

A Note on Cyclical Discount Factors and Labor Market Volatility

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Abstract

In this paper, I examine how time-varying discount factors can contribute to labor market volatility in a Diamond-Mortensen-Pissarides matching model. I find that the procyclical discount factor of either entrepreneurs or workers can magnify labor market volatility. Quantitatively, the entrepreneur's discount factor has a larger effect than the worker's discount factor. To account for the observed labor market volatility, an extremely large variation in discount factors is necessary.

Keywords: cyclical discount factors, Diamond-Mortensen-Pissarides model, labor market volatility

JEL Classifications: E24, E32, J64

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1 Introduction

Since the seminal work of Shimer (2005), many solutions have been proposed to resolve the so-called labor market volatility puzzle. Shimer observed that a standard Diamond-Mortensen-Pissarides (DMP henceforth) model cannot account for the quantitative variation in vacancy and unemployment during the business cycle.¹ Potential solutions to this puzzle include: (i) sticky wages (Hall (2005), Shimer (2005), and Gertler and Trigari (2008)), (ii) different calibration strategies (Hagedorn and Manovskii (2008)), (iii) on-the-job search (Nagypál (2007)), and (iv) incomplete markets and long-term contracts (Rudanko (2008, 2009)), to name a few. This paper contributes to this literature.

In this paper, I consider the variation in discount factors over the business cycle. In particular, I assume that the discount factors of either entrepreneurs (who own the firms and post vacancies) or workers (who work at the firm) are procyclical. I find that procyclical discount factors can magnify the labor market volatility. Quantitatively, entrepreneurs' discount factors have a larger effect than workers' discount factors. However, to account for the volatility observed in the data, an extremely large degree of variation is required.

I offer two interpretations of the entrepreneur's procyclical discount factor: (i) a cyclical financial constraint, and (ii) a cyclical stochastic discount factor by the entrepreneurs/investors. These interpretations represent a link between financial market volatility and labor market volatility. Similarly to (ii), the worker's procyclical discount factor can be interpreted as a cyclical stochastic discount factor by the workers. From this viewpoint, cyclical discount factors of entrepreneurs seem empirically more relevant considering the recent studies by Guvenen (2007) and Parker and Vissing-Jørgensen (2009). In particular, Guvenen suggests that stockholders' income risks are less insured than non-stockholders' income risks. Parker and Vissing-Jørgensen show that wealthy households' consumption is more volatile than that of non-wealthy households. Hall and Woodward (2008) document that entrepreneurs bear a large non-diversifiable risk.

The paper is organized as follows. Section 2 describes the model and calibration. Section 3

¹See also Andolfatto (1996) for an earlier observation.

presents the result. Section 4 concludes.

2 Model

I employ a standard DMP model with exogenous separation (Pissarides (1985)). The calibration largely follows Shimer (2005).

2.1 Model setup

Time is discrete. The economy is made up of entrepreneurs and workers, with a each population equal to 1. Entrepreneurs own the firm and post vacancies optimally so that their utility is

$$U_f = \sum_{t=0}^{\infty} B_t^f c_t^f,$$

where c_t^f is the consumption of the entrepreneurs (a superscript or subscript f represents “firm”) at time t . The cumulative discount factor B_t^f evolves according to

$$B_{t+1}^f = B_t^f \beta_f(z_t),$$

where z_t is the aggregate state at time t . The aggregate state is assumed to take one of two values: $z_t \in \{g, b\}$. The output of a job-worker match, y , is affected by the aggregate state. In particular, we assume that $y(g) > y(b)$. z_t follows a Markov process and the transition probability from state z to state z' is represented by $\pi_{zz'}$.

Workers supply labor and maximize

$$U_w = \sum_{t=0}^{\infty} B_t^w c_t^w,$$

where c_t^w is the consumption of the workers (a superscript or subscript w represents “worker”) at time t . The cumulative discount factor B_t^w evolves according to

$$B_{t+1}^w = B_t^w \beta_w(z_t).$$

The entrepreneurs post v_t amount of vacancies at time t . It costs ξ units of consumption goods to post one unit of vacancy. Vacancies and unemployed workers meet according to the matching

function

$$M(v_t, u_t) = \chi v_t^{1-\eta} u_t^\eta,$$

where u_t is the number of unemployed workers. From this matching function, the probability that a vacancy finds a worker is

$$\lambda_f(\theta_t) = \frac{M(v_t, u_t)}{v_t} = \chi \theta_t^{1-\eta}$$

and the probability that an unemployed worker finds a vacant job is

$$\lambda_w(\theta_t) = \frac{M(v_t, u_t)}{u_t} = \chi \theta_t^{-\eta}.$$

Here, $\theta_t = v_t/u_t$ is the vacancy-unemployment ratio.

I solve the model using a recursive formulation.² It turns out that for all of the endogenous variables, I can use z as the state variable. The value of a filled job, $J(z)$ is

$$J(z) = y(z) - w(z) + \beta_f(z)E[\sigma V(z') + (1 - \sigma)J(z')|z],$$

where $w(z)$ is the wage in state z and $E[\cdot|z]$ is the conditional expectation given the current state z . The value of a vacancy is

$$V(z) = -\xi + \beta_f(z)E[\lambda_f(\theta)J(z') + (1 - \lambda_f(\theta))V(z')|z]. \quad (1)$$

Since the entrepreneurs can optimally choose the amount of vacancy, the free-entry condition $V(z) = 0$ has to hold in equilibrium. This condition determines θ , and therefore θ is a function of z : $\theta(z)$.

The value functions for the workers are

$$W(z) = w(z) + \beta_w(z)E[\sigma U(z') + (1 - \sigma)W(z')|z]$$

for employed workers and

$$U(z) = h + \beta_w(z)E[(1 - \lambda_w(\theta(z)))U(z') + \lambda_w(\theta(z))W(z')|z]$$

for unemployed workers. Here, I assume that an unemployed worker can receive h units of consumption goods.

²A similar formulation is used in Krusell, Mukoyama, and Şahin (2009).

I assume that the wage is determined by a generalized Nash bargaining solution: $w(z)$ is chosen to maximize $(W(z) - U(z))^\gamma (J(z) - V(z))^{1-\gamma}$. Since $W(z) - U(z)$ and $J(z) - V(z)$ are linear in $w(z)$, the Nash bargaining solution results in the simple surplus-sharing rules

$$W(z) - U(z) = \gamma S(z)$$

and

$$J(z) - V(z) = (1 - \gamma)S(z), \quad (2)$$

where

$$S(z) = (W(z) - U(z)) + (J(z) - V(z)) \quad (3)$$

is the total surplus. Therefore, w is indeed a function of z .

From (1), (2), and $V(z) = 0$, it follows that

$$\xi = \beta_f(z)[\pi_{zg}\lambda_f(\theta(z))(1 - \gamma)S(g) + \pi_{zb}\lambda_f(\theta(z))(1 - \gamma)S(b)]. \quad (4)$$

There are two equations (for each z) here. These two equations can be rewritten as

$$S(g) = \frac{\pi_{bb}X(g) - \pi_{gb}X(b)}{P} \quad (5)$$

and

$$S(b) = \frac{\pi_{gg}X(b) - \pi_{bg}X(g)}{P}, \quad (6)$$

where $X(z) = \xi / (\beta_f(z)\lambda_f(\theta(z))(1 - \gamma))$ and $P = \pi_{gg}\pi_{bb} - \pi_{bg}\pi_{gb} = \pi_{bb} - \pi_{gb}$.

From (3) and the value functions,

$$S(z) = y(z) - h + (\pi_{zg}S(g) + \pi_{zb}S(b))[\beta_f(z)(1 - \sigma)(1 - \gamma) + \beta_w(z)(1 - \sigma - \lambda_w(z))\gamma] \quad (7)$$

holds (there are two equations here). Therefore, the four equations (5), (6), and (7) can be solved for four unknowns: $S(z)$ and $\theta(z)$ for each z .

Once these are found, we can calculate the wage as

$$w(z) = y(z) - (1 - \gamma)S(z) + \beta_f(z)(1 - \sigma)(\pi_{zg}(1 - \gamma)S(g) + \pi_{zb}(1 - \gamma)S(b)).$$

Unemployment follows

$$u' = u + \sigma(1 - u) - \lambda_w(\theta(z))u.$$

Vacancy can be calculated as

$$v = \theta(z)u.$$

2.2 Calibration

A large part of calibration follows Shimer (2005). In particular, I assume that $\eta = 0.72$, $\gamma = 0.72$, and $h = 0.4$. I set one period as 6 weeks. For the business cycles, I set $y(g) = 1.02$, $y(b) = 0.98$, and $\pi_{gg} = \pi_{bb} = 0.9375$. ξ is set so that $\theta = 1$ holds in the deterministic steady state (that is, all the cyclical parameters are set at their average values) of the model. In particular, from (4) and (7),

$$\xi = \frac{\beta_f \lambda_f (1 - \gamma)(y - h)}{1 - \beta_f(1 - \sigma)(1 - \gamma) - \beta_w(1 - \sigma - \lambda_w)\gamma} \quad (8)$$

has to be satisfied in the deterministic steady state. Noting that $\lambda_w = \lambda_f = \chi$ when $\theta = 1$, this equation (with average values of parameters β_f , β_w , and y) pins down the value of ξ . The baseline value of β_f and β_w are set at 0.995, but these will be changed in the experiments.

3 Results

In the first experiment, I make β_f procyclical. In particular, I keep $\beta_f(g) = 0.995$ and vary $\beta_f(b)$. $\beta_w(g)$ and $\beta_w(b)$ are also set at 0.995. Figure 1 shows the differences in the unemployment rates (the average when $z = g$ and the average when $z = b$) for various values of $\beta_f(b)$. For each $\beta_f(b)$, ξ is set so that (8) is satisfied. $\beta_f(b) = 0.995$ is the standard model with constant and homogeneous discount factors. As is shown in Shimer (2005), the standard model does not generate high volatility in the labor market. It can be seen that lowering $\beta_f(b)$ helps to increase labor market volatility, although to attain a significant effect an extremely low $\beta_f(g)$ is necessary.

What does a low $\beta_f(b)$ mean? I offer two interpretations here. First, it can represent the financial constraints faced by the entrepreneurs. If an entrepreneur depends on outside funds, and if $1 + r$ is the gross cost of the funds, then she discounts the future profit by $\beta_f = 1/(1 + r)$. If r is

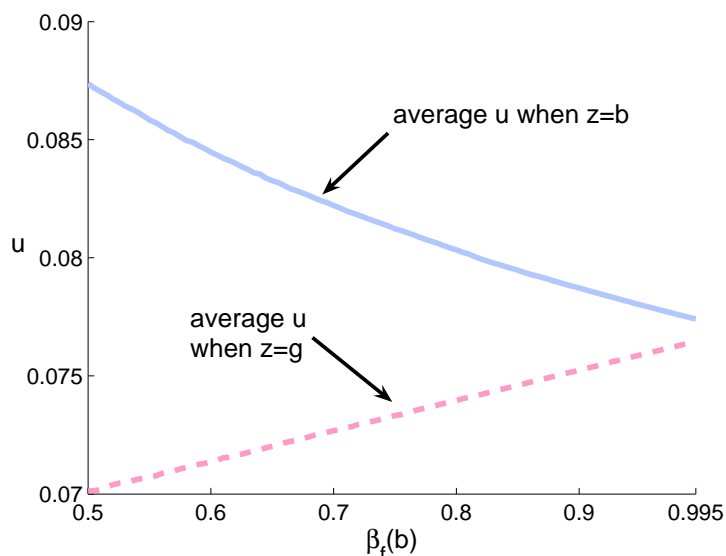


Figure 1: Average unemployment rates when $z = g$ and $z = b$.

cyclical, it shows up in a cyclical β_f . A low $\beta_f(b)$ means that the entrepreneur's cost of raising funds is high during recessions. Second, if the entrepreneur has to self-finance and has a concave utility function $U(c_t^f)$, the stochastic discount factor of the entrepreneur/investor is $\beta_f U'(c_{t+1}^f)/U'(c_t^f)$ and the future is discounted more heavily when c_t^f is low. For example, when $U(c) = c^{1-\nu}/(1-\nu)$ for $\nu > 0$, $\beta_f U'(c_{t+1}^f)/U'(c_t^f) = \beta_f (c_{t+1}^f/c_t^f)^{-\nu}$ and 1% change in consumption is translated into $\nu\%$ change in the stochastic discount factor. Although a linear utility is assumed here, the stochastic β_f can be seen as a shortcut in analyzing a stochastic discount factor with concave utility.

Figure 2 shows a case where β_f is constant at 0.995 and $\beta_w(b)$ is changed instead. It can be seen that lowering $\beta_w(b)$ magnifies labor-market volatility but the quantitative impact is smaller than the case of cyclical $\beta_f(b)$. Recent empirical evidence in Guvenen (2007) and Parker and Vissing-Jørgensen (2009) seems to suggest that a cyclical $\beta_f(b)$ is more important in light of the second interpretation above. Guvenen's result suggests that stockholders' income shocks are less insured than those of non-stockholders. This implies that it is more likely that the marginal utility of stockholders³ fluctuates more than that of non-stockholders.

³As Hall and Woodward (2008) argue, in the standard venture capital contract, entrepreneurs retain a large

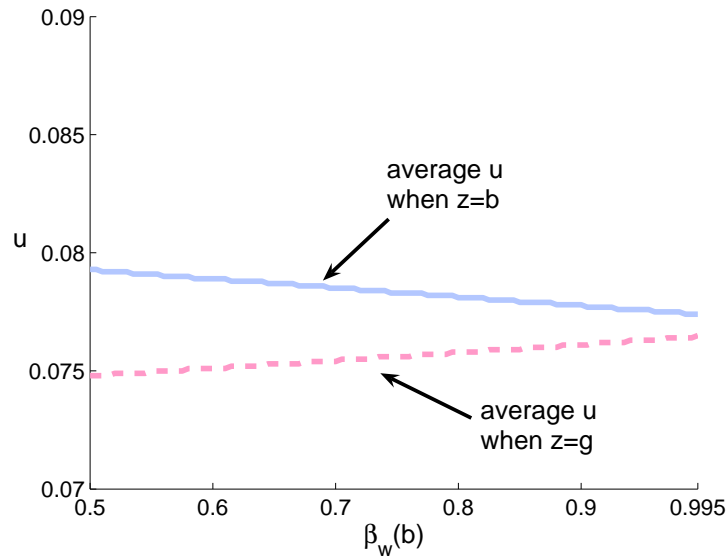


Figure 2: Average unemployment rates when $z = g$ and $z = b$.

4 Conclusion

This paper analyzes the effects of cyclical discount factors on labor market dynamics. It is shown that procyclical discount factors, of either entrepreneurs or workers, magnify unemployment volatility. Quantitatively, the entrepreneur's discount factor has a larger impact than the worker's discount factor. To achieve a realistic labor market volatility, an extreme amount of variation in discount factors is necessary.

fraction of equity ownership in the companies they found.

References

- [1] Andolfatto, David (1996). “Business Cycles and Labor Market Search,” *American Economic Review* 86, 112-132.
- [2] Gertler, Mark and Antonella Trigari (2006). “Unemployment Fluctuations with Staggered Nash Wage Bargaining,” New York University and IGER, Università Bocconi.
- [3] Guvenen, Fatih (2007) “Do Stockholders Share Risk More Effectively than Non-stockholders?” *Review of Economics and Statistics* 89, 275-288.
- [4] Hagedorn, Marcus and Iourii Manovskii (2008). “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 1692-1706.
- [5] Hall, Robert E. (2005). “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review* 95, 50-65.
- [6] Hall, Robert E. and Susan Woodward (2008). “The Burden of the Nondiversifiable Risk of Entrepreneurship,” NBER Working Paper 14219.
- [7] Krusell, Per; Toshihiko Mukoyama; and Ayşegül Şahin (2009). “Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations,” mimeo. Princeton University, University of Virginia, and Federal Reserve Bank of New York.
- [8] Nagypál, Éva (2007). “Labor-Market Fluctuations and On-the-Job Search,” mimeo. Northwestern University.
- [9] Pissarides, Christopher A. (1985). “Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages,” *American Economic Review* 75, 676-690.
- [10] Parker, Jonathan A. and Annette Vissing-Jørgensen (2009). “Who Bears Aggregate Fluctuations? Estimates and Implications for Consumption Inequality,” *American Economic Review* 99, 399-405.

- [11] Rudanko, Leena (2008). "Idiosyncratic and Aggregate Risk in a Frictional Labor Market," mimeo. Boston University.
- [12] Rudanko, Leena (2009). "Labor Market Dynamics under Long Term Wage Contracting," *Journal of Monetary Economics* 56, 170-183.
- [13] Shimer, Robert J. (2005). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review* 95, 25-49.