Online Appendix

Appendix A. Data details

Appendix A.1. Survey of Income and Program Participation (SIPP)

Appendix A.1.1. Data description

SIPP is a dataset of household-based panel surveys administrated by the US Census Bureau. We use the following seven panels from the SIPP for our analysis: 1990, 1991, 1992, 1993, 1996, 2001, and 2004. These panels have a sample of 14,000–52,000 individuals. Each panel is a nationally representative sample of households interviewed every four months. Individuals are asked to provide their employment information as detailed as on a weekly basis. With these SIPP panels, we identify the workers' job and occupation switches on an annual basis.

As noted in Stinson (2003), the 1990–1993 panels had substantial miscoding in their job IDs. Thus, we use the revised job IDs provided by the US Census Bureau. We do not use the panels before 1990, because no revised job IDs are provided. We are not able to use the 2008 panel, because the US Census Bureau's data-cleaning procedure has made occupational switches within firms unidentifiable for that panel.

Appendix A.1.2. Sample selection

We select observations where an individual is between ages 23 and 55. We drop observations where an individual works in the public sector or is self-employed. We also drop observations where no occupation information is available. We only focus on individuals who report valid employment status.

Appendix A.1.3. Data cleaning

In the SIPP, workers are asked to list up to two employers for each week. When a worker has two occupations at the same time, we select the occupation for the greater number of hours worked. We drop the observations with managerial occupations to eliminate the flows due to promotions. Those managerial occupations include the following:

- Legislators
- Chief executives and general administrators, public administration
- Administrators and officials, public administration
- Administrators, protective services
- Financial managers
- Personnel and labor relations managers
- Purchasing managers
- Managers, marketing, advertising, and public relations
- Administrators, education, and related fields

- Managers, medicine and health
- Postmasters and mail superintendents
- Managers, food serving and lodging establishments
- Managers, properties and real estate, funeral directors
- Managers, service organizations, n.e.c.
- Managers and administrators, n.e.c.

Appendix A.1.4. Attrition

One of the major problems in longitudinal survey data is that individuals can drop from the sample over time. The SIPP is not exempt from this problem either, which creates biases in the decomposition results. Therefore, we run a robustness check by running the decomposition with the balanced panels of the SIPP in Appendix C.4.

Appendix A.1.5. Identifying job and occupational switches

We follow Xiong (2008) to identify occupational switches of workers. We first define the three broad occupational groups as listed in Appendix A.4. When a person reports multiple occupations, we use the one for the job that reports the largest number of hours worked in the month. Keeping the monthly frequency of the SIPP panel, we then identify the occupational switches by comparing the occupation of the worker in the current month and 12 months ago. The identified switches are then aggregated to the annual frequency.

The identified occupational switches are classified into within-firm and across-firm switches by using the Job ID. The within-firm switches are the switches where the worker stays in the same firm. The across-firm switches are the switches where the worker moves to a different firm.

The literature widely acknowledges measurement errors in occupational codes can lead to spurious transitions, as highlighted in studies such Kambourov and Manovskii (2009) and Moscarini and Thomsson (2007). Our approach here is similar to that in Carrillo-Tudela et al. (2022): We use a high degree of aggregation (i.e., three broad occupational groups) to minimize the coding errors. Additionally, since 1986, the SIPP interviewing process has incorporated a practice known as "dependent interviewing," wherein if a worker confirms no change in job type or employer from the previous interview, the occupational code from the prior interview is retained. This method significantly reduces erroneous occupational transitions, particularly among those switching jobs within the same firm.

Appendix A.2. Sample of Integrated Labor Market Biographies (SIAB)

Appendix A.2.1. Data description

We utilize the Sample of Integrated Labour Market Biographies (SIAB) spanning from 1975 to 2017 for our analysis of German labor markets. This dataset is provided by the Institute for Employment Research (IAB) in Germany. It constitutes a 2% sample of the Integrated Employment Biographies (IEB) population, encompassing employees covered by social security, individuals engaged in marginal part-time employment (since 1999), recipients of unemployment insurance benefits, and those officially registered as job-seeking or participating in active labor market policy programs. Excluded from this dataset are the self-employed, civil servants, individuals in military service, and those not actively participating in the labor force. It contains information on the starting and ending dates of each employment spell with an employer identification number and occupation classification code.

Appendix A.2.2. Sample selection

We select individuals who have German citizenship and have never worked or resided in East Germany. We then select observations where the individual is between ages 23 and 55. We drop observations where no occupation information is available.

Appendix A.2.3. Data cleaning

We look at a worker's labor market information at the beginning of each calendar year. If a worker has multiple jobs, we select an occupation that is associated with the highest wage per day to identify the main occupation. We drop the observations of managerial occupations to eliminate the flows due to promotions. Those managerial occupations include the following:

- Foremen, master mechanics
- Entrepreneurs, managing directors
- Members of Parliament, ministers
- Senior government officials
- Association leaders, officials

Appendix A.2.4. Attrition

Workers may disappear from the social security records for various reasons (leave the labor force, migrate abroad, become a public servant or self-employed, or pass away). The IAB is adding new individuals to the sample every year to keep it as 2% of the entire population in Germany.

Appendix A.2.5. Identifying job and occupational switches

To identify occupational switches of workers, we first define the three broad occupational groups as listed in Appendix A.4 following follow Böhm et al. (2024). We then look at the worker's labor market status and information at the beginning of a calendar year. When a worker reports multiple occupations, we use the one for the job that reports the highest wage per day. Keeping the annual frequency of the SIAB panel, we then identify the occupational switches by comparing the occupation of the worker in the current year and the previous year. We classify within-firm and across-firm switches by using the establishment IDs.

Appendix A.3. Current Population Survey (CPS)

Appendix A.3.1. Data description

CPS, administered by the US Census Bureau, is conducted with a sample of around 60,000 households and consists of the basic monthly questions focusing on labor force participation and supplemental questions, such as the annual March income supplement. Each individual shows up in the records at most eight times: respondents are contacted monthly for the first four consecutive months, followed by an eight-month gap, and then the monthly interview resumes for the last four months. We use the Public Use Microdata File of the Basic Monthly CPS files from January 1994 to October 2019, which are obtained from the DataWeb FTP of the US Census Bureau. The respondents are matched based on Drew et al. (2014).

Appendix A.3.2. Sample selection

For comparability with the SIPP estimates, we restrict our focus to males between the ages of 23 and 55. We drop observations where an individual works in the public sector or is self-employed. We drop the observations of managerial occupations. We also drop observations where no occupation information is available.

Appendix A.4. Occupational groups

Appendix A.4.1. US

We classify occupations into the three broad groups, as defined by Acemoglu and Autor (2011). For the SIPP and CPS, we aggregate the US Census' 1990/2000 Occupational Classification codes into these three broader categories:

1. Nonroutine cognitive: professional, technical, management, business , and financial occupations.
2. Routine: clerical, administrative support, sales workers, craftsmen, foremen, operatives, installation, maintenance and repair occupations, production and transportation occupations, laborers.
3. Nonroutine manual: service workers.

Appendix A.4.2. Germany

For the SIAB, we follow Böhm et al. (2024) to group three-digit occupations (120 occupations according to the KLDB1988 classification) into nine categories and define the three groups, which correspond to those in Acemoglu and Autor (2011), as follows:

1. Nonroutine cognitive: managers, professionals, and technicians.
2. Routine: craftspeople, sales personnel, office workers, production workers, operations, and laborers.
3. Nonroutine manual: service personnel.

Appendix B. Decomposition method

Let ℓ_{it} be the stock of employment of occupation i at time t. Further, let

$$E_t \equiv \sum_{i=c,r,m} \ell_{it}$$

be the employment. The employment share at time t for occupation i is

$$\frac{\ell_{it}}{E_t}.$$

We want to decompose

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right)$$

into net flows:

$$\log(\ell_{it}) = \log\left(\sum_{j=c,r,m,k=s,d} f_{t-1,t}^{ji,k} + f_{t-1,t}^{Ui}\right) = \log\left(\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}\right).$$

Here, U includes unemployment, out-of-labor force, and dropped/added sample. s is for the same firm, and d is for the different firm. Thus,

$$\begin{aligned}
\log(\ell_{i,t+1}) - \log(\ell_{it}) &= \log\left(\frac{\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ji,k} + f_{t,t+1}^{Ui}}{\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}}\right) \\
&= \log\left(1 + \frac{\sum_{j\neq i,k=s,d}(f_{t,t+1}^{ji,k} - f_{t,t+1}^{ij,k}) + (f_{t,t+1}^{Ui} - f_{t,t+1}^{iU})}{\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}}\right) \\
&\approx \frac{\sum_{j\neq i,k=s,d}(f_{t,t+1}^{ji,k} - f_{t,t+1}^{ij,k}) + (f_{t,t+1}^{Ui} - f_{t,t+1}^{iU})}{\ell_{it}}.
\end{aligned}$$

Note also that

$$\begin{aligned}
\log(E_{t+1}) - \log(E_t) &\approx \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \ell_{it} \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \left(\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}\right) \frac{E_{t+1} - E_t}{E_t}.
\end{aligned}$$

Let

$$\Delta_{t,t+1}^E \equiv \frac{E_{t+1} - E_t}{E_t}.$$

Combining the above, we have

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) = \frac{1}{\ell_{it}}\left[\sum_{j\neq i}(f_{t,t+1}^{ji,s} - f_{t,t+1}^{ij,s}) + \sum_{j\neq i}(f_{t,t+1}^{ji,d} - f_{t,t+1}^{ij,d}) + (f_{t,t+1}^{Ui} - f_{t,t+1}^{iU}) - \ell_{it}\Delta_{t,t+1}^E\right].$$

To calculate the cumulative changes from period t to period $t+T$, note

$$\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) = \sum_{\tau=0}^{T-1}\left[\log\left(\frac{\ell_{i,t+\tau+1}}{E_{t+\tau+1}}\right) - \log\left(\frac{\ell_{i,t+\tau}}{E_{t+\tau}}\right)\right].$$

Then, we can apply the decomposition formula to obtain

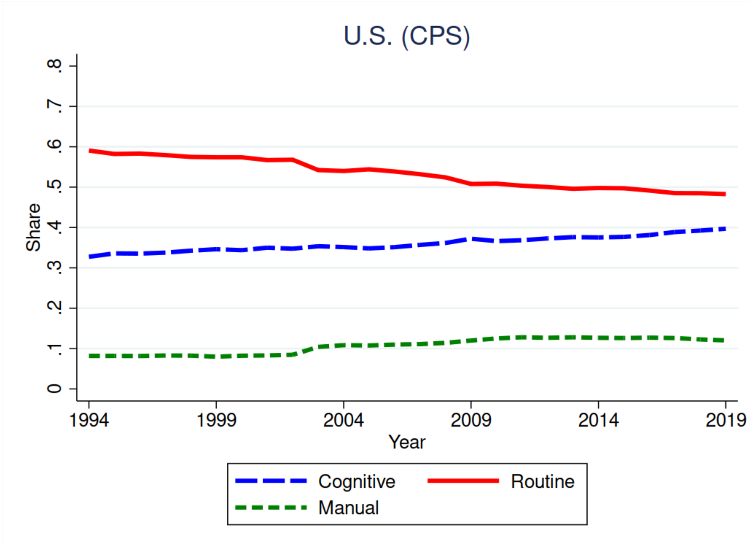
$$\begin{aligned}
&\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) \\
&\quad = \left[\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,s}_{t+\tau,t+\tau+1} - f^{ij,s}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}} + \sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,d}_{t+\tau,t+\tau+1} - f^{ij,d}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}\right. \\
&\qquad \left. + \sum_{\tau=0}^{T-1}\frac{f^{Ui}_{t+\tau,t+\tau+1} - f^{iU}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}} - \sum_{\tau=0}^{T-1}\Delta^{E}_{t+\tau,t+\tau+1}\right].
\end{aligned}$$

Appendix C. Robustness of the empirical results

Appendix C.1. Occupational Employment Shares from the CPS

In this subsection, we show the results of the changes in occupational employment share using the Current Population Survey (CPS) data. Figure C.1 shows the pattern. The results are consistent with the SIPP results in Figure 1 in the main text.

Figure C.1: Occupational Employment Shares in the US, CPS, 1994–2019



Data Source: CPS

Appendix C.2. Decomposition results with detailed external flows

In this section, we provide detailed decomposition results for the US and Germany. Table C.1 corresponds to Table 1 in the main text, but with the breakdowns of the external reallocation to the job-to-job (EE) flows, the flows into/exit from unemployment (U), and the flows into/exit from out of labor force (OLF). We include the effect from the size of employment Δ^E to the last term. Due to the low frequency of observations out of the unemployment state in the earlier period, we start in 1977 for the German data to comply with the disclosure policy of the SIAB.

We found flows out of labor force (OLF) are the most important component of the external reallocation to cognitive occupations both in the US and Germany. The second largest component is the job-to-job (EE) flow in both the US and Germany. On the other hand, the flows from unemployment (U) are the smallest component of the external reallocation in the US. In Germany, the flows into unemployment (U) negatively contribute to the increase in the cognitive share.

Table C.1: Decompositions of Occupational Employment Share Changes for the US and Germany

Occupational employment share					
	(3)	(4)		(5)	
US (SIPP)	log (ΔShare)	Internal		External	
1989–2007			EE	U	OLF
Cognitive	0.173	0.006	0.046	0.008	0.114
Routine	−0.140	−0.001	−0.009	0.017	−0.147
Manual	0.230	−0.013	−0.053	0.024	0.271
Germany (SIAB)	log (ΔShare)	Internal		External	
1977–2017			EE	U	OLF
Cognitive	0.614	0.154	0.130	−0.152	0.481
Routine	−0.273	−0.032	−0.022	−0.278	0.059
Manual	0.387	−0.040	−0.054	−0.206	0.686

Data Source: SIPP (US); SIAB (Germany). *Note*: The numbers in the table are rounded.

Appendix C.3. Effects of demographics and industry composition

To see the extent to which the differences in the demographic composition (age, education, and industry) can explain the differences in the reallocation patterns between the US and Germany, we conduct the following experiments. We first calculate the stock and the flow variables in the decomposition formula (1) for the US by age, education, and industry. We use four groups for age (23-29, 30-39, 40-49, and 50-55), two groups for education (university graduates and others), and three groups for industry (agriculture and mining, manufacturing, services). We then take the weighted average of the stock and flow variables so that the age, education, or industry characteristics of the US become the same as that of Germany for each year during the period 1989–2007.

Table C.2 summarizes the results of the experiments. We found the differences in age and industry composition can not entirely explain the differences in the internal-external reallocation patterns between the US and Germany.

Table C.2: Age, Skill, and Industry Composition for the US

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
US	1989	2007	log (ΔShare)	Internal	External
Cognitive	0.252	0.300	0.173	0.006	0.167
Routine	0.619	0.538	−0.140	0.001	−0.140
Manual	0.128	0.162	0.230	−0.013	0.243
US: Age	1989	2007	log (ΔShare)	Internal	External
Cognitive	0.253	0.301	0.172	0.008	0.164
Routine	0.619	0.539	−0.138	−0.000	−0.137
Manual	0.128	0.160	0.224	−0.018	0.242
US: Education	1989	2007	log (ΔShare)	Internal	External
Cognitive	0.135	0.176	0.265	0.045	0.219
Routine	0.715	0.628	−0.129	−0.007	−0.123
Manual	0.150	0.196	0.267	−0.017	0.284
US: Industry	1989	2007	log (ΔShare)	Internal	External
Cognitive	0.273	0.335	0.202	0.011	0.192
Routine	0.574	0.473	−0.193	−0.007	−0.186
Manual	0.153	0.192	0.229	0.002	0.227
Germany	1975	2017	log (ΔShare)	Internal	External
Cognitive	0.129	0.250	0.662	0.166	0.496
Routine	0.745	0.560	−0.285	−0.035	−0.250
Manual	0.126	0.190	0.408	−0.036	0.444

Data Source: SIPP (US); SIAB (Germany). *Note*: The numbers in the table are rounded.

One exception is the differences in educational composition, which could increase the internal inflow for cognitive occupations significantly. In the counterfactual experiment, the internal net inflow for cognitive occupations in the US is 0.045 (17% of the total cognitive reallocation), whereas the same number is 0.166 for Germany (25% of the total cognitive reallocation). On the other hand, we find even educational composition cannot explain the differences in the internal net inflow for routine occupations. In the same experiment, the internal net inflow for routine occupations in the US is −0.007 (5% of the total routine reallocation), whereas the same number is -0.035 for Germany (12% of the total routine reallocation).

Appendix C.4. Balanced panel for SIPP

To check the robustness of our results in Table 1 and Figure 2 for the sample attrition issue of the SIPP sample, we create a balanced panel for the SIPP and run the decomposition again. That is, we select individuals who report their labor market status without any missing observations over the sample period of each SIPP panel and use the created balanced panel data for our analysis.

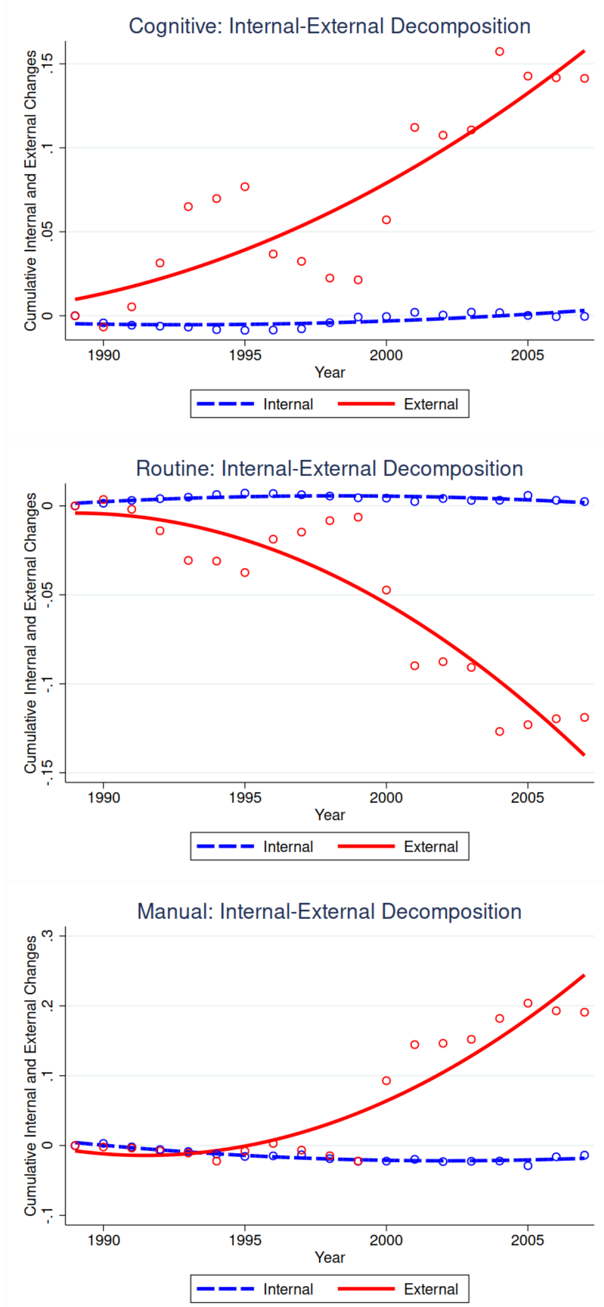
Our internal-external decomposition results do not change the patterns even for the balanced panel case, as seen in Table C.3 and Figure C.2.

Table C.3: Decompositions of Occupational Employment Share Changes for the US, Balanced Panel

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
US	1989	2007	log (ΔShare)	Internal	External
Cognitive	0.290	0.333	0.141	−0.000	0.141
Routine	0.598	0.532	−0.116	0.002	−0.119
Manual	0.113	0.135	0.177	−0.014	0.191

Data Source: SIPP; 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels.

Figure C.2: Cumulative Changes in Occupational Employment in the US, SIPP, 1989–2007, Balanced Panel



Data Source: SIPP; 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels.

Appendix D. A simple model of heterogeneous labor

Two periods exist, with a measure N workers (thus, the total labor supply is fixed) and measure one homogeneous firms. The firm operates both periods. For simplicity, assume that the discount rate is zero for both workers and firms.

The firm's production function at each period is

$$f(n_c, n_r, s_a) = n_c^{\mu}(n_r + s_a)^{1-\mu},$$

where n_c is the number of workers engaging in the cognitive task, and n_r is the number of workers engaging in the routine task. We abstract from manual tasks for simplicity. The labor market is competitive.

Initially, both the firm and the worker believe s_a will be 0 for both periods. Between periods 1 and 2, an unexpected shock (an "MIT shock") occurs that makes all firms' s_a become $\bar{s}_a > 0$. After observing this event, both firms and households reoptimize.

Now, deviating from the baseline model, imagine a model where labor is heterogeneous before tasks are assigned. In particular, only skilled workers (indexed by s) can perform cognitive tasks, and all routine tasks are performed by unskilled workers (indexed by u). One can think of this situation as a corner solution where (in the equilibrium we look at) the wages of cognitive occupation are strictly higher than the wages of routine occupation so that all skilled workers choose to be in a cognitive occupation even though they can perform routine tasks as well.

On the labor-supply side, each worker has to pay training costs at the beginning of each period to be qualified as a skilled worker. The cost is xw_c, where w_c is the wage as a cognitive worker, and x is idiosyncratic and distributed following the distribution function $F(x) = \Pr[X \leq x]$. For each worker, the value of x is the same for both periods. Thus, at the beginning of period 1 (note the MIT shock is not anticipated), a worker decides to become skilled if

$$2w_c(1 - x) \geq 2w_r,$$

which means if $x \leq x^*$, where

$$x^* \equiv 1 - \frac{w_r}{w_c}.$$

Here, w_r is the wage for routine tasks. Suppose the distribution for x is uniform: $F(x) = x$. Then, the skilled labor supply is

$$N_s = N\left(1 - \frac{1}{p}\right) \tag{D.1}$$

and unskilled labor supply is

$$N_u = N\frac{1}{p}, \tag{D.2}$$

where

$$p \equiv \frac{w_c}{w_r} \geq 1$$

is the skill premium. The relative supply curve is

$$\frac{N_s}{N_u} = p - 1. \tag{D.3}$$

On the demand side, the firm's first-order conditions for the first period are

$$w_r = (1-\mu)\left(\frac{n_r}{n_c} + \frac{s_a}{n_c}\right)^{-\mu} \tag{D.4}$$

and

$$w_c = \mu\left(\frac{n_r}{n_c} + \frac{s_a}{n_c}\right)^{1-\mu}. \tag{D.5}$$

Therefore,

$$p = \frac{\mu}{1-\mu}\left(\frac{n_r}{n_c} + \frac{s_a}{n_c}\right).$$

Because $s_a = 0$ in the first period, the first-period relative demand function is

$$p = \frac{\mu}{1-\mu}\frac{n_r}{n_c}. \tag{D.6}$$

In equilibrium, $N_s = n_c$ and $N_u = n_r$, and thus, from (D.3) and (D.6), the equilibrium price satisfies

$$p = \frac{\mu}{1-\mu}\frac{1}{p-1}.$$

From this equation, we can solve for p as

$$p = \frac{1+\sqrt{1+4\mu/(1-\mu)}}{2}$$

and then we can solve for (N_s, N_u, w_c, w_r) from other conditions.

In the second period, after the MIT shock, the firm and the consumers reoptimize. First, consider the US economy, where no firing taxes are in place. In this case, all firms can fire all workers at the end of period 1, let them make the skill decision, and rehire with the optimal choice. Supply decisions are not affected by changes in s_a. The relative demand curve is now

$$p = \frac{\mu}{1-\mu}\left(\frac{N_u}{N_s} + \frac{\bar{s}_a}{N_s}\right). \tag{D.7}$$

One can easily see that because $\bar{s}_a/N_s > 0$, the relative demand curve shifts up, and thus, both p and N_s/N_u go up in equilibrium. The four unknowns (N_s, N_u, w_c, w_r) can be solved from equations (D.1), (D.2), (D.4), and (D.5). Inspecting these equations, one can easily see the following:

Proposition 1. *When the firing cost is not present, with automation, $N_s = n_c$ and w_c go up, and $N_u = n_r$ and w_u go down.*

Thus, this model generates the labor market polarization based on automation, as in our baseline model (with homogeneous workers). The homogeneous-skills version of the model can easily be obtained by setting $w_r = w_c = w$ in (D.4) and (D.5) and adding the (unified) labor market equilibrium condition $n_c + n_r = N$ (three equations with three unknowns w, n_c, and n_r). The difference is that the movement of wages is now ambiguous.

Note that, in this framework, polarization may occur by supply factors, such as the shift in the $F(\cdot)$ function. If the cost of becoming skilled becomes lower, for example, the equilibrium N_s goes up, and N_u goes down. However, in this case, the wage implications are different. When the supply factor is dominant, the skill premium would fall as N_s/N_u goes up. For example, when $F(x) = mx$ for $x \in [0, 1/m]$ and $m \geq 1$, when m goes up becoming skilled becomes cheaper. The relative labor-supply curve can be rewritten as

$$\frac{N_s}{N_u} = \frac{p-1}{1-p(1-1/m)}.$$

Combining this equation with the relative demand curve (D.6), one can see N_s/N_u is increasing in m and p is decreasing in m. Empirically, p has been going up in the US since the 1970s, although we have observed some decline in the last few years. The evidence seems to support the shift in demand (such as automation), although the supply factor may have played some role.

Now, let us consider the case with firing taxes. Suppose the firm has to pay $\tau > 0$ firing tax per worker fired. If the firm wants to fire a worker and rehire after training, the firm has to pay τ in addition to w_c per switched worker. The worker still has to pay the training cost, and the total training cost is $\kappa[I(n'_c - n_c)]^2$, where $\kappa > 0$ is the parameter, n_c is the period 1 cognitive workers, n'_c is the period 2 cognitive workers, and $I \in [0, 1]$ is the fraction of internally reallocated workers (which is determined by the firm). Using this notation, the firing tax can be written as $(1-I)\tau(n'_c - n_c) = (1-I)\tau(n_r - n'_r)$.

The firm decides n'_c, n'_r, and I given w'_c and w'_r, where prime ($'$) indicate the period 2 variable. The firm cannot force the workers to train; thus, the worker decides to obtain skills by the rule

$$w'_c(1-x) \geq w'_r.$$

The labor-supply rules analogous to (D.1) and (D.2) hold.

The firm's first-order conditions are now

$$\begin{aligned}
w'_c &= \mu\left(\frac{n'_r}{n'_c} + \frac{s_a}{n'_c}\right)^{1-\mu} - (1-I)\tau - 2\kappa I^2(n'_c - n_c), \\
w'_r &= (1-\mu)\left(\frac{n'_r}{n'_c} + \frac{s_a}{n'_c}\right)^{-\mu}, \\
\tau(n'_c - n_c) &= 2\kappa I(n'_c - n_c)^2.
\end{aligned}$$

Then, in the equilibrium,

$$\begin{aligned}
p &= \frac{\mu}{1-\mu}\left(\frac{N'_u}{N'_s} + \frac{\bar{s}_a}{N'_s}\right) - \frac{\tau}{1-\mu}\left(\frac{N'_u}{N'_s} + \frac{\bar{s}_a}{N'_s}\right)^{\mu}, \\
I &= \frac{\tau}{2\kappa(N'_c - N_c)}.
\end{aligned}$$

It follows that the relative demand curve shifts down with $\tau > 0$, and hence, both p and N_s/N_u are lower with the higher firing tax. From the condition on I, the following holds.

Proposition 2. *When $\tau > 0$, some workers switch from routine occupations to cognitive occupations by going through reassignment within the firm when automation occurs. The fraction of within-firm reallocation, I, is increasing in τ and decreasing in κ.*

This proposition shows that, even when the tasks are tied to workers of different skill types, when the endogenous choice of skills is taken into account, a qualitatively similar outcome is obtained as in the homogeneous-skills case.

Appendix E. Alternative specification of automation cost

In this section, we present a variant of the baseline model in which we set the adoption cost to decline at a constant rate to describe the diffusion process of technology. Specifically, the adoption cost is replaced by

$$\Gamma(\underline{s}_a, \overline{s}_a; t) = \rho_a^t \bar{c}_a,$$

which is now time variant. On the transition path, firms decide whether to adopt, depending on the current Γ. The value functions for the firms not yet automated are modified as

$$\begin{aligned} & V_t(\mathbf{n}, s_h; \underline{s}_a) \\ = & \max_{\mathbf{n}' \geq \mathbf{0}, d \in \{0,1\}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\ & \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' - d\Gamma(\underline{s}_a, \overline{s}_a; t) \\ & \quad +\beta \mathbb{E}_{s'_h} [dW_{t+1}(\mathbf{n}', s'_h; \overline{s}_a) + (1-d) V_{t+1}(\mathbf{n}', s'_h; \underline{s}_a) | s_h], \end{aligned}$$

where $d = 1$ if firms plan to adopt, and $d = 0$ otherwise. Other model ingredients are similar to the main text. We set $\bar{c}_a = 0.190$ and $\rho_a = 0.990$.

Appendix E.1. Model fit

Figures E.3-E.10 present the model fit for the alternative specification. Overall, the results are similar to the baseline model, whereas the graphs are not as smooth as in the main text.

Figure E.3: Occupation Share in Data versus Model: US

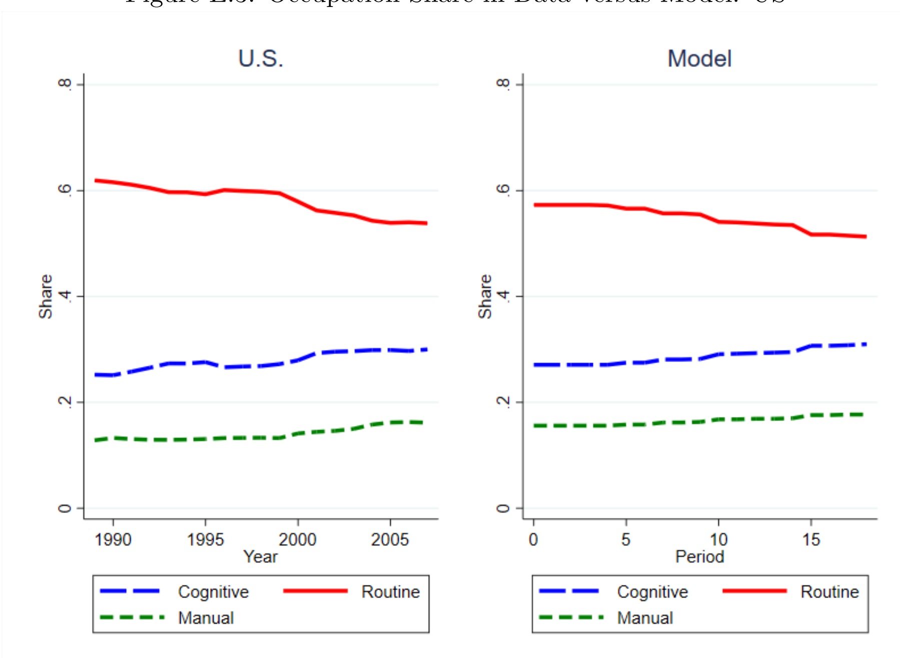


Figure E.4: Occupation Share in Data versus Model: Germany

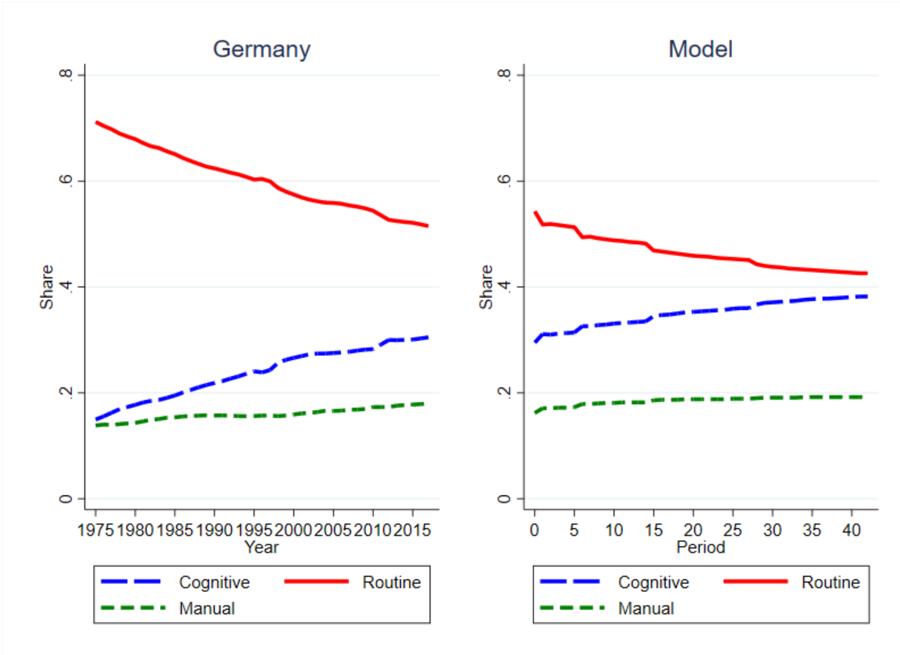


Figure E.5: Cumulative Share Changes of Cognitive in Data versus Model: US

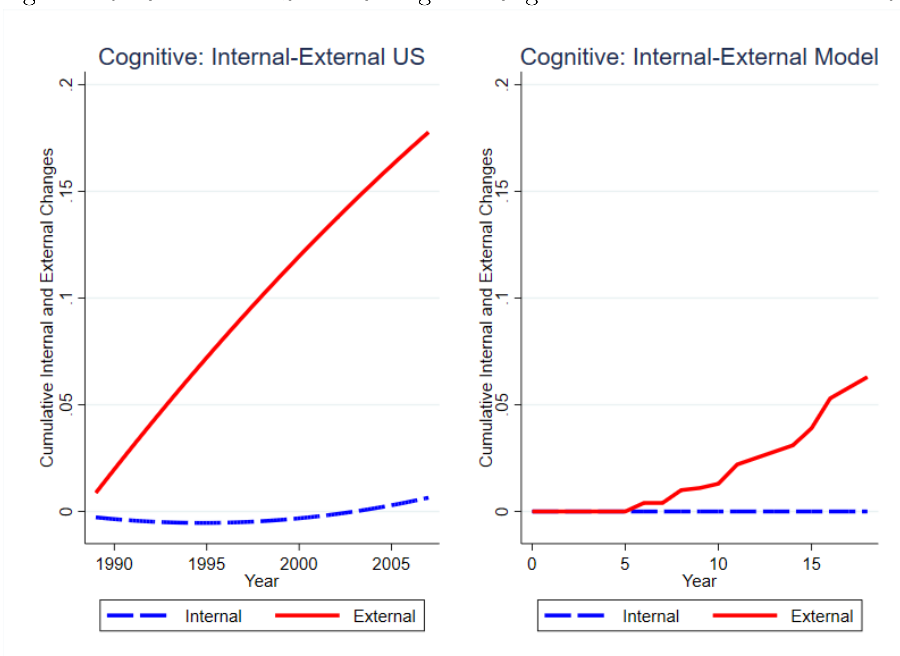


Figure E.6: Cumulative Share Changes of Cognitive in Data versus Model: Germany

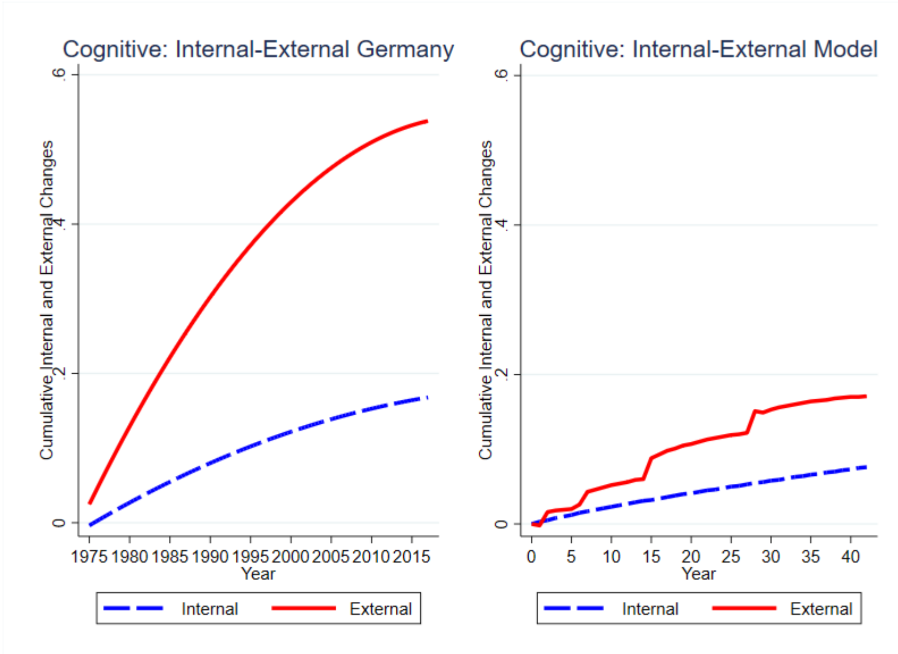


Figure E.7: Cumulative Share Changes of Routine in Data versus Model: US

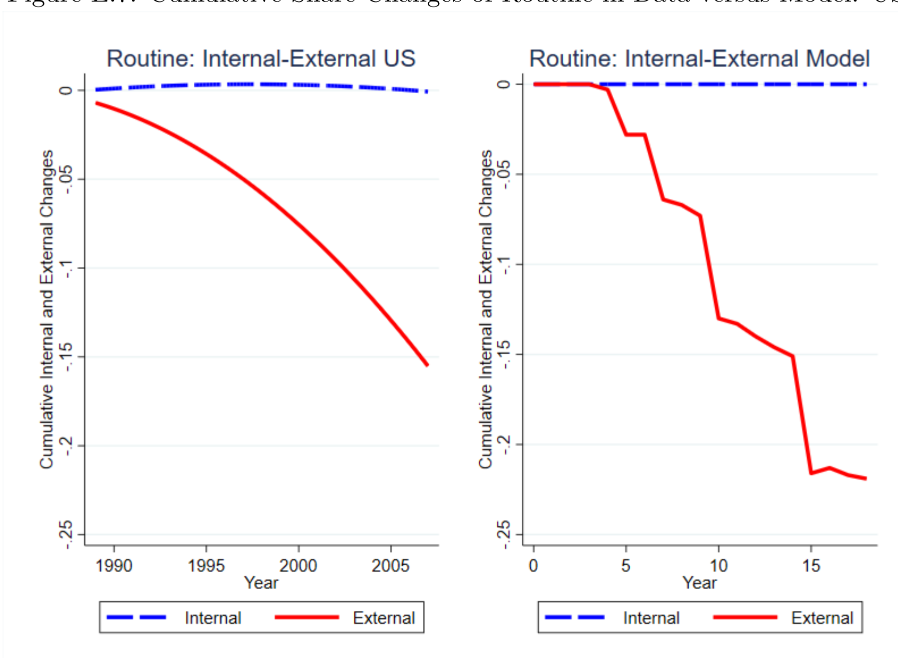


Figure E.8: Cumulative Share Changes of Routine in Data versus Model: Germany

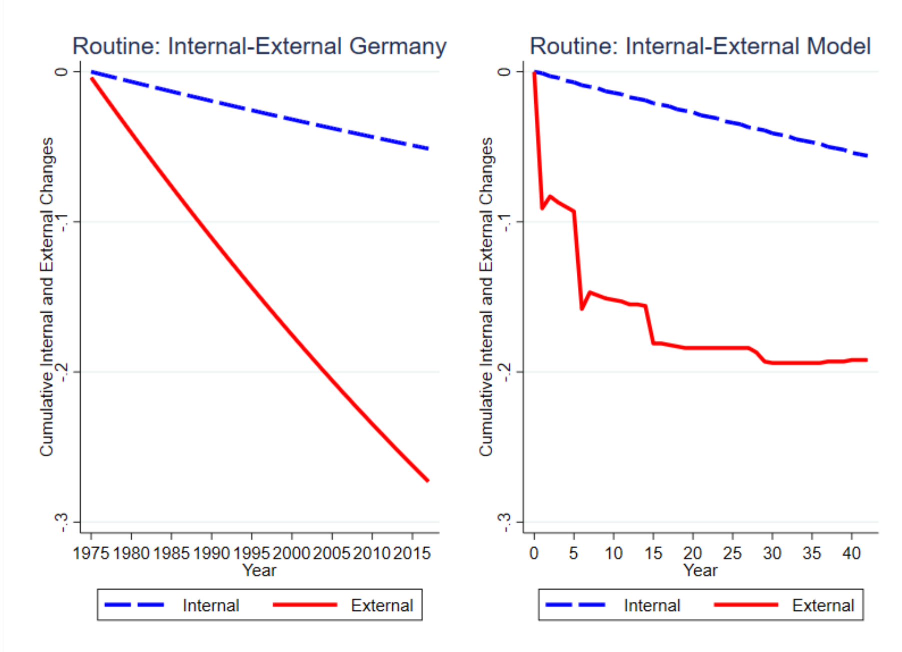


Figure E.9: Cumulative Share Changes of Manual in Data versus Model: US

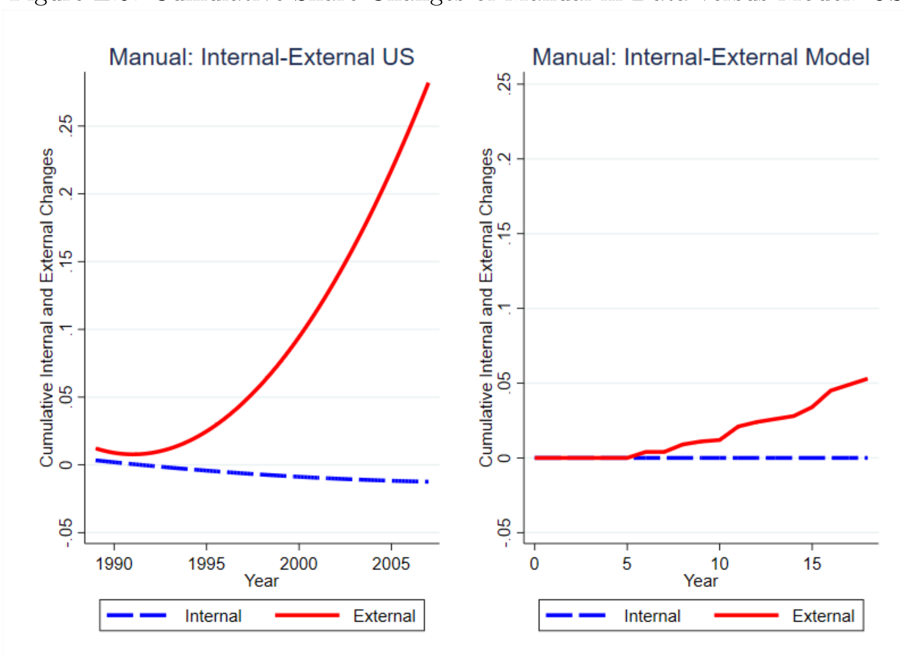


Figure E.11: Counterfactual Occupation Share: Reducing κ by Half

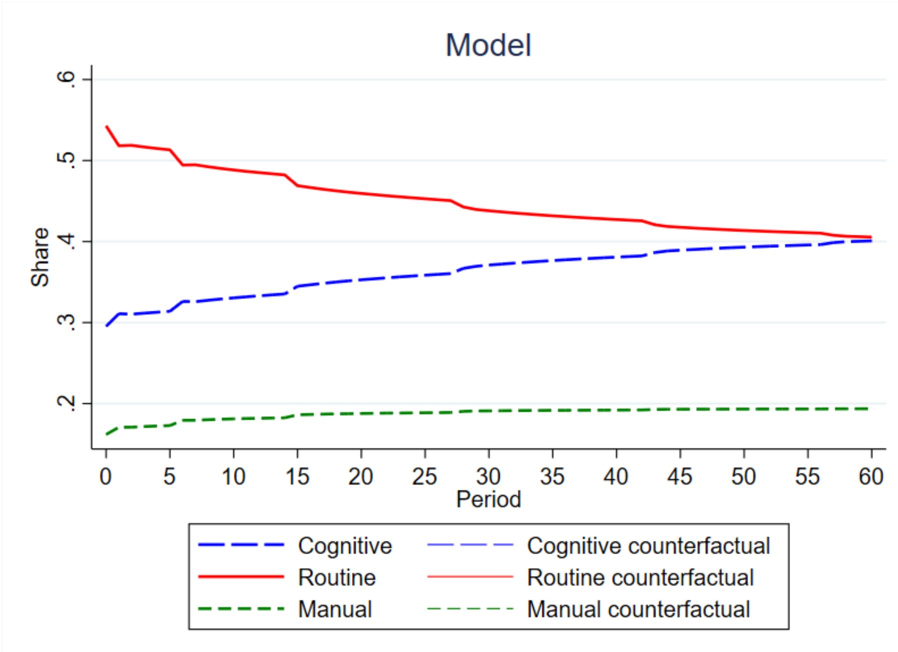
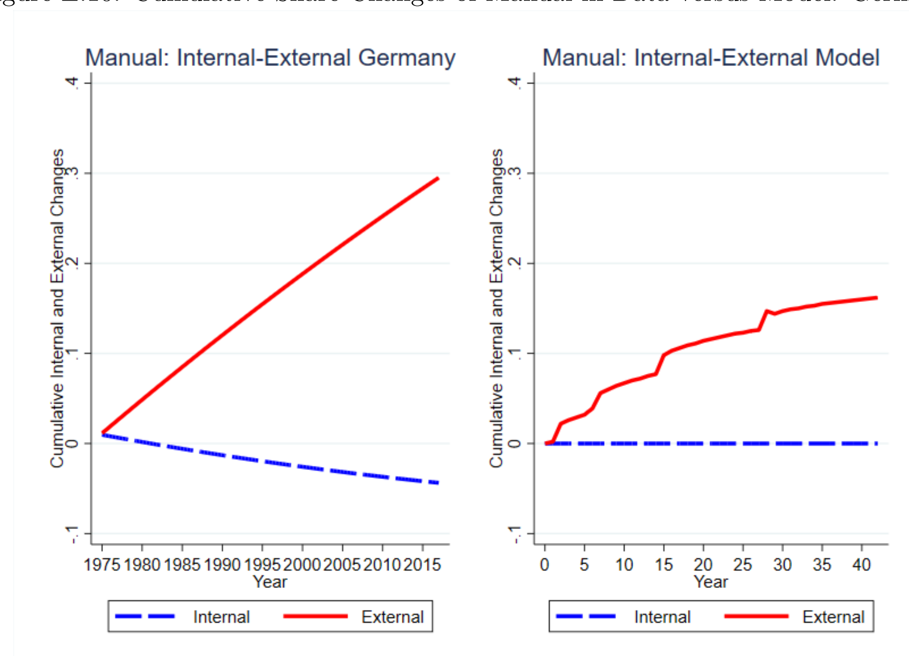


Figure E.10: Cumulative Share Changes of Manual in Data versus Model: Germany



Appendix E.2. Counterfactual on the reorganization cost parameter κ

This subsection repeats the counterfactual exercise for reducing κ by half with the alternative specification in Figures E.11-E.14. The results with the alternative specification are also similar to those in the main text. Once again, the values of aggregate output, aggregate labor, and labor productivity are almost identical between the baseline and the counterfactual.

Figure E.12: Counterfactual Flow of Cognitive: Reducing κ by Half

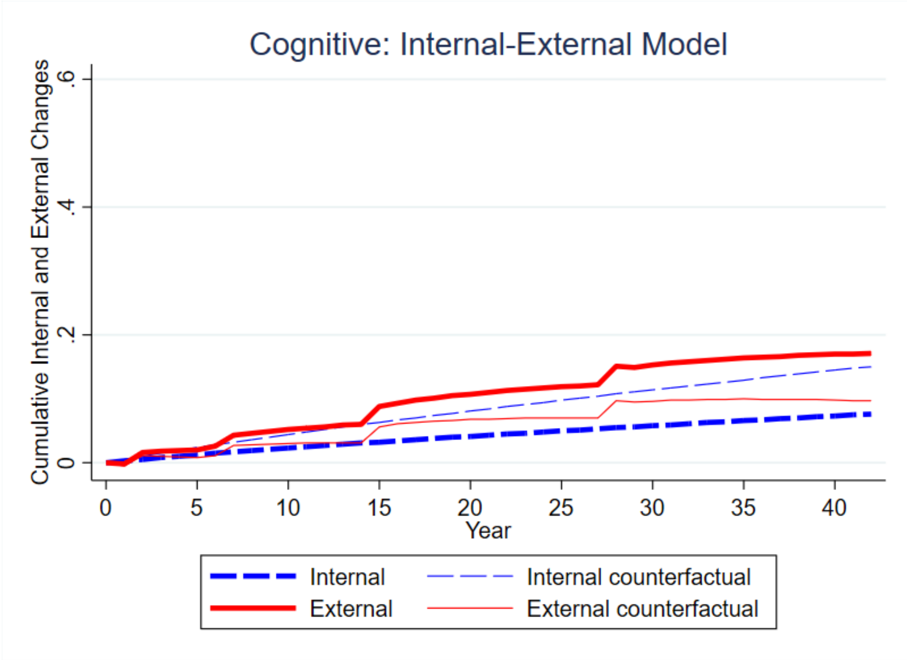


Figure E.13: Counterfactual Flow of Routine: Reducing κ by Half

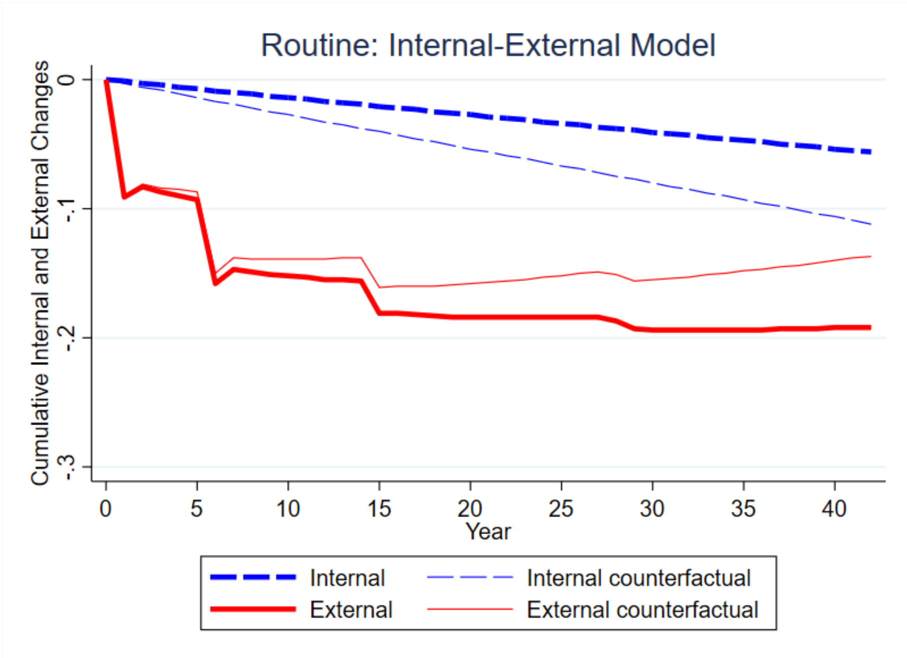


Figure E.14: Counterfactual Flow of Manual: Reducing κ by Half

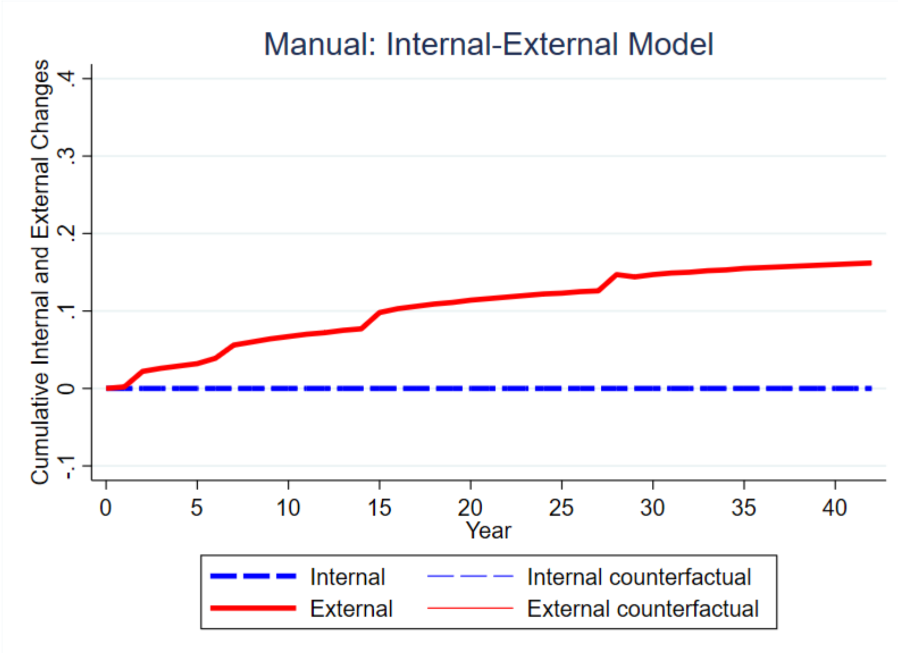


Table E.4: Counterfactual Productivity: Reducing τ by Half

Variable	Baseline	Counter-Factual
Aggregate Output	1.000	1.100
Aggregate Labor	1.000	1.168
Labor Productivity	1.000	0.950

Appendix E.3. Counterfactual on firing tax parameter τ

Counterfactual results repeated with the alternative specification for reducing τ by half in Figures E.15-E.18 and Table E.4 are similar again to those in the main text.

Figure E.15: Counterfactual Occupation Share: Reducing τ by Half

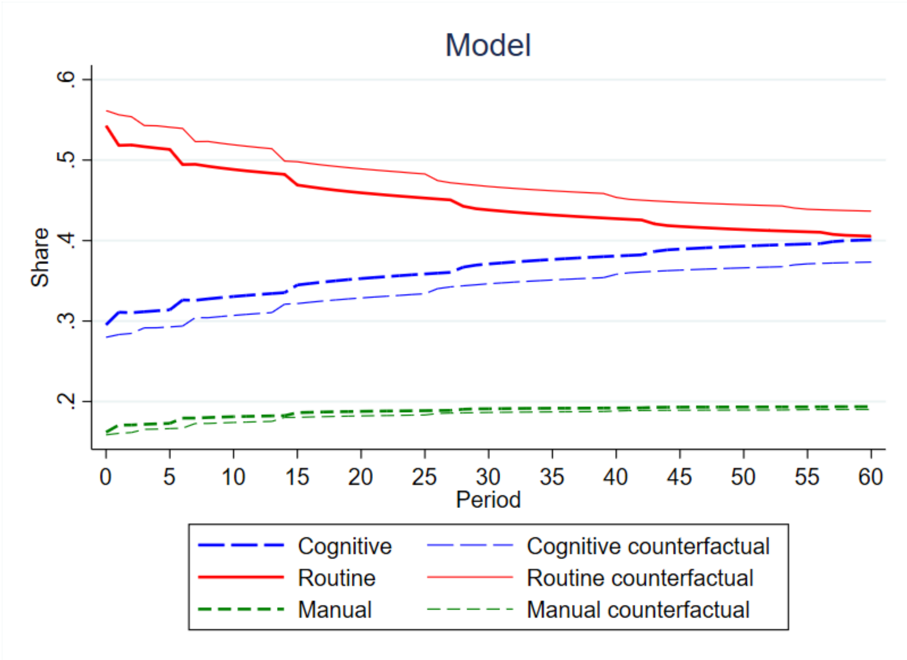


Figure E.16: Counterfactual Flow of Cognitive: Reducing τ by Half

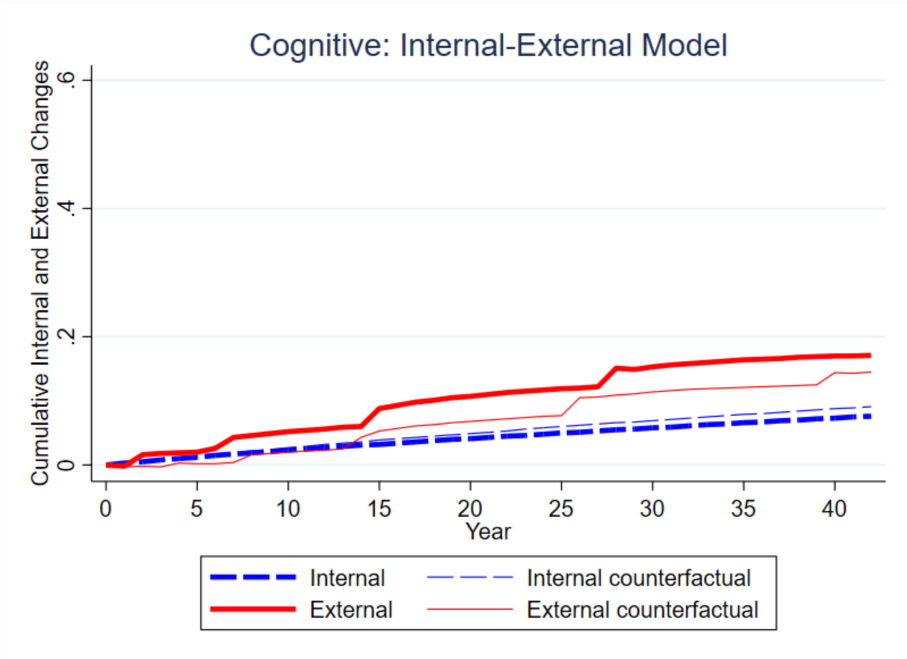


Figure E.17: Counterfactual Flow of Routine: Reducing τ by Half

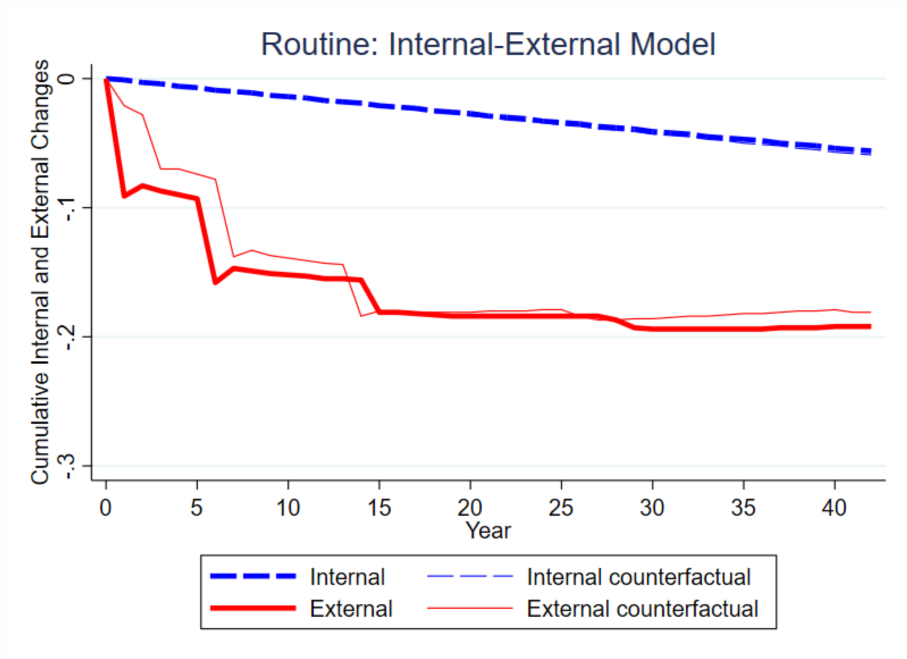
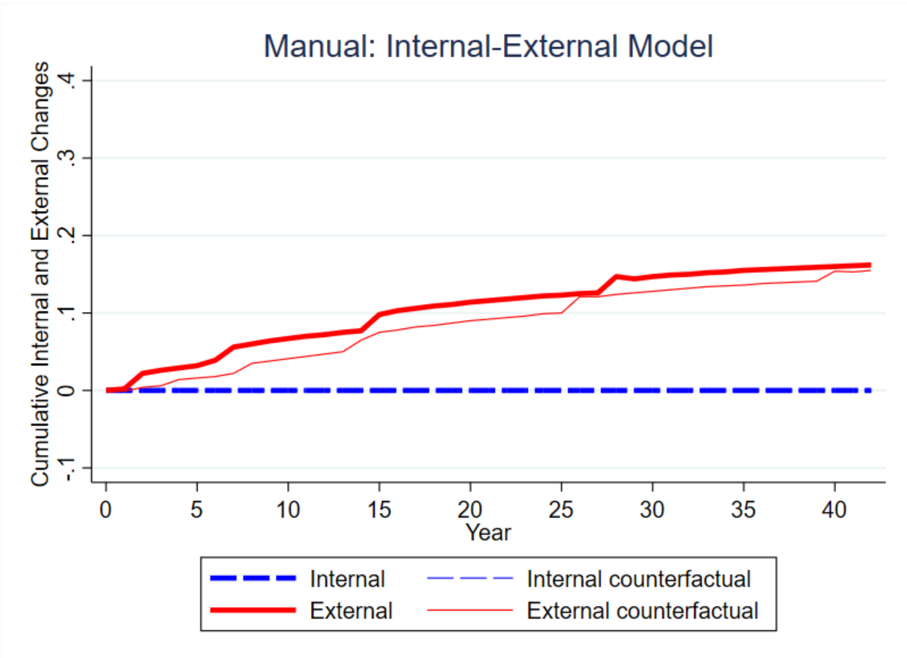


Figure E.18: Counterfactual Flow of Manual: Reducing τ by Half



Appendix F. Computing the transition dynamics

We initially compute the steady state where $s_a = \underline{s}_a$ for all firms. In that steady state, no firm has a possibility of "automating" and moving to $s_a = \overline{s}_a$.

We assume the economy is initially in a steady state where all firms have $s_a = \underline{s}_a$ and expect it to stay constant forever. Then, at a point in time (called time 0), the economy unexpectedly shifts to a new regime where a firm can endogenously switch to $s_a = \overline{s}_a$ when doing so is profitable. In particular, after time 0, with probability p, the firm (at any point in time) can decide whether it automates with adoption cost $\Gamma(\underline{s}_a, \overline{s}_a)$. The regime switch is permanent, and all economic agents understand the nature of the switch. At the firm level, the transition from $\underline{s}_a$ to $\overline{s}_a$ is one time and permanent: once they change s_a to $\overline{s}_a$, it stays at that value. The aggregate economy experiences a gradual transition from the steady state where all firms have $s_a = \underline{s}_a$ to another steady state where all firms have $s_a = \overline{s}_a$. We interpret this transition as the process of labor market polarization, driven by automation at each firm.

To analyze the macroeconomic dynamics of this transition, we first compute the initial and final steady states. As in the previous section, let $\mathbf{n} = (n_m, n_c, n_r)$ be the previous period's occupational employment, and let $\mathbf{n}' = (n'_m, n'_c, n'_r)$ be the current period's employment decision. In the initial steady state, where no firms automate, a firm's dynamic programming problem is

$$\begin{aligned} \underline{V}(\mathbf{n}, s_h; \underline{s}_a) &= \max_{\mathbf{n}', x'} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'), 0\} + \max\{n_r - (n'_r + x'), 0\}) \\ &\quad -\kappa x'^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w\mathbf{1}\cdot\mathbf{n}' \\ &\quad +\beta \mathbb{E}_{s'_h}[\underline{V}(\mathbf{n}', s'_h; \underline{s}_a)|s_h]], \end{aligned}$$

subject to

$$\begin{aligned} n'_m &\geq 0, \\ n'_c &\geq x', \\ n'_r &\geq 0, \\ 0 &\leq x' \leq n_r. \end{aligned}$$

Note the time notation is not included, because the only element of the model that is affected by calendar time is the automation decision (which is absent here). Here, we have already eliminated the notation of $\hat{n}'_i$ and $\tilde{n}'_i$ using the new notation of x'.

x' can be solved analytically once $\mathbf{n}$ and $\mathbf{n}'$ are given. Denote the solution as $x'(\mathbf{n}, \mathbf{n}')$. Then the problem can be rewritten as:

$$\begin{aligned} &\underline{V}(\mathbf{n}, s_h; \underline{s}_a) \\ = \; &\max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\ &\quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - \underline{w}\mathbf{1}\cdot\mathbf{n}' + \beta \mathbb{E}_{s'_h}[\underline{V}(\mathbf{n}', s'_h; \underline{s}_a)|s_h]]. \end{aligned}$$

At the final state, where all firms have completed the automation, the Bellman equation is

$$\begin{aligned} &\overline{W}(\mathbf{n}, s_h; \overline{s}_a) \\ = \; &\max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\ &\quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s}_a) - \overline{w}\mathbf{1}\cdot\mathbf{n}' + \beta \mathbb{E}_{s'_h}[\overline{W}(\mathbf{n}', s'_h; \overline{s}_a)|s_h]]. \end{aligned}$$

After computing the initial and final steady states, we compute the transition dynamics. Let $d = 1$ if firms plan to adopt, and $d = 0$ otherwise. The value functions for the firms not yet automated are written as

$$\begin{aligned}
& V_t(\mathbf{n}, s_h; \underline{s}_a) \\
= & \max_{\mathbf{n}'\geq \mathbf{0}, d\in\{0,1\}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w_t \mathbf{1}\cdot\mathbf{n}' \\
& \quad +\beta \mathbb{E}_{s'_h}[p\{d(W_{t+1}(\mathbf{n}', s'_h; \overline{s}_a) - \Gamma(\underline{s}_a, \overline{s}_a)) + (1-d)V_{t+1}(\mathbf{n}', s'_h; \underline{s}_a)\} + (1-p)V_{t+1}(\mathbf{n}', s'_h; \underline{s}_a)|s_h],
\end{aligned}$$

and the firms that are already automated solve the Bellman equation

$$\begin{aligned}
& W_t(\mathbf{n}, s_h; \overline{s}_a) \\
= & \max_{\mathbf{n}'\geq \mathbf{0}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s}_a) - w_t \mathbf{1}\cdot\mathbf{n}' \\
& \quad +\beta \mathbb{E}_{s'_h}[W_{t+1}(\mathbf{n}', s'_h; \overline{s}_a)|s_h].
\end{aligned}$$

In addition, the distributions of firms are defined as below. Let $m_t^V(\mathbf{n}, s_h; \underline{s}_a)$ and $m_t^W(\mathbf{n}, s_h; \overline{s}_a)$ be the measures of non-automated and automated firms in the period t, and let M_t^V and M_t^W be the total mass of the corresponding firms. The mass is defined as

$$\begin{aligned}
M_t^V &= \sum_{\mathbf{g_n}} \sum_{g_h} m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \\
M_t^W &= \sum_{\mathbf{g_n}} \sum_{g_h} m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a).
\end{aligned}$$

The counterparts at the initial steady state are denoted by $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ and $\underline{M}^V$. At the final steady state, they are $\overline{m}^W(\mathbf{n}, s_h; \overline{s}_a)$ and $\overline{M}^W$. We assume $M_t^V = M_t^W = \underline{M}^V = \overline{M}^W = 1$, as we shut down entry-exit.

We compute these objects using the following steps.

Appendix F.1. Preparation

We discretize the labor and shock, and the grid points are denoted by $(n_m^{g_m}, n_c^{g_c}, n_r^{g_r}) = \mathbf{n}^{\mathbf{g_n}}$, respectively, and $s_h^{g_h}$ where integer $g_{\cdot} \in \{1, ..., g_{\cdot}^{max}\}$. Later, we redistribute the weight of an off-grid point $\mathbf{n}$ to the neighboring grid points, such as $\mathbf{n}^{\mathbf{g_n}}$, by the following discrete measure G such that

$$G(\mathbf{n}, \mathbf{n}^{\mathbf{g_n}}) = \begin{cases} \frac{\prod |n_j^{g'_j} - n_j|}{\prod_j |n_j^{g'_j} - n_j^{g_j}|} & \text{if } n_j \text{ is between } n_j^{g_j} \text{ and } n_j^{g'_j} \text{ including endpoint for all } j = m, c, r, \\ 0 & \text{otherwise}, \end{cases}$$

where g'_j is either $g_j - 1$ or $g_j + 1$. The transition probability from $s_h^{g_h}$ to $s_h^{g'_h}$ is denoted by $P(s_h^{g'_h}|s_h^{g_h})$.

Whereas $(\beta, \eta, \phi, \tau, \kappa)$ are given from outside model, ξ is pinned down within the model. First, assuming $\tau = 0$ and $\underline{w}$=1, we solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s}_a)$ by value function iteration. Next, simulating the above firms' decision rule repeatedly as

$$\underline{m}^{V,new}(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g'_h}; \underline{s}_a) = \sum_{\mathbf{g_n}} \sum_{g_h} G(\underline{\mathbf{n}}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g'_h}|s_h^{g_h}) \underline{m}^{V,old}(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a),$$

we can obtain an invariant distribution of firms $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$. Then, the labor demand is computed as $\underline{N} = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)\underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)$. Then, by the intra-temporal optimality,

$$\xi = \frac{\underline{w}}{\underline{N}^{\frac{1}{\eta}}}.$$

Appendix F.2. Computing the initial and final steady states

Setting $\tau > 0$, we guess the GE wage $\underline{w}$, solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s}_a)$ by value function iteration, and compute the invariant distribution $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ by using the obtained decision rule similarly to the previous subsection. Then, we check if $\underline{w}$ equates the demand and supply of labor as

$$\left(\frac{\underline{w}}{\xi}\right)^{\eta} = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)\underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a).$$

If excess demand exists, we increase $\underline{w}$, and vice versa. Then, we repeat it until $\underline{w}$ equates the demand and supply of labor. We apply the same steps for $\overline{w}$ and $\overline{W}$.

Appendix F.3. Backward induction

First, we guess the path of w_t on the transition. Given w_t, we solve for V_t and W_t, and corresponding decision rules $\mathbf{n}_t'(\mathbf{n}, s_h; \underline{s}_a)$ and $\mathbf{n}_t'(\mathbf{n}, s_h; \overline{s}_a)$ by backward induction from T to 1, whereas we set $W_{T+1} = \overline{W}$ and $V_{T+1} = \overline{V}$. The latter is a hypothetical non-automated value function at the final steady state and obtained by solving

$$\begin{aligned}
&\overline{V}(\mathbf{n}, s_h; \underline{s}_a) \\
= &\max_{\mathbf{n}' \geq \mathbf{0}, d \in \{0,1\}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}_j'(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}_j'(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
&+ f(\mathbf{n}', s_h; \underline{s}_a) - \overline{w}\mathbf{1} \cdot \mathbf{n}' \\
&+ \beta \mathbb{E}_{s_h'} [p\{d(\overline{W}(\mathbf{n}', s_h'; \overline{s}_a) - \Gamma(\underline{s}_a, \overline{s}_a)) + (1-d)\overline{V}(\mathbf{n}', s_h'; \underline{s}_a)\} + (1-p)\overline{V}(\mathbf{n}', s_h'; \underline{s}_a) | s_h] \Big],
\end{aligned}$$

At each t, we solve for V_t and W_t and the decision rules, and proceed to $t-1$.

Appendix F.4. Simulating forward

Using the decision rules obtained above for $t = 1, ..., T$, we can compute $m_t^V(\mathbf{n}, s_h; \underline{s}_a)$, $m_t^W(\mathbf{n}, s_h; \overline{s}_a)$ as follows. Let $\phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)$ be the indicator of firms adopting at the grid point $(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h})$. First,

$$m_t^V(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g_h'}; \underline{s}_a) = \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g_h'} | s_h^{g_h})(1 - \phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)) m_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a).$$

Second,

$$\begin{aligned}
m_t^W(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g_h'}; \overline{s}_a) &= \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g_h'} | s_h^{g_h}) \phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) m_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) \\
&+ \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g_h'} | s_h^{g_h}) m_{t-1}^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a),
\end{aligned}$$

where the first term on the right-hand side represents the non-automated firms that become automated at the end of period t, and the second is the automated firms from the last period. As for the period 1 measure, we set $m_1^V(\mathbf{n}, s_h; \underline{s}_a) = \underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ and $m_1^W(\mathbf{n}, s_h; \overline{s}_a) = 0$.

Appendix F.5. Updating the guess

We check if w_t for each t equates the demand and supply of labor as

$$\left(\frac{w_t}{\xi}\right)^{\eta} = \sum_{\mathbf{g_n}}\sum_{g_h} \mathbf{1}\cdot\mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) + \sum_{\mathbf{g_n}}\sum_{g_h} \mathbf{1}\cdot\mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a)m_t^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a),$$

where the first term on the right-hand side is the labor demand from non-automated firms, and the second term is the demand from automated firms. If excess demand exists, increase w_t, and vice versa. Then, we go back to the backward induction until w_t equates the demand and supply of labor for $t = 1, ..., T$.

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