The Basic of the DMP Model

Toshihiko Mukoyama

Federal Reserve Board

August 2013

The views expressed here are solely the responsibility of the authors' and should not be interpreted as reflecting views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System.

Why Diamond-Mortensen-Pissarides (DMP) model?

- Explicit treatment of unemployment (want to work but can't find a job).
- Determination of flows rather than (/in addition to) stocks.
- Rich policy implications, in particular on the labor demand side.
- Textbook: Pissarides (2000), but I will work with discrete time.

The fundamental assumption: matching function

- The matching function:

$$\text{The number of match (hire) at } t+1 = M(v_t, u_t).$$

  The right-hand side is the matching function, whose inputs are unemployment u_t and vacancy v_t.
  - Note: In Galí's HB paper formulation, the left-hand side is the number of match (hire) at t.
- The matching functions are usually assumed to be constant returns to scale, and let the probability of a worker finding a job be

$$\frac{M(v_t, u_t)}{u_t} = M\left(\frac{v_t}{u_t}, 1\right) = M(\theta_t, 1) = p(\theta_t).$$

  Let the probability of a vacancy finding a worker is

$$\frac{M(v_t, u_t)}{v_t} = M\left(1, \frac{u_t}{v_t}\right) = M\left(1, \frac{1}{\theta_t}\right) = q(\theta_t).$$

The fundamental assumption: matching function

- The unemployment dynamics, assuming that there are only employment and unemployment (and total population 1)

$$u_{t+1} = (1 - p(\theta_t))u_t + \sigma(1 - u_t),$$

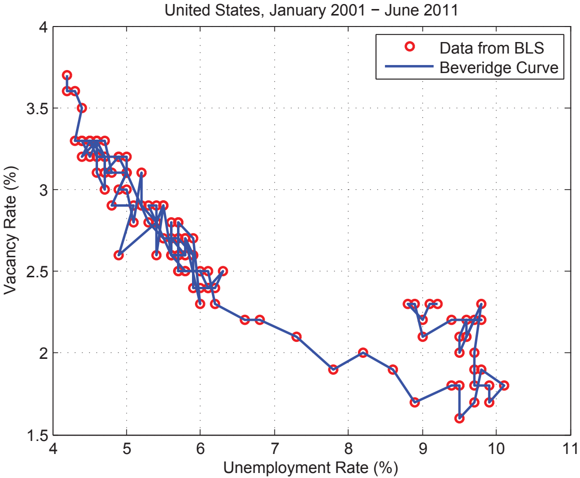
where σ is the separation probability (exogenous: see Mortensen-Pissarides 1994 for how to endogenize this).

- The steady-state unemployment:

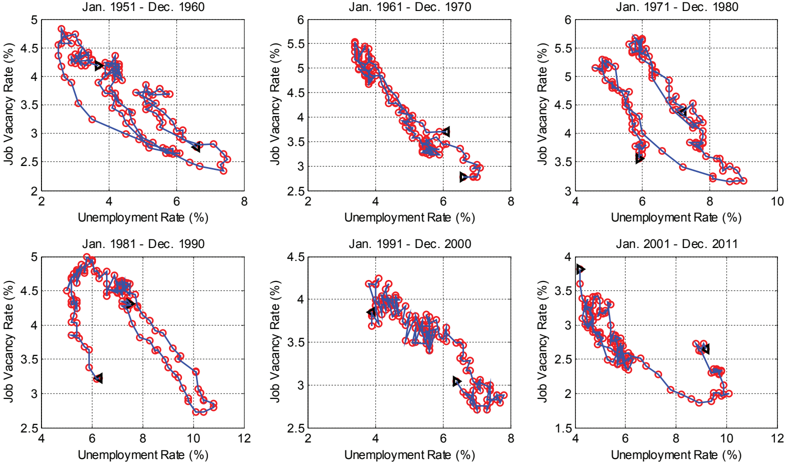
$$u = \frac{p(v/u)}{\sigma + p(v/u)}.$$

Thus this approach embeds a negative relationship between v and u ("Beveridge curve"), as long as σ doesn't move around much and $M(\cdot, \cdot)$ is stable.

Beveridge curves



Beveridge curves



The basic DMP model (Pissarides 1985)

- Q: how is v_t (or θ_t) determined in equilibrium? The firm's decision depends on
  - Cost: cost of hiring a worker (similar to investment/adjustment cost)
  - Benefit: future profit from hiring a worker.
- We'll clearly see this in the "optimal hiring" equation

$$\kappa = \beta q(\theta_t) E[J_{t+1}]$$

where κ is the cost of posting one vacancy, β is discount factor, and J_{t+1} is the present value of hiring a worker. This can be rewritten as

$$\frac{\kappa}{q(\theta_t)} = \beta E[J_{t+1}].$$

The RHS is the cost of hiring one worker, and the LHS is the benefit.

The basic DMP model

- J_t satisfies

$$J_t = z_t - w_t + \beta(1-\sigma)E[J_{t+1}],$$

where z_t is the product per worker and w_t is the real wage.

- Using this, the optimal hiring equation can be rewritten as

$$\frac{\kappa}{q(\theta_t)} = \beta E\left[z_{t+1} - w_{t+1} + (1-\sigma)\frac{\kappa}{q(\theta_{t+1})}\right].$$

- Later we'll detail (in Galí's paper) how the wages are determined—but once it is done, this will look like

$$\frac{\kappa}{q(\theta_t)} = \beta E\left[(1-\gamma)(z_{t+1} - b) + (1-\sigma-\gamma p(\theta_{t+1}))\frac{\kappa}{q(\theta_{t+1})}\right],$$

where b is the worker's value of not working and γ is a parameter in Nash bargaining. This is one equation, one unknown. After log-linearization, the solution will look like

$$\hat{\theta}_t = \Omega \hat{z}_t,$$

where Ω is a function of parameters. Done!

The basic DMP model

- This is really all—the rest of the model is basically used for "properly" formulating the wage setting (Nash bargaining).
- I don't really view it as very important (as long as it is explicitly formulated) because the wage setting rule can be anything.
- Please see the separate note for the complete formulation.

On business cycles (Shimer 2005, Gertler-Trigari 2009)

- Going back to optimal hiring equation

$$\frac{\kappa}{q(\theta_t)} = \beta E\left[z_{t+1} - w_{t+1} + (1-\sigma)\frac{\kappa}{q(\theta_{t+1})}\right],$$

  one can see how it generates business cycle fluctuations. The easy special case is when $\sigma = 1$:

$$\frac{\kappa}{q(v_t/u_t)} = \beta E[z_{t+1} - w_{t+1}].$$

  In booms, $E[z_{t+1} - w_{t+1}]$ goes up, so v_t has to go up until the LHS goes up sufficiently.
- "Shimer puzzle": v_t doesn't move as much as in the data.
  - The RHS solutions
    - Make w_{t+1} more sticky. Then $E[z_{t+1} - w_{t+1}]$ will move more.
    - Make β cyclical. (SDF for the firm, actually.)
  - The LHS solutions
    - Make the LHS less procyclical (note: $\kappa/q(v/u)$ is procyclical for a given v). E.g. "cost per hire" (Pissarides 2009)
    - Make κ countercyclical. (Financial frictions etc.)

On inflation (Krause and Lubik 2007)

- In the competitive market (if the worker is hired one period ahead),

$$E[z_{t+1}] = E[w_{t+1}].$$

  In other words, MPL=MC.

- Here, the optimal hiring equation looks like

$$\begin{aligned} E[z_{t+1}] &= E\left[w_{t+1} + J_t - (1-\sigma)J_{t+1}\right] \\ &= E\left[w_{t+1} + \frac{\kappa}{\beta q(\theta_t)} - \frac{(1-\sigma)\kappa}{\beta q(\theta_{t+1})}\right] \end{aligned}$$

  Here, the MC part is like "user cost of capital"—it includes the "depreciation" (or "capital loss") term. This extra part can (potentially) play a role in the inflation dynamics once this is embedded in a New Keynesian model.

One more note

- In the standard macro-labor literature, it is common to work with $\theta_t = v_t/u_t$. Galí works with $x_t = h_{t+1}/u_t$ (in his timing h_t instead of h_{t+1} but anyway), where $h_{t+1} = M(v_t, u_t)$ is the total hire.
- Of course, there is one-to-one relationship between x_t and θ_t. In fact, $x_t = p(\theta_t)$, so x_t is an increasing function of θ_t.
- Then, the cost of hiring one worker $\kappa/q(\theta_t)$ is

$$\frac{\kappa}{q(\theta_t)} = \frac{\kappa}{q(p^{-1}(x_t))} = G(x_t),$$

where $G(\cdot)$ is an increasing function. The optimal hiring equation for a small firm can be rewritten as

$$G(x_t) = \beta E\left[z_{t+1} - w_{t+1} + (1-\sigma)G(x_{t+1})\right].$$

In sum

- This formulation treats employment in a similar way as investment. Another way of looking at the search-matching friction (from the firm's side) is that it is a linear adjustment cost that varies endogenously. Yet another way of looking at this is that having a worker is like having intangible capital.
- At the frontier, there are a lot of criticisms, alternative formulations, and modifications being done, but at this point the most natural approach to formulating unemployment seems to be using this framework with some wage stickiness.
- As I demonstrated, this model ends up being "one equation, one unknown." We can use this as a "module" for a larger model where unemployment is just a part of it. For example, in Galí's paper we go over, there are concave utility, sticky prices, sticky wages, monetary policy, and labor force participation decisions. I will try to clarify where the integration of this model interacts with the original part and where it can be considered as a "separate part."