Chapter 2: A Framework for Macroeconomics

May 2026

Outline

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Introduction and Goals

Three Goals of the Chapter

1. Review major macroeconomic time series with a focus on **long-run behavior**, primarily using US data
2. Gradually introduce the **basic framework**—the neoclassical growth model—that will be the core tool throughout the textbook
    - Each graph interpreted from the perspective of this framework
    - Hard, if not impossible, to account for the data without it
3. Serve as a **stepping stone** into the rest of the text: preview of topics and chapters

Three Goals of the Chapter

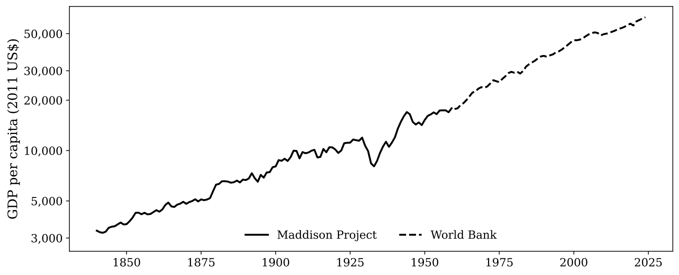
Key features of the framework:

- Goes back to Solow's growth model, then adds conscious choices (saving, labor supply, technology development)
- Microeconomic approach: explicit optimization, cross-sectional data
- No presumption of perfect markets: emphasis on identifying specific frictions
- **Quantitative**: account for magnitudes, not just qualitative features

Output Grows Steadily

Fact 1: Output Per Capita Grows at a Roughly Constant Rate

GDP per capita in the US (log scale), 1840–2018



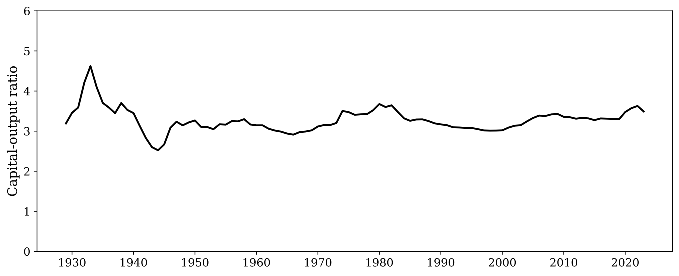
- Almost constant growth over more than a century ($\approx 1.86\%$/year; $R^2 > 0.98$)
- Swings (including the Great Recession) are barely visible from a bird's-eye view

Goal: "Account" for output growth by examining the production side—how basic inputs and their prices have evolved.

Resources Behind Output—and Their Prices

Fact 2: The Capital-Output Ratio Is Roughly Constant

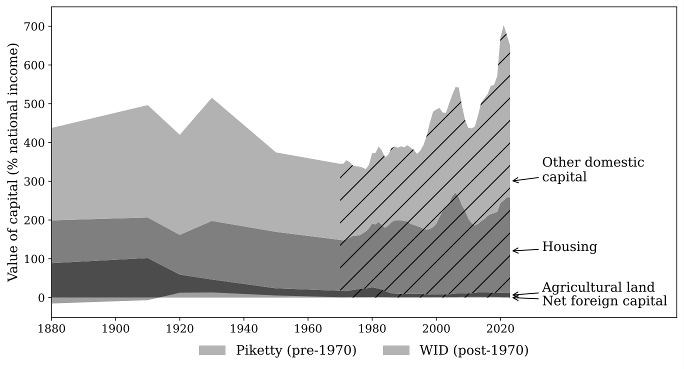
Capital-output ratio in the US, 1929–2022



- Clear stability at a value of around 3
- Jaggedness during the Great Depression, but stable thereafter
- This had puzzled economists: seemed to suggest a rigid technology with no role for labor
- Solow (1956) showed this constancy is natural in a neoclassical framework

Wealth-Output Ratio

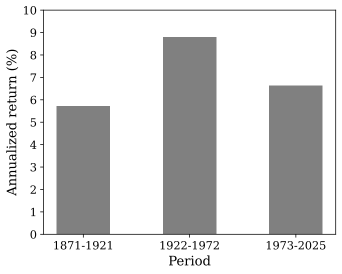
Wealth-output ratio in the US



- Broader interpretation of capital: wealth (includes land, housing)
- Also shows marked stability, though total is 4–5 rather than 3
- Large changes in **composition**

Fact 3: The Return on Capital Is Roughly Constant

Return on capital—annualized stock market returns, three periods

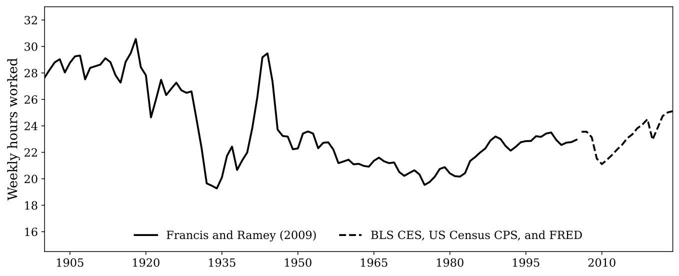


- Under competitive markets: stock-market returns ≈ marginal cost of investment = price of capital
- No strong trend across three long periods
- Short-run fluctuations are large (due to asset valuations), so longer-run averages are more appropriate

User cost of capital (Hall and Jorgenson, 1969): interest rate + depreciation + fall in value. Also stationary.

Fact 4: Hours Worked Per Capita Are Roughly Constant (Postwar)

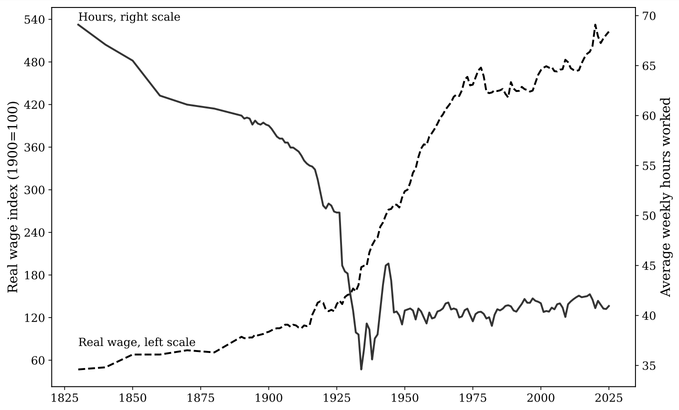
Average weekly hours worked in the US (age 14+)



- Since the beginning of the 20th century: hours fell from $\sim$28 to $\sim$23 hours/week
- Since the end of WWII: hours look stable
- "US hours are stationary" is accurate for the **postwar** period
- Large departures during Great Depression and WWII

Fact 5: Wages Have Risen at or above Output Growth Rate

Average weekly hours worked in US manufacturing, and real wages, 1830–2023



- Clear downward trend in hours over the long time period
- Wages have risen dramatically

A Neoclassical Picture Emerges

Solow's Interpretation

The data on capital and output had puzzled economists. Solow (1956) noted:

- Steady rise in output; equally steady rise in capital; stationary (or declining) hours
- Price of capital stationary; wages trending upward
- **Natural interpretation**: Labor has become more expensive relative to capital $\Rightarrow$ firms use less labor relative to capital
- Capital-labor ratio has risen at the rate of output growth

Why hasn't the return to capital fallen? Technological change directed toward labor (making labor more productive) would increase the value of capital on the margin. But a **neoclassical** production function (decreasing marginal returns) ensures that as K/L rises, the marginal value of capital falls back. These two forces offset, keeping the return to capital stationary.

The Aggregate Production Function

Solow posited that aggregate output is generated by:

$$y_t = F_t(k_t, \ell_t)$$

- k_t: aggregate capital per capita; ℓ_t: hours worked per capita
- F_t: production function that may shift over time (technological progress)
- CRS: doubling inputs doubles output
- Neoclassical: decreasing marginal returns to each input

The Cambridge Capital Controversy (1950s): Joan Robinson and Piero Sraffa (Cambridge, UK) vs. Samuelson and Solow (MIT). Whether an aggregate production function can exist rigorously when output aggregates many goods. Existence conditions are knife-edge. But **usefulness** of the aggregate production function is well-established empirically—Cambridge, Mass. "won" on pragmatic grounds.

Growth Accounting

Solow's Growth Accounting: Derivation

Assumptions: (i) $y(t) = F(k(t), \ell(t), t)$, CRS, neoclassical; (ii) perfect competition for inputs.

Differentiating:

$$dy = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial k} dk + \frac{\partial F}{\partial \ell} d\ell$$

Dividing by y, using profit maximization ($r = \partial F / \partial k$, $w = \partial F / \partial \ell$), and letting $1 - \alpha$ denote labor's share, α capital's share (from CRS $\Rightarrow$ zero pure profits):

$$\frac{dy}{y} = \underbrace{\frac{\partial F}{\partial t} \frac{dt}{F}}_{\text{Solow residual}} + \alpha \frac{dk}{k} + (1 - \alpha) \frac{d\ell}{\ell}$$

Output growth = weighted sum of input growth rates + residual.

TFP and Labor-Augmenting Technology

Hicks-neutral (TFP): $F(k, \ell, t) = zF(k, \ell)$

$$\frac{dy}{y} = \frac{dz}{z} + \alpha \frac{dk}{k} + (1-\alpha)\frac{d\ell}{\ell}$$

z is total factor productivity: a common factor multiplying both inputs.

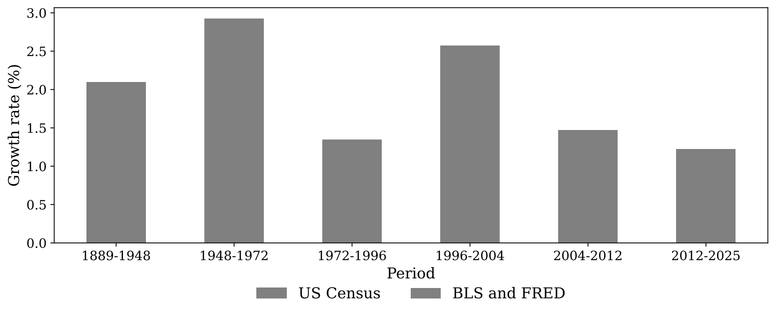
Labor-augmenting (Harrod-neutral): $F(k, \ell, t) = F(k, z\ell)$

$$\frac{dy}{y} = (1-\alpha)\frac{dz}{z} + \alpha \frac{dk}{k} + (1-\alpha)\frac{d\ell}{\ell}$$

Key result (Uzawa, 1961): For balanced growth in a model with a limited input (labor hours constant), technological change **must** be labor-augmenting. This is not a choice but a **necessity**.

Labor Productivity

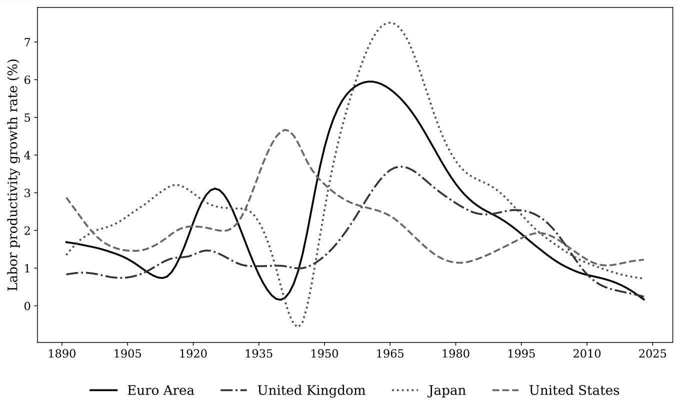
Labor productivity in the US, sub-periods



- Average growth rate: a little below 2% per year
- Significant ups and downs across sub-periods
- Currently in a “down” period

Labor Productivity: Cross-Country

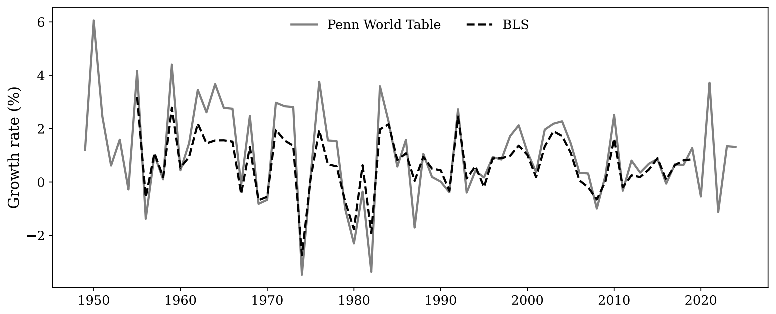
Labor productivity for a selection of countries (HP-filtered)



- Stable, positive labor productivity growth across countries
- Hovering around 2–2.5% per year
- Similar patterns across advanced economies

Total Factor Productivity

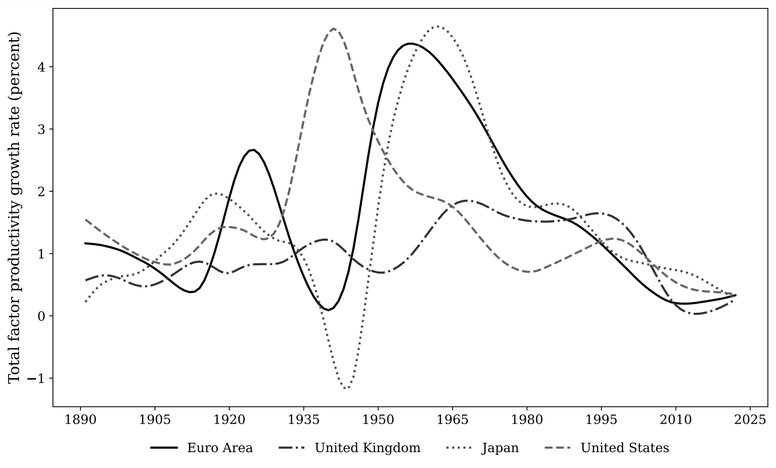
TFP in the US, two measures



- TFP growth at a little below 2% per year
- Significant movements up and down

Total Factor Productivity

Historical TFP for a broader set of countries (HP-filtered)



- Similar patterns across countries
- Smoothed data shows long-run trends more clearly

The Dynamic System

Capital Accumulation

Tomorrow's capital = today's capital + investment − depreciation:

$$k_{t+1} = (1-\delta)k_t + i_t$$

Dividing by y_{t+1}, letting γ_t denote net growth rate of output:

$$\frac{k_{t+1}}{y_{t+1}}(1+\gamma_t) = (1-\delta)\frac{k_t}{y_t} + \frac{i_t}{y_t}$$

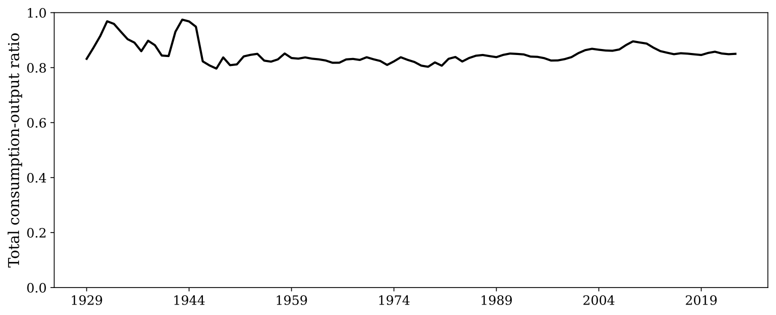
Balanced growth path: If k, i, y all grow at the same constant rate γ and $i_t/y_t = s$ (constant):

$$\frac{k}{y} = \frac{s}{\gamma+\delta}$$

Numerical check: $\delta \approx 0.08$, $\gamma \approx 0.02$, $s \approx 0.30 \Rightarrow k/y = 3$ ✓

The Consumption-Output Ratio

Ratio of total consumption to output



- Large movements early in the 20th century
- Thereafter: small movements
- $1 - c/y \approx$ investment-output ratio (net exports ≈ 0 for US)
- Overall: assumption of a **constant saving rate** is a good first approximation

The Solow Model: Convergence

Solow's dynamic system (with labor-augmenting technology, constant ℓ, constant saving rate s):

$$k_{t+1} = sF(k_t, (1+\gamma)^t \ell) + (1-\delta)k_t$$

Variable transformation: $\tilde{k}_t \equiv k_t/(1+\gamma)^t$ (stationary if k grows at rate γ):

$$(1+\gamma)\tilde{k}_{t+1} = sF(\tilde{k}_t, \ell) + (1-\delta)\tilde{k}_t$$

Unique steady state: $\bar{\tilde{k}}$ solves $(1+\gamma)\bar{\tilde{k}} = sF(\bar{\tilde{k}}, \ell) + (1-\delta)\bar{\tilde{k}}$.

Convergence: Under neoclassical conditions, for *any* initial k_0, the economy converges to the balanced growth path.

- When k low: high marginal productivity $\Rightarrow$ fast accumulation
- When k high: low marginal productivity $\Rightarrow$ slow accumulation
- This de-mystifies why $k/y \approx 3$: deviations are self-correcting

From Solow to the Workhorse Model

The convergence property makes this framework the **workhorse model** for macroeconomics:

- Business cycles: add stochastic shocks to the dynamic system
- Supply shocks or demand shocks—propagation differs, but all paths eventually return toward balanced growth
- Convergence is ensured by the neoclassical production function

Unresolved issue: Some assumptions are mere mechanical descriptions:

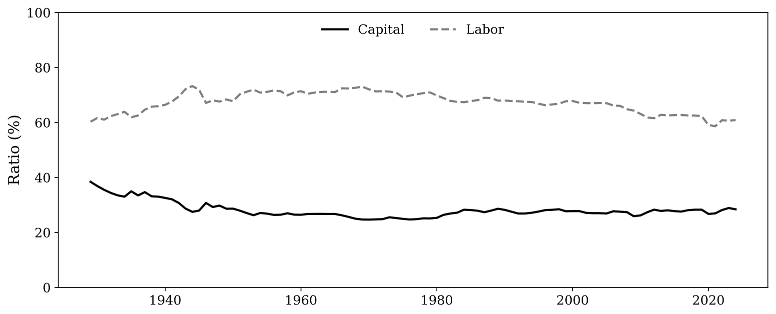
- Investment is a constant fraction s of output
- Hours worked are constant at ℓ

These should be the results of **conscious choices** by households. We need to "rationalize" s and ℓ based on utility maximization.

Factor Shares

Fact 6: Factor Shares Are Roughly Constant

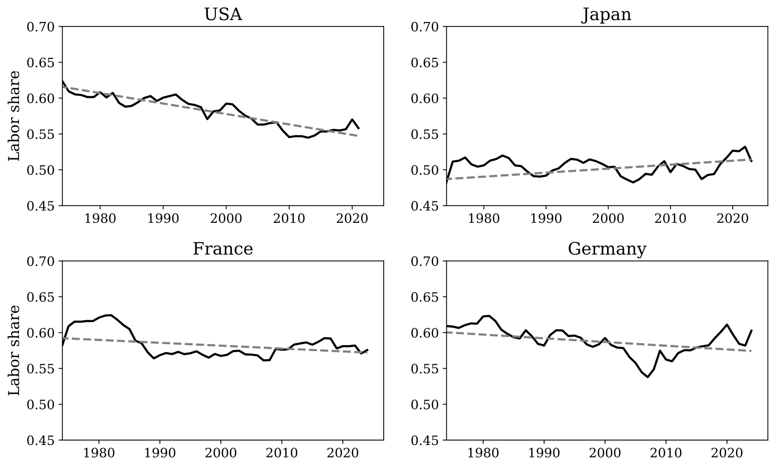
US factor shares over time



- On balanced growth: rk grows at the rate of output (r constant, k grows at γ); $w\ell$ grows at the rate of output (w grows at γ, ℓ constant)
- Therefore shares should be constant $\Rightarrow$ confirmed by the data
- Remarkable constancy over the full sample

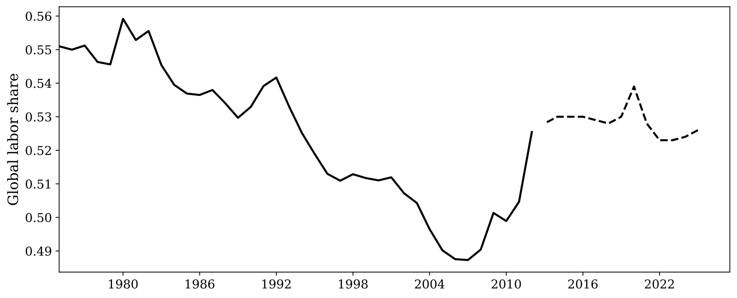
The Recent Decline in the Labor Share

Labor share for US, Japan, France, Germany



The Recent Decline in the Labor Share

Global labor share



- Restricting the time interval: noticeable **downward trend** in recent decades
- Observed in US, Japan, France, Germany, and globally
- Subject to much scrutiny and research
- For now: maintain stylized fact as “close to constant over time”

The Cobb-Douglas Production Function

Virtually all applied macro studies use:

$$F(k, \ell) = Ak^{\alpha}\ell^{1-\alpha}$$

- α: constant capital share; $1 - \alpha$: constant labor share
- Under perfect competition: $F_k k/F = \alpha$ (independent of k and ℓ)
- Factor shares constant regardless of business-cycle position
- Although shares are not literally constant, movements are relatively minor
- Cobb-Douglas generates a decent approximation—hence its wide use

Summing Up the Stylized Facts

The Seven Kaldor Facts

1. Output per capita has grown at a **roughly constant rate**
2. The capital-output ratio has remained **roughly constant**
3. The consumption-output ratio has been **roughly constant**
4. The wage rate has grown at a roughly constant rate **equal to the growth rate of output**
5. The real interest rate has been **roughly constant** (over longer periods)
6. Labor share of output has remained **roughly constant**
7. Hours worked per capita have been **roughly constant** (postwar)

These facts are consistent with a neoclassical framework: CRS production function with labor-augmenting technical change, decreasing marginal products, constant labor supply, constant depreciation rate, constant saving ratio.

Rationalizing Saving and Labor Supply

Time and People

Time: Discrete (mostly) or continuous; infinite horizon (finite T creates non-stationarity; T is virtually irrelevant for decisions far from T).

People: Utility-maximizing agents.

- "Rationalize" observed choices as optimal given well-defined problems
- Allows welfare statements and normative policy comparisons
- Baseline: **representative agent, dynastic** (lives forever = cares about offspring)
- Also common: life-cycle models, overlapping generations, heterogeneous agents

The Euler Equation

Preferences: Time-additive: $u(c_t) + \beta u(c_{t+1})$

- Same u in both periods $\Rightarrow$ consumption in both periods are normal goods
- Concavity $\Rightarrow$ consumption smoothing
- $\beta < 1$: impatience (or probability of death)

Setting MRS = relative price (gross real interest rate R_{t+1}):

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = R_{t+1}$$

Equivalently:

$$u'(c_t) = \beta R_{t+1} u'(c_{t+1})$$

Interpretation: Marginal utility loss of saving one unit today = discounted marginal utility gain from consuming R_{t+1} units tomorrow.

Balanced Growth Requires Power Utility

Key question: Can balanced growth (constant consumption growth, constant R) be consistent with the Euler equation?

Result: Balanced growth arises in equilibrium **if and only if** the utility function is a power function:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \qquad \sigma > 0$$

The case $\sigma \to 1$ yields $u(c) = \log c$.

Why? With power utility: $u'(c_t)/u'((1+\gamma)c_t)$ is constant. So a constant R is consistent with constant consumption growth.

"Only if": For any other u, saving rates would go to 0 or 1 over time as the economy grows. Balanced growth as observed in the data would not be possible.

$\Rightarrow$ Power utility (CRRA) is used in almost all applications.

Labor vs. Leisure

Add leisure to utility: $u(c, 1-\ell)$ where $\ell + l = 1$.

MRS = wage: $u_2(c_t, 1-\ell_t)/u_1(c_t, 1-\ell_t) = w_t$.

Requirement: Constant hours along balanced growth path where c and w grow at rate γ.

Result: Balanced growth with constant hours **if and only if**:

$$u(c, 1-\ell) = \frac{(c \cdot v(1-\ell))^{1-\sigma} - 1}{1-\sigma}$$

where $v(l)$ is strictly increasing and $c \cdot v(l)$ is strictly quasiconcave.

Intuition: At higher wages, the substitution effect (work more) exactly cancels the income effect (enjoy more leisure).

Allowing for Declining Hours

The longer-run data (and cross-country evidence) suggest hours fall slowly ($\sim$1/3 % per year).

Result: Balanced growth with hours declining at a constant rate **if and only if**:

$$u(c, 1-\ell) = \frac{(c^{1-\nu} g(c^{\nu}\ell))^{1-\sigma} - 1}{1-\sigma}$$

with $\nu > 0$ and $g(\cdot)$ decreasing.

- Hours grow at $(1+\gamma)^{-\nu}$; consumption at $(1+\gamma)^{1-\nu}$
- $\nu = 0$: collapses to the constant-hours case
- $\nu > 0$: income effect slightly dominates substitution effect
- Interpretation: at lower income, people work more because consumption is more important

Preview: The Rest of the Text

Part I: Core Methods (Chapters 3–10)

- **Chapter 3**: The Solow model in detail; convergence
- **Chapter 4**: Dynamic optimization (Lagrangian, Bellman, contraction mapping)
- **Chapter 5**: Dynamic competitive equilibrium; welfare theorems
- **Chapter 6**: Welfare analysis; CES price index
- **Chapter 7**: Uncertainty; stochastic dynamic programming
- **Chapter 8**: Empirical strategies (calibration, estimation, VARs, identification)
- **Chapter 9**: Continuous-time analytical techniques
- **Chapter 10**: Computational tools (VFI, linearization, Blanchard-Kahn)

These methods are core material used repeatedly throughout the applied chapters.

Part II: Applied Chapters (11–25)

- **Ch 11**: Consumption (MPC, incomplete markets, heterogeneous agents)
- **Ch 12**: Labor supply (intensive/extensive margins, wage elasticities)
- **Ch 13**: Growth (cross-country, endogenous growth, innovation)
- **Ch 14**: Real business cycles (Kydland-Prescott, DSGE)
- **Ch 15**: Government (taxation, optimal policy, time consistency)
- **Ch 16**: Asset prices (consumption CAPM, equity premium puzzle)
- **Ch 17**: Money (fiat money, inflation, exchange rates)
- **Ch 18**: Nominal frictions (New Keynesian model, monetary policy)

Part II: Applied Chapters (continued)

- **Ch 19**: Credit market frictions (financial accelerator, Great Recession)
- **Ch 20**: Frictional labor markets (DMP model, Shimer puzzle)
- **Ch 21**: Inequality (wages, wealth, HANK model)
- **Ch 22**: Heterogeneous firms (misallocation, granularity, markups)
- **Ch 23**: International macroeconomics (open-economy business cycles)
- **Ch 24**: Sovereign debt and default risk
- **Ch 25**: Sustainability (climate change, integrated assessment, energy)

Virtually every chapter builds directly on the market version of the neoclassical growth model developed in this chapter and formalized in Chapters 3–5.

Summary

Key Takeaways

1. The Kaldor facts (constant growth rate, constant k/y, constant c/y, rising wages, stable r, stable factor shares, stable hours) motivate the **neoclassical growth framework**
2. **Growth accounting**: Solow residual is the dominant source of growth
3. **Convergence**: neoclassical production function ensures the economy returns to its balanced growth path
4. **CRRA utility** is the unique preference specification consistent with balanced growth (constant saving rate, constant or slowly declining hours)
5. The **Euler equation** is the central optimality condition connecting consumption, saving, and interest rates
6. The recent **decline in the labor share** is an active area of research and a departure from the classic Kaldor facts