

Why Does “Taking Log and Compute the Cyclical Component” Yield “Percentage Deviation from the Trend”?

Toshihiko Mukoyama

February 2023

Consider the variable of interest: X_t , $t = 1, \dots, T$. This variable is given as the time series taken from the data (e.g. GDP from time 1 to t). Denote the *trend* of X_t as \tilde{X}_t . The *percentage deviation of the trend*, x_t , is defined as

$$x_t \equiv \frac{X_t - \tilde{X}_t}{\tilde{X}_t}.$$

Let us compute the log deviation from the trend (the *cyclical component*) as

$$y_t \equiv \log(X_t) - \log(\tilde{X}_t).$$

Because $\log(1 + w) \approx w$ when w is small,

$$\log(X_t) - \log(\tilde{X}_t) = \log\left(\frac{X_t}{\tilde{X}_t}\right) = \log\left(1 + \frac{X_t - \tilde{X}_t}{\tilde{X}_t}\right) \approx \frac{X_t - \tilde{X}_t}{\tilde{X}_t}$$

Therefore,

$$y_t \approx x_t.$$

In the business cycle analysis, we frequently take log of a variable, HP-filter it, and compute the statistics of the cyclical component. The outcome of this procedure, therefore, can be interpreted as calculating the statistical characteristics of the percentage deviation from the trend.