $\tilde{K}(K,G,G')$ is the unique solution K' to the equation:

$$u'(C(K,G,K',G')) = \omega(K',G').$$

These functions are independent of the value function. Then, we set

$$T_{\mathbf{w}}\omega(K,G) = \beta(1-\delta_K + q(G)f'(K))u'(\tilde{C}(K,G)).$$

This is the minimum value of feasible $\beta(1-\delta_K + q(G)f'(K))u'(C)$ for given K and G.

4. Update ω.

   If $||\omega - T_{\mathbf{w}}\omega||_\infty < tol$, we stop updating ω and move to step 5. Otherwise, set $\omega = T_{\mathbf{w}}\omega$ and return to step 2.

5. Construct a grid for λ. We set $\lambda_{min} = \min_{K,G}\omega(K,G)$, where the minimum is over the grid. We set $\lambda_{max} = 6$ and verify this bound is not binding. Then we construct an equally spaced grid for λ with 110 points.

   In all cases when $\lambda > \lambda_{max}$, we set[30]

$$v(K,G,\lambda) = v(K,G,\lambda_{max}) + \frac{1}{1-\beta}(\log(\lambda_{max}) - \log(\lambda)).$$

6. We construct functions $\mathcal{C}(K,G,\lambda)$, $\mathcal{U}(K,G,\lambda)$ and $\lambda'(K,G,\lambda)$ to satisfy the following:

$$\beta(1-\delta_K + q(G)f'(K))u'(\mathcal{C}(K,G,\lambda)) = \lambda,$$

$$\mathcal{U}(K,G,\lambda) = u(\mathcal{C}(K,G,\lambda)),$$

$$\lambda'(K,G,\lambda) = u'(\mathcal{C}(K,G,\lambda)).$$

[30]This approximation follows from the log-preferences assumption. Similar approximations exist for CRRA and CARA preferences.

7. Initialize v as described in K.1.

8. Find Tv

   For all points on the grid (K, G, λ) such that $\lambda \geq \omega(K, G)$, we set

$$Tv(K, G, \lambda) = \max_{K', G'} \mathcal{U}(K, G, \lambda) + \beta v(K', G', \lambda'(K, G, \lambda)),$$

   subject to

$$\lambda'(K, G, \lambda) \geq \omega(K', G')$$

   and

$$\mathcal{C}(K, G, \lambda) + K' + G' \leq \Theta(G) f(K) + (1 - \delta_K)K + (1 - \delta_G)G - \tau(G, G').$$

9. If $||v - Tv||_\infty < tol$, we end. If not, we update $v = Tv$ and return to step 8.

K.3 Market economy: Non-commitment (section 3.3)

In this case, we need to find three objects: the government-policy function ν, the private-saving function M and the government's value function v. We use the notation from section 3.3. As in K.1, for all off-grid values of K and G, we approximate v, ν, and M by linear interpolation.

The algorithm is then as follows:

1. Set the grids for K and G as in section K.1.

2. Initialize the government's policy function $\nu(K, G) = G_{min}$.

3. Initialize the private saving function $M(K, G) = K_{min}$.

4. Set the government's value function to be the fixed point of:

$$v(K, G) = u(C(K, G, M(K, G), \nu(K, G))) + \beta v(M(K, G), \nu(K, G)).$$

5. Find updates for ν and M.

   For all (K, G) on the grid:

(a) Set the new ν function as

$$\nu'(K,G) \in \arg\max_{G'} u(C(K,G,\tilde{M}(K,G,G'),G') + \beta v(\tilde{M}(K,G,G'),G'),$$

where $\tilde{M}(K,G,G')$ is computed as the solution K' of the problem:

$$u'(C(K,G,K',G')) = \beta(1-\delta_K + q(G')f'(K'))u'(C(K',G',M(K',G'),\nu(K',G'))).$$

Note that given the (linearly interpolated) ν and M, $\tilde{M}$ is always computed exactly for any (K,G,G'), so the condition above always holds exactly.

(b) Set the new M function as $M'(K,G) = \tilde{M}(K,G,\nu'(K,G))$.

6. If $||\nu - \nu'||_\infty + ||M - M'||_\infty < tol$, end. Otherwise: update $\nu = \nu'$ and $M = M'$ and go to step 4.

Additional References for Online Appendix

[1] Abreu, Dilip, David Pearce, and Ennio Stacchetti (1990). "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica* 58, 1041–1063

[2] Feenstra, Robert C., Robert E. Lipsey, Haiyan Deng, Alyson C. Ma, and Hengyong Mo (2005). "World Trade Flows: 1962-2000," *NBER working paper* No 11040