

Chapter 15: Government and Public Policies

Chapter authors: Marina Azzimonti, Jonathan Heathcote, and Kjetil Storesletten

April 2026

Outline

1. Introduction
2. Public Finance: Data Overview
3. Effects of Distortionary Taxes
 - 3.1 Long-Run Distortions
 - 3.2 Tax Incidence
 - 3.3 Tax Reform
 - 3.4 The Laffer Curve
 - 3.5 Theories of Government Spending
4. Government Debt and Ricardian Equivalence
5. Ramsey Taxation
 - 5.1 Time Consistency
6. Debt and Pensions with Overlapping Generations
7. Taxes and Transfers as Redistribution
 - 7.1 A Macro Model of Progressivity
8. Summary

Introduction

Why Does the Government Intervene?

Under the First Welfare Theorem (complete, competitive markets, no externalities): competitive equilibrium allocations are Pareto optimal.

Three main rationales for government intervention:

1. **Public goods & externalities**

- National defense: no way to restrict enjoyment
- Education, healthcare: large positive externalities

2. **Incomplete markets & private information frictions**

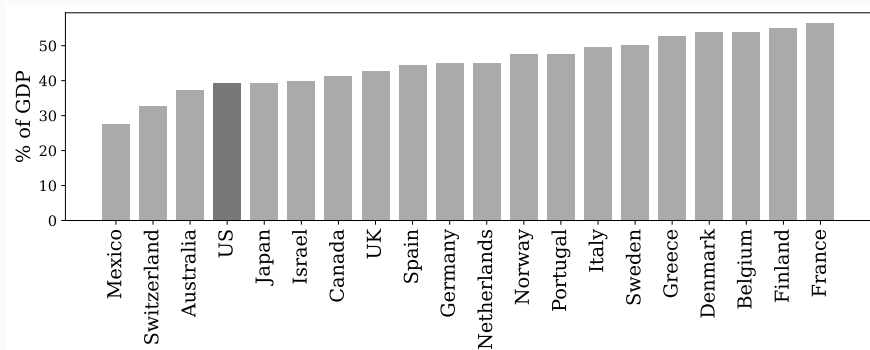
- Hard to buy private unemployment insurance
- Financial crises: government bailouts, tax cuts

3. **Redistribution**

- Market economies generate substantial inequality
- Taxing the rich, transferring to the poor

The Fundamental Trade-off

Government spending (avg. 2010–2019), % of GDP



- High public consumption/transfers necessitates **high taxes**
- High taxes \Rightarrow distortions to private choices \Rightarrow lower efficiency
- Societies differ in how they view the trade-off between:
 - Benefits of equity / public good provision
 - Efficiency costs of distortionary taxes

Public Finance: Data Overview

The Government Budget Constraint

$$\underbrace{G_t + T_t + i_t B_{t-1}}_{\text{Expenditures}} = \underbrace{Rev_t + B_t - B_{t-1}}_{\text{Revenue + Borrowing}}$$

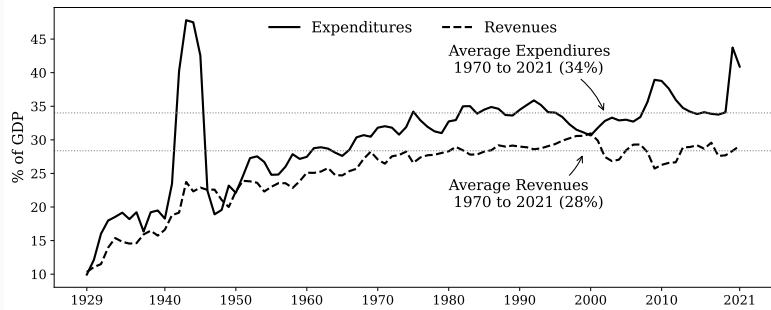
- G_t : government consumption and investment (education, defense, parks)
- T_t : transfers to individuals and corporations (food stamps, agricultural subsidies, Social Security)
- $i_t B_{t-1}$: interest payments on outstanding debt
- Rev_t : tax revenues (income, payroll, sales, excise, corporate, property)
- $B_t - B_{t-1}$: net borrowing

Deficit: expenditures (incl. interest) $>$ revenues \Rightarrow debt rises.

Surplus: expenditures $<$ revenues \Rightarrow debt falls.

US Revenues and Expenditures

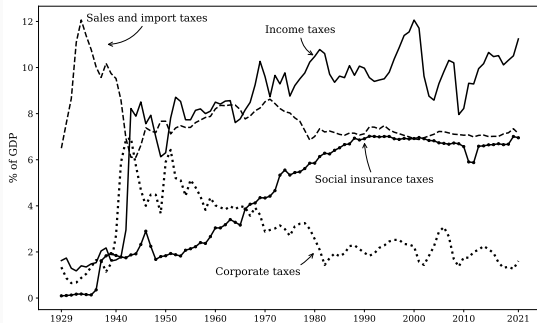
Revenues and outlays, as percentages of GDP, 1929–2021



1. Both revenue and spending exhibit **upward trends** (1929–1970), then stabilize
2. Expenditures tend to exceed revenues: typically runs deficits
 - 1970–2021 avg: expenditures $\approx 34\%$, revenues $\approx 28\%$
3. Expenditures **jump** during wars and recessions (WWII, Great Recession, COVID-19), while revenues typically fall

Sources of Tax Revenue

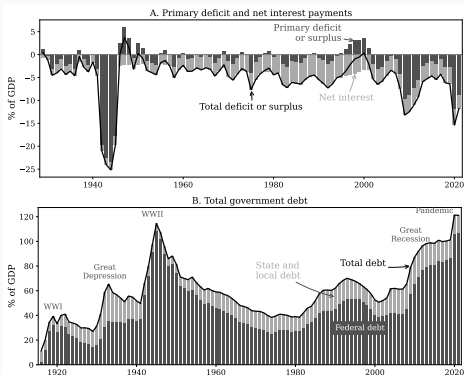
Taxes by category, as percentages of GDP, 1929–2021



- **Income taxes:** rose significantly, now $\approx 11\%$ of GDP
- **Social insurance taxes:** rose significantly, now $\approx 7\%$ of GDP
- **Sales and import taxes:** relatively constant at $\approx 8\%$
- **Corporate taxes:** declined substantially, now $< 2\%$ of GDP

Important: The theoretical distinction (labor income tax, capital income tax, etc) does not map cleanly into empirical categories.

Deficits, Interest Payments, and Debt



- U.S. borrowed heavily during wars (WWII) and recessions
- **Tax smoothing** visible: revenues stable, spending volatile
- Persistent deficits even outside of recessions
- Most U.S. debt issued by Federal government (states have balanced budget rules)
- Debt/GDP $\approx 100\%$ as of 2023

Debt Sustainability

Debt-to-GDP ratio evolves as:

$$b_t = b_{t-1} \frac{1 + r_t}{1 + \gamma_t} + d_t$$

where b_t = debt/GDP, r_t = real interest rate, γ_t = real GDP growth, d_t = primary deficit/GDP.

Debt rises if and only if:

$$d_t > \frac{\gamma_t - r_t}{1 + \gamma_t} b_{t-1}$$

- When $r_t > \gamma_t$ and primary deficit = 0: debt/GDP rises on its own
- When $r_t < \gamma_t$: debt/GDP falls even with moderate deficits
- The value of r relative to γ is critical for public finance dynamics

Is US Debt Sustainable?

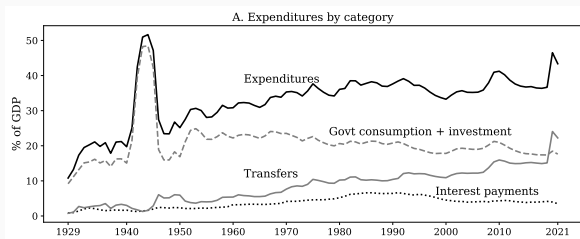
US in 2023: $b \approx 1.0$, $\gamma \approx 0.03$, $r \approx 0.02$

- Maximum primary deficit consistent with stable debt:
 $\frac{0.03-0.02}{1.03} \times 1.0 \approx 1\%$ of GDP
- Actual primary federal deficit: $\approx 3\%$ of GDP (CBO projects $\sim 3\%$ over next decade)
- \Rightarrow Debt-to-GDP ratio will **continue to rise**

Will debt explode? With constant $r < \gamma$, any size primary deficit is consistent with *some* stable debt ratio. E.g., $d = 0.03$ is consistent with $b = 309\%$ of GDP.

But: As debt rises, equilibrium r is likely to **rise** (investors demand higher returns). Once $r > \gamma$, stabilization requires primary *surpluses*.

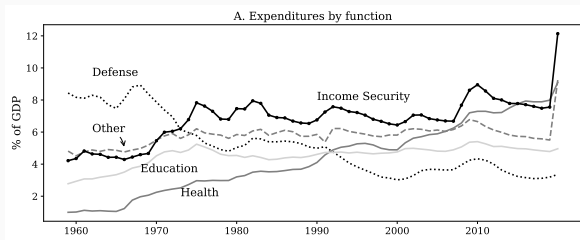
Composition of Expenditures: By Category



- Early in sample: government consumption & investment is the largest component and drives the trend
- Over time: **transfers expand significantly**, overtaking G during COVID-19
- Transfers include both redistributive and insurance programs
- Interest payments variable over time

Growth in government size coincides with creation of Social Security, unemployment insurance (Great Depression), and increased public investment (post-WWII).

Composition of Expenditures: By Function



- **Defense:** peaked at 8% of GDP in 1959, now $\approx 3\%$
- **Health** (Medicare, Medicaid): steady rise, now exceeds 8%
- **Income security** (unemployment insurance, retirement, disability, welfare): fluctuates $\approx 7\%$
 - Rises in recessions, falls in booms \Rightarrow **“automatic stabilizer”**
- **Education:** relatively constant at $\approx 5\%$

Effects of Distortionary Taxes

Neoclassical Growth Model with Taxes: Setup

- **Households:** infinitely-lived, discount factor β , utility $u(c_t, \ell_t)$, save in capital
- **Firms:** CRS production $y_t = f(k_t, \ell_t)$, competitive
- **Government:** finances G_t and transfers T_t using proportional taxes
 - $\tau_{c,t}$: consumption tax
 - $\tau_{\ell,t}$: labor income tax
 - $\tau_{k,t}$: capital income tax (net of depreciation)

Resource constraint:

$$C_t + G_t + K_{t+1} = f(K_t, L_t) + (1 - \delta)K_t$$

Government budget constraint:

$$G_t + T_t = \tau_{c,t}C_t + \tau_{\ell,t}w_tL_t + \tau_{k,t}(r_t - \delta)K_t$$

Household budget constraint:

$$(1 + \tau_{c,t})c_t + k_{t+1} = (1 - \tau_{\ell,t})w_t\ell_t + k_t + (1 - \tau_{k,t})(r_t - \delta)k_t + T_t$$

A government policy is a sequence $\{\tau_{c,t}, \tau_{k,t}, \tau_{\ell,t}, G_t, T_t\}_{t=0}^{\infty}$.

No government debt for now (introduced later).

Competitive Equilibrium: Definition

Definition 15.1: A competitive equilibrium given policy $\{\tau_{c,t}, \tau_{k,t}, \tau_{\ell,t}, G_t, T_t\}_{t=0}^{\infty}$ is allocations $\{C_t, L_t, K_{t+1}\}$ and prices $\{w_t, r_t\}$ such that:

- (i) $\{C_t, L_t, K_{t+1}\}$ maximizes $\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$ subject to
- $$(1 + \tau_{c,t})c_t + k_{t+1} = (1 - \tau_{\ell,t})w_t \ell_t + k_t + (1 - \tau_{k,t})(r_t - \delta)k_t + T_t,$$
- $k_0 = K_0$, and $k_{t+1} \geq 0$
- (ii) $\{L_t, K_t\}$ solves firm profit maximization:

$$\max_{k_t, \ell_t} \{f(k_t, \ell_t) - w_t \ell_t - r_t k_t\}$$

with $\ell_t = L_t$ and $k_t = K_t$ in equilibrium

- (iii) Government budget constraint

$$G_t + T_t = \tau_{c,t}C_t + \tau_{\ell,t}w_t L_t + \tau_{k,t}(r_t - \delta)K_t$$

is satisfied at each date t

Optimal saving (Euler equation):

$$\frac{1 + \tau_{c,t+1}}{1 + \tau_{c,t}} \frac{u_1(c_t, l_t)}{u_1(c_{t+1}, l_{t+1})} = \beta [1 + (1 - \tau_{k,t+1})(r_{t+1} - \delta)]$$

Optimal labor supply:

$$-u_2(c_t, l_t) = \frac{1 - \tau_{\ell,t}}{1 + \tau_{c,t}} \cdot w_t \cdot u_1(c_t, l_t)$$

Firm FOCs: $w_t = f_2(k_t, l_t)$ and $r_t = f_1(k_t, l_t)$

How Taxes Distort Decisions

Planner's FOCs (maximize utility s.t. resource constraint):

$$\frac{u_1(C_t, L_t)}{u_1(C_{t+1}, L_{t+1})} = \beta [1 + f_1(K_{t+1}, L_{t+1}) - \delta]$$
$$-u_2(C_t, L_t) = f_2(K_t, L_t) \cdot u_1(C_t, L_t)$$

Comparing with household FOCs:

- τ_k depresses after-tax return to saving:

$$1 + (1 - \tau_k)(r - \delta) < 1 + (r - \delta)$$

- Rising τ_c ($\tau_{c,t+1} > \tau_{c,t}$) works in the same direction as τ_k
- τ_ℓ depresses return to working:

$$\frac{1 - \tau_\ell}{1 + \tau_c} w < w$$

- τ_c also depresses the return to working
- All three taxes reduce capital, labor supply, and output vs. first best

Steady-State: Capital-Labor Ratio

In steady state with constant tax rates and Cobb-Douglas

$$f(k, \ell) = k^\alpha \ell^{1-\alpha};$$

From Euler equation:

$$1 = \beta [1 + (1 - \tau_k)(r - \delta)]$$

Since $r = f_1 = \alpha(k/\ell)^{\alpha-1}$:

$$\frac{k}{\ell} = \left(\frac{\alpha(1 - \tau_k)}{\rho + \delta(1 - \tau_k)} \right)^{\frac{1}{1-\alpha}}$$

where $\rho = (1 - \beta)/\beta$ is the rate of time preference.

- Higher $\tau_k \Rightarrow$ lower $k/\ell \Rightarrow$ higher r , lower w
- τ_ℓ and τ_c have **no impact** on k/ℓ or pre-tax prices r , w in steady state

Steady-State: Labor Supply (GHH Preferences)

Greenwood-Hercowitz-Huffman (GHH) utility:

$$u(c, \ell) = \log \left(c - \frac{\ell^{1+1/\phi}}{1 + 1/\phi} \right)$$

Key property: consumption drops out of labor FOC \Rightarrow **no income effects** on labor supply.

Steady-state labor supply:

$$\ell = \left(\frac{1 - \tau_\ell}{1 + \tau_c} \right)^\phi w^\phi$$

- ϕ : elasticity of hours to after-tax wages
- Higher τ_ℓ or $\tau_c \Rightarrow$ lower ℓ
- Higher $\tau_k \Rightarrow$ lower w (via lower k/ℓ) \Rightarrow also lower ℓ

Note: With income effects (e.g., separable utility), the effect of τ_ℓ on ℓ is **ambiguous** (substitution vs. income effect).

Steady-State Output

With GHH preferences, we can solve for hours worked as a function of k/ℓ because $w = f_2 = (1 - \alpha)(k/\ell)^\alpha$. With

$$\frac{k}{\ell} = \left(\frac{\alpha(1 - \tau_k)}{\rho + \delta(1 - \tau_k)} \right)^{\frac{1}{1-\alpha}}$$

we can solve for the steady-state output

$$y = \ell \left(\frac{k}{\ell} \right)^\alpha = \left(\frac{1 - \tau_\ell}{1 + \tau_c} \right)^\phi (1 - \alpha)^\phi \left(\frac{\alpha(1 - \tau_k)}{\rho + \delta(1 - \tau_k)} \right)^{\frac{\alpha(1+\phi)}{1-\alpha}}$$

- **All three tax rates** reduce steady-state output
- Output is decreasing in each tax rate
- τ_k has an **amplified effect**: depresses k/ℓ directly, which lowers w , which in turn depresses ℓ

Tax Incidence: Worker–Capitalist Framework

With GHH preferences, the representative agent steady state is **identical** at the aggregate level to a two-type economy:

- **Workers:** rent labor, own no capital
- **Capitalists:** own and rent capital, do not work

(consumption appears in neither the SS saving FOC nor the SS labor FOC.)

With no transfer ($T = 0$):

Worker consumption:

$$c_w = \frac{1 - \tau_\ell}{1 + \tau_c} w\ell = \frac{1 - \tau_\ell}{1 + \tau_c} (1 - \alpha)y$$

Capitalist consumption:

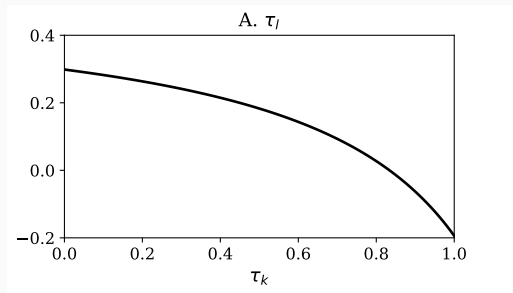
$$c_k = \frac{1 - \tau_k}{1 + \tau_c} (r - \delta)k = \frac{1 - \tau_k}{1 + \tau_c} \cdot \frac{\alpha\rho}{\rho + (1 - \tau_k)\delta} y$$

τ_ℓ directly depresses c_w ; τ_k directly depresses c_k ; τ_c hits both. All taxes indirectly depress both via lower y .

Budget-Balancing Tax Pairs

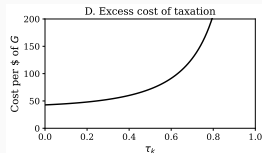
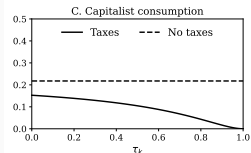
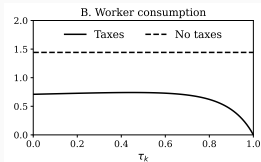
With $T = 0$ and $G = gY$, the government budget constraint gives the locus of budget-balancing pairs (τ_ℓ, τ_k) :

$$\tau_\ell = \frac{1}{1 - \alpha} \left(g - \tau_k \rho \frac{\alpha}{\rho + (1 - \tau_k)\delta} \right)$$



- As τ_k rises, the required τ_ℓ falls (substituting capital for labor taxation)
- As $\tau_k \rightarrow 1$, output collapses and both consumptions go to zero

Excess Cost of Taxation



Excess cost (per dollar of G) is defined as how much total consumption is reduced by taxes, net of public consumption financed:

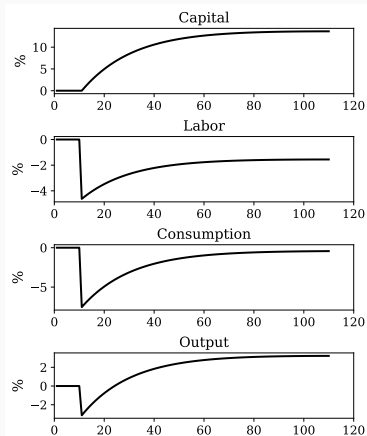
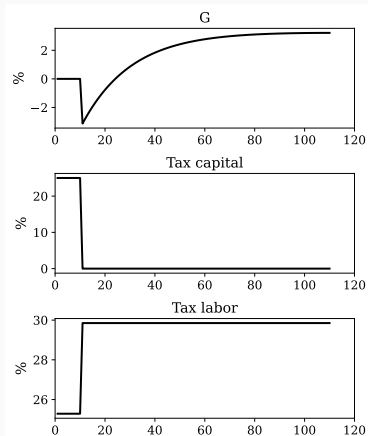
$$\text{Excess Cost} = \frac{(c_{w,\tau=0} - c_w)/(1 + \phi) + c_{k,\tau=0} - c_k - gy}{gy}$$

($c_{w,\tau=0} - c_w$ is divided by $1 + \phi$ to account for the utility from leisure)

- Always **positive** (taxes are distortionary)
- **Increasing and convex** in τ_k
- Minimized at $\tau_k = 0$

Tax Reform: Eliminating Capital Income Taxes

- Initial steady state: $\tau_k = 0.25$, $\tau_\ell = 0.2529$
- At period 11: unexpected, permanent switch to $\tau_k = 0$, $\tau_\ell = 0.2985$
- G_t varies during transition to balance budget date-by-date



Tax Reform: Interpreting the Transition

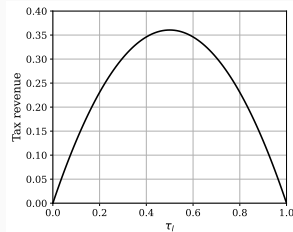
- **Capital:** accumulates slowly toward new (higher) steady state
- **Labor supply:** drops on impact (higher τ_ℓ), gradually recovers as rising capital raises wages
- **Output:** falls short-run (labor drop), recovers long-run (capital accumulation)
- **Consumption:** falls initially (more saving + lower output), then recovers

Distributional implications:

- **Capitalists:** always better off (capital income grows)
- **Workers:** may be *significantly* worse off (higher τ_ℓ)
- Whether reform is beneficial depends on weights
- More desirable if agents derive utility from G

Lesson: Steady-state analysis alone is **insufficient**; transitions can be painful.

The Laffer Curve



Revenue-maximizing tax rates (with $\tau_k, \tau_\ell \geq 0$):

$$\tau_\ell^* = \frac{1}{1 + \phi}, \quad \tau_k^* = 0$$

- At $\tau_\ell = 0$: no taxes levied \Rightarrow no revenue
- At $\tau_\ell = 1$: no one works \Rightarrow nothing to tax \Rightarrow no revenue
- Higher Frisch elasticity $\phi \Rightarrow$ lower revenue-maximizing τ_ℓ
- Revenue is declining in τ_k at $\tau_k = 0 \Rightarrow \tau_k = 0$ is revenue-maximizing
- Above τ_ℓ^* : “wrong side of the Laffer curve”

Theories of G : Public Goods

So far, G generated no benefits—revenues “thrown into the ocean.” Two theories:

1. Public goods: Government produces goods valued by society (defense, parks, sanitation).

$$u(c, g), \quad u_2(c, g) > 0, \quad u_{22}(c, g) \leq 0$$

- **First best** (lump-sum taxes): equate marginal utilities:
 $u_1(c, g) = u_2(c, g)$
- With distortionary taxes: also account for deadweight losses

2. Public infrastructure: roads, bridges, schools.

$$f(k, k_g, \ell) = Ak^\alpha \ell^{1-\alpha} k_g^\theta$$

Law of motion: $k_{g,t+1} = i_{g,t} + (1 - \delta_g)k_{g,t}$

- k_g : public capital; θ : elasticity of output w.r.t. public capital
- $\theta > 0 \Rightarrow$ increasing returns to scale
- Estimates of θ vary widely:
 - Low: 0.05 (Leeper, Walker, and Yang, 2010)
 - High: 0.39 (Aschauer, 1989)
- With non-distortionary revenue: optimal to equate marginal products:

$$f_{k,t+1} - \delta = f_{g,t+1} - \delta_g$$

Government Debt and Ricardian Equivalence

Setup: Two-Period Model with Lump-Sum Taxes

Add government debt. Abstract from capital to simplify.

Government: transfer T_0 in period 0, financed by debt B_0 , repaid by lump-sum tax τ_1 in period 1.

Household budget constraints:

$$c_0 + b_0 = w_0 \ell_0 + T_0$$

$$c_1 = w_1 \ell_1 + (1 + r_1)b_0 - \tau_1$$

b_0 : household bond purchases. Wages w_0, w_1 and return r_0 exogenous.

Ricardian Equivalence: Proof

Combine budget constraints in present value:

$$c_0 + \frac{c_1}{1+r_1} = w_0 \ell_0 + \frac{w_1 \ell_1}{1+r_1} + \underbrace{T_0 - \frac{\tau_1}{1+r_1}}_{=0}$$

Since $\tau_1 = (1+r_1)T_0$ (government must repay): the tax and transfer terms **cancel exactly**.

Result: Lifetime budget constraint is **unaffected** by the size of T_0 .

- Optimal allocation of consumption and hours: **identical** to zero-tax case
- Household saves exactly T_0 more \Rightarrow absorbs extra government bonds
- Extra savings provide exactly enough income to pay τ_1

This is **Ricardian Equivalence**: the timing of lump-sum taxes is irrelevant. Result extends to infinite-horizon economies.

Ricardian Equivalence: Assumptions

The result hinges on:

1. Taxes are **lump-sum** (not distortionary)
2. Households face **no credit constraints**
3. The households who get the transfers are the **same ones** who must repay the debt

Does **not** hinge on there being no capital.

Key question: What happens when taxes are *distortionary*? Then timing matters—some tax paths are better than others. This motivates the **Ramsey problem**.

Ramsey Taxation

Why Not Lump-Sum Taxes?

In a representative agent economy: lump-sum taxes are distortion-free.

But in practice:

- Households differ widely in income
- Some too poor to afford a moderate lump-sum tax
- Many view it as **unfair** if the poor pay the same as the rich

⇒ Literature focuses on **proportional taxes**

Even with proportional taxes, different types of income can be taxed at different rates, and rates can vary over time. Government can save or borrow using debt.

The Ramsey problem: Maximize welfare by choosing optimal tax rates and debt, with **commitment** at $t = 0$.

Two-Period Ramsey Model: Setup

Household: $u(c_0, \ell_0) + \beta u(c_1, \ell_1)$

Production: $y_t = A_t \ell_t$ (no capital). Competitive $\Rightarrow w_t = A_t$

Storage technology: 1 unit at $t = 0 \rightarrow 1 + r$ units at $t = 1$ (r exogenous)

Tax instruments: τ_0, τ_1 on labor income; $\tau_{b,1}$ on wealth income at $t = 1$.

Household budget constraints:

$$c_0 = (1 - \tau_0)w_0\ell_0 - b_0$$

$$c_1 = (1 - \tau_1)w_1\ell_1 + (1 + r(1 - \tau_{b,1}))b_0$$

Household Optimality Conditions

First-order conditions for the household:

$$u_{c,0}w_0(1 - \tau_0) = -u_{\ell,0}$$

$$u_{c,1}w_1(1 - \tau_1) = -u_{\ell,1}$$

$$u_{c,0} = \beta(1 + r(1 - \tau_{b,1}))u_{c,1}$$

Resource feasibility (single equation):

$$c_0 + g_0 + \frac{c_1 + g_1}{1 + r} = A_0\ell_0 + \frac{A_1\ell_1}{1 + r}$$

The Primal Approach: Implementability Constraint

Key idea: Substitute household FOCs into the lifetime budget constraint to eliminate prices and taxes.

$$(1 - \tau_t)w_t = -u_{\ell,t}/u_{c,t}$$

$$1 + r(1 - \tau_{b,1}) = u_{c,0}/(\beta u_{c,1})$$

After substitution and simplification using the household budget constraint:

$$u_{c,0}c_0 + \beta u_{c,1}c_1 = -u_{\ell,0}l_0 - \beta u_{\ell,1}l_1$$

This is the **implementability constraint**. It embeds household optimality *and* budget constraint. Government budget is automatically satisfied by Walras' Law.

Result: Any allocation satisfying the resource feasibility and the implementability condition can be implemented by some feasible tax plan.

The Ramsey Problem

Maximize household welfare subject to resource and implementability constraints:

$$\max_{c_0, c_1, \ell_0, \ell_1} u(c_0, \ell_0) + \beta u(c_1, \ell_1) + \lambda[\text{RC}] + \mu[\text{IC}]$$

Four FOCs:

$$u_{c,0} - \lambda + \mu u_{c,0} + \mu u_{cc,0} c_0 + \mu u_{\ell c,0} \ell_0 = 0$$

$$u_{\ell,0} + \lambda A_0 + \mu u_{\ell,0} + \mu u_{c\ell,0} c_0 + \mu u_{\ell\ell,0} \ell_0 = 0$$

$$\beta u_{c,1} - \frac{\lambda}{1+r} + \beta \mu u_{c,1} + \beta \mu u_{cc,1} c_1 + \beta \mu u_{\ell c,1} \ell_1 = 0$$

$$\beta u_{\ell,1} + \frac{\lambda A_1}{1+r} + \beta \mu u_{\ell,1} + \beta \mu u_{c\ell,1} c_1 + \beta \mu u_{\ell\ell,1} \ell_1 = 0$$

Four FOCs + RC + IC = 6 equations, 6 unknowns
($c_0, c_1, \ell_0, \ell_1, \lambda, \mu$).

Separable Utility: Simplification

With

$$u(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\ell^{1+1/\phi}}{1+1/\phi}$$

Cross-derivatives vanish. Second derivatives simplify: $u_{cc}c = -\sigma u_c$
and $u_{\ell\ell} = u_\ell/\phi$.

The four FOCs become:

$$u_{c,0}(1 + \mu - \mu\sigma) = \lambda$$

$$u_{\ell,0}(1 + \mu + \mu/\phi) = -\lambda A_0$$

$$\beta u_{c,1}(1 + \mu - \mu\sigma) = \lambda/(1 + r)$$

$$\beta u_{\ell,1}(1 + \mu + \mu/\phi) = -\lambda A_1/(1 + r)$$

Ramsey Result 1: Zero Capital Income Tax

Comparing the Euler equations (1st and 3rd FOCs):

$$\beta(1+r)u_{c,1} = u_{c,0}$$

⇒ No intertemporal wedge. Comparing with household Euler equation:

$$\tau_{b,1} = 0$$

Result (Chamley-Judd): The government should commit to **neither tax nor subsidize** income from savings.

Ramsey Result 2: Tax Smoothing

Comparing labor and consumption FOCs:

$$u_{c,t} A_t \frac{1 + \mu - \mu\sigma}{1 + \mu + \mu/\phi} = -u_{\ell,t}, \quad t = 0, 1$$

Matching with household FOCs using $w_t = A_t$:

Result (Barro, Lucas-Stokey):

$$\tau_0 = \tau_1 = \frac{\mu(\sigma + 1/\phi)}{1 + \mu + \mu/\phi}$$

Labor tax rate should be **constant** over time; positive when g_0 or $g_1 > 0$.

Intuition: Distortions from taxation are **convex** in the tax rate

Time Consistency

Assume the Ramsey plan has $b_0 > 0$ (household saves in period 0).

At $t = 1$: household wealth b_1 is already determined.

⇒ Taxing wealth income is effectively a **lump-sum tax!**

A benevolent planner *at* $t = 1$ would deviate from the Ramsey plan:

- Set $\tau_1 = 0$ (avoid distorting labor)
- Fund all spending from $\tau_{b,1}$ (non-distortionary at that point)

⇒ The Ramsey plan is **time inconsistent** (Kydland and Prescott, 1977).

Time Consistency: Implications

- The Ramsey plan is still useful: tells us what's optimal *with commitment*
- **Institutions** can provide commitment:
 - Constitutional limits on tax changes
 - Independent fiscal authorities
- Large literature characterizing best **time-consistent** policies
- Time inconsistency arises in many settings:
 - Sovereign debt default
 - Political economy: changing coalitions, redistribution motives depress investment

Debt and Pensions with Overlapping Generations

OLG Model: Setup

Return to two-period OLG endowment economy, extended with government debt and pensions. Small open economy with exogenous r .

- Population growth at rate n : $N_t = (1 + n)^t$
- Young share: $(1 + n)/(2 + n)$
- Only the young work; labor income: $y_t = (1 + \gamma)^t \omega$
- Aggregate labor income: $Y_t = (1 + n)^t (1 + \gamma)^t \omega$

PAYG pension: pays p_t to every old individual, financed by tax τ_p :

$$p_t = (1 + n) \tau_p y_t$$

Government: spends $G_t = gY_t$, taxes labor at τ_t , issues debt B_t .

OLG: Government and Household Budget

Government budget (in GDP ratios, pension self-financing):

$$G_t + p_t N_{t-1} + (1+r)B_{t-1} = (\tau_p + \tau_t)Y_t + B_t$$
$$\Rightarrow g + \frac{1+r}{(1+n)(1+\gamma)} b_{t-1} = \tau_t + b_t$$

Household budget constraints:

$$c_{y,t} + a_t = (1 - \tau_t - \tau_p)y_t$$

$$c_{o,t+1} = (1+r)a_t + p_{t+1}$$

Euler equation:

$$u'(c_{o,t+1}) = (1+r)\beta u'(c_{y,t})$$

Lifetime budget:

$$c_{y,t} + \frac{c_{o,t+1}}{1+r} = \left(1 - \tau_t - \tau_p + \frac{(1+n)(1+\gamma)}{1+r} \tau_p \right) y_t$$

PAYG Pension = Forced Government Saving

Consider a individual who is forced to purchase $b_{p,t} = \tau_p y_t$ government bonds at return r_p :

$$c_{y,t} + a_t + b_{p,t} = (1 - \tau_t)y_t$$

$$c_{o,t+1} = (1 + r)a_t + (1 + r_p)b_{p,t}$$

Setting $1 + r_p = (1 + n)(1 + \gamma) \approx 1 + n + \gamma$, these budget constraints are **identical** to the ones under PAYG pension.

Return on PAYG pension = growth rate of output.

- If $n + \gamma > r$: pension **increases** lifetime income for the young
⇒ Pareto improving (dynamic inefficiency)
- If $n + \gamma < r$ (“normal” case): pension is effectively a **tax on the young**
- Initial old generation always gains (receives benefits without contributing)

Ricardian Equivalence Breaks Down in OLG

Experiment: one-time tax holiday at t in economy with zero initial debt.

- $\tau_t = 0$, $B_t = gY_t$
- Repay by raising τ_{t+1} , no further debt

Generation t : gains

Generation $t + 1$: loses (higher present-value taxes)

⇒ Aggregate consumption rises at t , falls at $t + 2$.

⇒ **Ricardian equivalence fails.**

Why: Debt reshuffles the tax burden **across generations** (not just across time for the same agent).

Pension Promises as Implicit Debt

Total effective US government debt:

- **Narrow definition** (bonds outstanding): federal debt held by public $\approx 94\%$ of GDP (Q2 2023)
- **Broad definition** (includes implicit pension debt): present value of future federal pension promises = \$65.9 trillion, or $\approx 245\%$ of GDP
- **Total:** $94 + 245 = 339\%$ of GDP
- This excludes future Medicare costs!

Key difference: Nominal debt default ruled out by US Constitution (though inflation can erode it). Pension promises have **no constitutional protection**—government can reduce benefits or raise eligibility age.

Strain on Pension Systems

Two factors straining OECD pension systems:

1. **Population aging**: lower mortality + lower fertility \Rightarrow more retirees per worker (lower n in the model)
2. **Secular stagnation**: US GDP per capita growth fell from 2.3% (1950–2000) to 1.2% (2000–2020) (lower γ)

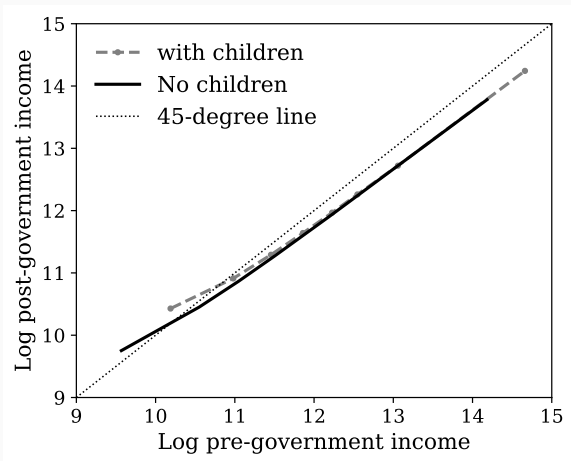
Both reduce the PAYG return $(1 + n)(1 + \gamma)$ relative to $1 + r$.

- US Social Security Trust Fund scheduled to deplete by **2033**
- Real-world pension funds are not purely PAYG, but public pension savings are relatively small
- Options: reduce benefits, raise retirement age, increase τ_p

Taxes and Transfers as Redistribution

The US Tax and Transfer System: Data

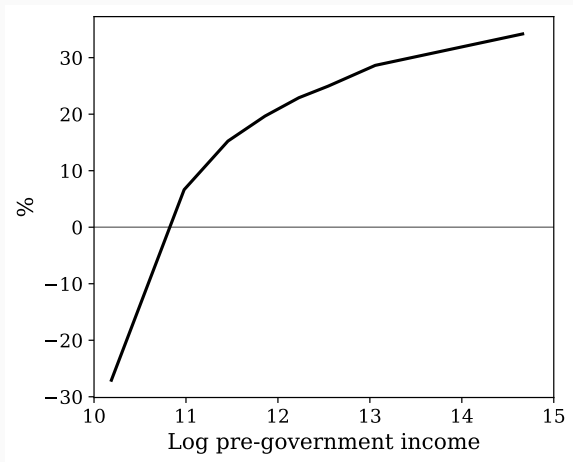
Log pre-government income vs. log post-government income



Relationship approximately linear in logs, except at lowest income percentiles.

The US Tax and Transfer System: Data

Average net tax rates by household income



Average net tax rate is **increasing** with income \Rightarrow US system is **progressive**.

Log-Linear Tax Function

The US tax-and-transfer system is well approximated by:

$$y - T(y) = \lambda y^{1-\tau}$$

- y : pre-government household income
 - $T(y)$: taxes minus transfers
 - $y - T(y)$: disposable income
 - λ : controls the level of taxation
 - τ : **progressivity parameter**
-
- $\tau > 0$: **progressive** (marginal rate $T'(y)$ exceeds average rate $T(y)/y$)
 - $\tau = 0$: flat tax ($T'(y) = T(y)/y = 1 - \lambda$)
 - $\tau < 0$: regressive

Model Setup

A static model. Unit continuum of individuals indexed by i , with heterogeneous productivity w_i :

$$u(c_i, l_i, G) = \log c_i - \frac{\ell_i^{1+1/\phi}}{1 + 1/\phi}$$

Tax system:

$$c_i = \lambda(w_i l_i)^{1-\tau}$$

Government budget must balance:

$$\int c_i di + G = Y = \int w_i l_i di$$

Balanced-growth class utility: tractable closed-form solution.

Equilibrium: Hours and Consumption

FOC for hours:

$$\log \ell_i = \frac{\log(1 - \tau)}{1 + 1/\phi}$$

- Hours are **falling in** τ : progressivity discourages work (higher marginal rate)
- Hours are **independent of** w_i : balanced growth class utility
- As $\tau \rightarrow 1$: disposable income becomes λ regardless of hours
 $\Rightarrow \ell \rightarrow 0$

Consumption:

$$\log c_i = \log \lambda + (1 - \tau) \frac{\log(1 - \tau)}{1 + 1/\phi} + (1 - \tau) \log w_i$$

Consumption is increasing in w_i , but the pass-through is dampened: a 1% increase in w_i translates to only a $(1 - \tau)\%$ increase in c_i .

Consumption Inequality and the Equity–Efficiency Trade-off

Pre-government earnings inequality: $\text{Var}(\log(w\ell)) = \text{Var}(\log w)$

Consumption inequality:

$$\text{Var}(\log c) = (1 - \tau)^2 \text{Var}(\log w)$$

Higher $\tau \Rightarrow$ lower consumption inequality.

Consumption-equivalent flow utility (in equilibrium):

$$c_w - \frac{\ell^{1+1/\phi}}{1 + 1/\phi} = \frac{1}{1 + \phi} c_w$$

The fundamental trade-off:

- Higher progressivity (τ) = more redistribution = less consumption inequality
- Higher progressivity (τ) = more distortion = less hours worked = less output

Summary

Key Takeaways (I)

1. **Data:** US spending $\approx 34\%$ GDP; transfers growing; persistent deficits; debt/GDP $\approx 100\%$
2. **Distortionary taxes:** All three types ($\tau_c, \tau_\ell, \tau_k$) reduce output; τ_k is especially costly (convex excess cost; amplification through wages)
3. **Tax reform:** Eliminating τ_k improves long-run efficiency but creates painful transitions and distributional tensions
4. **Laffer curve:** Revenue-maximizing rates: $\tau_\ell^* = 1/(1 + \phi)$,
 $\tau_k^* = 0$
5. **Ricardian equivalence:** Holds with lump-sum taxes and infinitely-lived agents; **fails** in OLG (debt shifts burden across generations)

Key Takeaways (II)

6. **Ramsey taxation:** Optimal $\tau_k = 0$; smooth τ_ℓ over time; use debt to absorb temporary shocks
7. **Time consistency:** Optimal plans not credible ex post; institutions needed for commitment
8. **OLG pensions:** PAYG = implicit debt with return $n + \gamma$; aging and slower growth strain systems
9. **Redistribution:** Progressive τ reduces inequality but distorts labor supply — fundamental equity-efficiency trade-off