Galí 2011 Handbook Chapter

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Household

$$\max_{C_t, C_t(i), L_t, U_t, U_t^0, N_t, B_t} E\left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t)\right],$$

$$\text{where } U(C_t, L_t) = \log(C_t) - \frac{\chi}{1+\varphi} L_t^{1+\varphi}$$

subject to

$$L_t = N_t + \psi U_t,$$

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}},$$

$$N_t = (1-\delta)N_{t-1} + x_t U_t^0,$$

$$U_t = (1-x_t)U_t^0,$$

$$\int_0^1 P_t(i)C_t(i) + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j)N_t(j)dj + \Pi_t.$$

Household

With standard optimization,

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

where

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}.$$

The Euler equation is

$$Q_t = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\}.$$

Final goods firm (pricing friction)

- Technology: (firm $i \in [0,1]$ produces differentiated good)

$$Y_t(i) = X_t(i)$$

- Flexible price:

$$\max_{P_t(i), Y_t(i)} P_t(i)Y_t(i) - P_t^I(i)(1-\tau)X_t(i)$$

subject to

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t.$$

Then

$$P_t(i) = \frac{\epsilon}{\epsilon - 1}(1-\tau)P_t^I$$

Let $\mu_p \equiv \log(\epsilon/(1-\epsilon))$: "flexible price markup."

- Calvo: the fairy visits with probability $(1-\theta_p)$.
  The final outcome:

$$\pi_t^p = \beta E_t[\pi_{t+1}^p] - \lambda_p \hat{\mu}_t^p,$$

where $\pi_t^p = p_t - p_{t-1}$, $\lambda_p = (1-\theta_p)(1-\beta\theta_p)/\theta_p$, and $\hat{\mu}_t^p = p_t - (p_t^I - \tau) - \mu^p$.

Intermediate goods firm (labor market friction)

- This part of Galí's paper has some "inconsistencies"—the issue is subtle but important.
- The main issue is that he assumes a decreasing returns to scale production function at the firm level:

$$Y_t^I(j) = A_t N_t(j)^{1-\alpha}$$

  for firm $j \in [0, 1]$. But once decreasing returns to scale is assumed, the firm should take into account that changing $N_t(j)$ will change the wages through bargaining (which Galí ignores).This can be done (see Elsby and Michaels 2013), but it is a bit cumbersome.
- The reason Galí assumes decreasing returns to scale is that otherwise, with wage stickiness and linear hiring cost, a firm with the lowest wage would "outbid" all other firms in the intermediate goods market.
- Others such as Gertler and Trigari (2009) assume convex hiring cost at the firm level to avoid this problem.

Intermediate goods firm (labor market friction)

- Below I will not fix the inconsistencies and follow Galí's paper.
- One easy fix that I can think of is instead of calling j "a firm," call it "a color" (or whatever innocuous label). Each color has a continuum of firms on $[0, 1]$, and the production function for a firm k with color j is

$$y_t^I(j, k) = A_t N_t(j)^{-\alpha} n_t(j, k),$$

  where the $N_t(j)^{-\alpha}$ is an externality and $N_t(j) = \int_0^1 n_t(j, k) dk$. If we assume that the Calvo fairy visits each color rather than each firm, in a symmetric equilibrium we will have the same production function as in the paper at the color level, while having constant returns at the firm level.
- The only change with this fix is that the firm level MPL is $A_t N_t(j)^{-\alpha}$ instead of $(1 - \alpha) A_t N_t(j)^{-\alpha}$. I will use gray to indicate the part that can change.

Intermediate goods firm (labor market friction)

- The intermediate good firm:

$$Y_t^I(j) = A_t N_t(j)^{1-\alpha}$$

where

$$N_t(j) = (1-\delta)N_{t-1}(j) + H_t(j).$$

- The unit hiring cost is a function of the aggregate hiring rate:

$$G_t = \Gamma x_t^{\gamma},$$

where

$$x_t = \frac{\int_0^1 H_t(j)dj}{U_t^0}.$$

As I explained, this is isomorphic to the matching function approach. Each firm takes G_t as given.

- The optimal hiring policy: given wage, maximize the present value of profit.

Intermediate goods firm (labor market friction)

- The optimal hiring condition:

$$MRPN_t(j) = \frac{W_t(j)}{P_t} + G_t - (1-\delta)E_t[\Lambda_{t,t+1}G_{t+1}]$$

or, equivalently,

$$G_t = MRPN_t(j) - \frac{W_t(j)}{P_t} + (1-\gamma)E_t[\Lambda_{t,t+1}G_{t+1}],$$

where

$$MRPN_t(j) = (1-\alpha)\left(\frac{P_t^I}{P_t}\right)A_t N_t(j)^{-\alpha}.$$

Let

$$B_t \equiv G_t - (1-\delta)E_t[\Lambda_{t,t+1}G_{t+1}].$$

Then, the above becomes $MRPN_t(j) = W_t(j)/P_t + B_t$. Log-linearizing:

$$\hat{\mu}_t^p = (a_t - \alpha\hat{n}_t) - [(1-\Phi)(\hat{w}_t - \hat{p}_t) + \Phi b_t].$$

Going back to the "three equations"

- Euler equation:

$$\hat{c}_t = E_t[\hat{c}_{t+1}] - (\hat{i}_t - E_t[\pi^p_{t+1}] - \rho),$$

but $\hat{y}_t \neq \hat{c}_t$ because

$$Y_t = C_t + G_t H_t.$$

- NKPC:

$$\pi^p_t = \beta E_t[\pi^p_{t+1}] - \lambda_p\{(a_t - \alpha \hat{n}_t) - [(1-\Phi)(\hat{w}_t - \hat{p}_t) + \Phi \hat{b}_t]\}.$$

- Monetary policy (Taylor rule):

$$\hat{i}_t = \rho + \phi_\pi \pi^p_t + \phi_y \hat{y}_t + \nu_t.$$

So...

- We are almost done ....

  .. except for wages and the participation decision.
- We consider two wage setting regimes.
  - Flexible wages: Nash bargaining every period.
  - Fixed wages: Calvo fairy allows Nash bargaining. (A bit like Erceg, Henderson, and Levin 2000.)

Flexible wages

- Period-by-period Nash bargaining

$$\max_{W_t(j)} \ \mathcal{S}_t^H(j)^{1-\xi}\mathcal{S}_t^F(j)^{\xi}$$

where $\mathcal{S}_t^H(j) \equiv \mathcal{V}_t^N(j) - \mathcal{V}_t^U(j)$ is the present-value surplus for a marginal worker and $\mathcal{S}_t^F(j)$ is the present-value marginal surplus for the firm (or color) j.The solution will end up being

$$\frac{W_t(j)}{P_t} = \xi MRS_t + (1-\xi)MRPN_t(j),$$

where

$$MRS_t = \frac{U_2(C_t, L_t)}{U_1(C_t, L_t)} = \chi C_t L_t^{\varphi}.$$

Flexible wages

- The value of having one employed worker:

$$\mathcal{V}_t^N(j) = \frac{W_t(j)}{P_t} - MRS_t + E_t\{\Lambda_{t,t+1}((1-\delta)\mathcal{V}_{t+1}^N(j) + \delta\mathcal{V}_{t+1}^U)\}$$

- The value of sending one worker to unemployment:

$$\mathcal{V}_t^U(j) = x_t \int_0^1 \frac{H_t(z)}{H_t}\mathcal{V}_t^N(z)dz + (1-x_t)(-\psi MRS_t + E_t\{\Lambda_{t,t+1}\mathcal{V}_{t+1}^U\})$$

- It is optimal to send to unemployment (participate) until $\mathcal{V}_t^U = 0$. Thus the optimal participation condition is

$$(1-x_t)\psi MRS_t = x_t \int_0^1 \frac{H_t(z)}{H_t}\mathcal{S}_t^H(z)dz,$$

  where $\mathcal{S}_t^H(j) \equiv \mathcal{V}_t^N - \mathcal{V}_t^U = \mathcal{V}_t^N$.
- On the firm side,

$$\mathcal{S}_t^F(j) = MRPN_t(j) - \frac{W_t(j)}{P_t} + (1-\delta)E_t\{\Lambda_{t,t+1}\mathcal{S}_{t+1}^F(j)\}.$$

Fixed wages

- Calvo fairy visits firm (or color) j with probability $(1-\theta_w)$.
- A new worker's wage has to be the same as the existing wage at firm (or color) j.
- Long story short, Nash bargaining will deliver

$$\pi_t^w = \beta(1-\delta)E_t[\pi_{t+1}^w] - \lambda_w(\hat{\omega}_t - \hat{\omega}_t^{tar}),$$

  where $\pi_t^w = w_t - w_{t-1}$, w_t is the average log nominal wage, $\omega_t = w_t - p_t$ is the average log real wage, and ω_t^{tar} is the "target (flexible price) log real wage." λ_w is a constant and a function of parameters, in particular θ_w.
- The participation condition (again, long story short) will become

$$\hat{c}_t + \varphi \hat{l}_t = \frac{1}{1-x}\hat{x}_t + \hat{g}_t - \Xi \pi_t^w.$$

  $\Xi = 0$ in the flexible wage case.

A few words on calibration and results

- Most part of the parameters can be set in the standard way.
- The two parameters that always come up as controversial is χ (disutility of labor) and ξ (bargaining power of firm).
- In this type of model, the search cost ψ is also an important parameter.
- These three are interrelated through (i) bargaining equation and (ii) participation condition, when we target participation rate and unemployment rate.
- This is because if ξ and ψ is fixed, χ determines the wage (because it affects the household's surplus) and thus the profit and unemployment through the hiring decision; and if ξ and χ is fixed, ψ determines the participation rate through participation condition.
- In general a low ξ (low share of firm) would imply a low χ (large total surplus) and a high ψ.

A few words on calibration and results

- How do χ and ψ affect the dynamics?
- A high value of χ makes the wage "naturally" more sticky, and make the profit more volatile $\rightarrow$ more amplification of unemployment. (Hagedorn and Manovskii 2008)
- A high value of ψ makes the participation decision more sensitive to the movement of job finding probability. (Shimer 2011)
- To get the cyclicality of unemployment right, one needs to make it attractive for firms to hire workers more during booms.
- To get the cyclicality of participation right, one needs to make participation attractive for households during booms.