Chapter 22: Heterogeneous Firms

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April 2026

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Introduction

Motivation

Standard macroeconomic models assume an **aggregate production function**:

$$Y = F(K, L)$$

- Justified if all firms have homogeneous production functions
- In reality, firms are heterogeneous in many dimensions:
    - Large vs. small
    - Young vs. old
    - Growing vs. contracting
    - Productive vs. unproductive
    - Differentiated products

Why Does Firm Heterogeneity Matter?

Key policy and research questions that require explicit firm heterogeneity:

- How should we encourage (or discourage) the **entry of new firms**?
- Should we support **growing firms**, and if so how?
- Should we be concerned about the growing prominence of **"mega-firms"**?
- What are the causes and consequences of the recent **decline in business dynamism**?
- Does the representative-firm assumption give **misleading answers** to standard policy questions?

Chapter Roadmap

1. A simple model showing how heterogeneity affects aggregation
2. Empirical facts on U.S. firm heterogeneity
3. Reallocation and misallocation
4. General equilibrium: Hopenhayn-Rogerson (1993) framework
5. Alternative market arrangements (monopolistic competition, oligopoly)
6. Business cycles and heterogeneous firms
7. Endogenous productivity: Klette-Kortum (2004) model

A Simple Model

Setup

- Unit mass of firms, $i \in [0, 1]$
- Homogeneous good, perfect competition
- Production function of firm i:

$$y_i = a_i F(\mathbf{x}_i)^\gamma, \qquad \gamma \in (0, 1)$$

- a_i: firm-specific productivity (heterogeneous across firms)
- $\mathbf{x}_i$: input vector for firm i
- $F(\mathbf{x}_i)$: constant returns to scale
- Overall production: **decreasing returns to scale** (because $\gamma < 1$)

Why Decreasing Returns?

Key modeling assumption: $\gamma < 1$ ensures DRS at the firm level

With constant returns ($\gamma = 1$):

- The most productive firm(s) take over the entire economy
- Either: monopoly/oligopoly $\Rightarrow$ contradicts perfect competition
- Or: only the most efficient firms operate $\Rightarrow$ replicates homogeneous case

With $\gamma < 1$:

- Firms of different productivities can coexist
- Each firm has an optimal finite scale
- Richer implications for aggregation

Two-Step Optimization

Step 1: Cost minimization (common across firms)

$$\min_{\mathbf{x}} \; \mathbf{px} \quad \text{s.t.} \quad F(\mathbf{x}) = 1$$

Let $\mathbf{x}^*$ be the solution, $c \equiv \mathbf{px}^*$ the unit cost.

Step 2: Optimal scale for firm i

Let $m_i = F(\mathbf{x}_i)$ be combined input. Then:

$$\max_{m_i} \; a_i m_i^\gamma - c m_i$$

First-order condition:

$$a_i m_i^{\gamma-1} = \frac{c}{\gamma}$$

Aggregation

From the FOC: $y_i = (c/\gamma)m_i$ for all i. Summing:

$$Y = \frac{c}{\gamma}M$$

where $Y = \int y_i \, di$ is total output and $M = \int m_i \, di = F(\mathbf{X})$.

Define **aggregate productivity**:

$$A \equiv \left(\int a_i^{\frac{1}{1-\gamma}} \, di \right)^{1-\gamma}$$

Then the **aggregate production function** is:

$$Y = AF(\mathbf{X})^{\gamma}$$

Heterogeneity matters through A—the distribution of a_i shapes aggregate output.

Lognormal Example

Suppose $\log(a_i) \sim N(\nu - \sigma^2/2,\ \sigma^2)$

- Average productivity: $\int a_i \, di = \exp(\nu)$
- Aggregate productivity:

$$A = \exp\left(\nu + \frac{\gamma}{1-\gamma}\frac{1}{2}\sigma^2\right)$$

Key insight: Even holding the average of a_i fixed, a higher dispersion σ raises A.

Intuition: Productive resources are endogenously allocated—productive firms use more input and have a greater presence in aggregate production. More dispersion $\Rightarrow$ more room to allocate to productive firms in the right tail.

Two Fundamental Questions

Throughout this chapter, we keep two questions in mind:

1. **How is the distribution of a_i determined?**
    - Exogenous vs. endogenous productivity
    - Entry, exit, and innovation
2. **How does the economy allocate resources to different firms?**
    - Efficient allocation vs. misallocation
    - Frictions, distortions, and policy

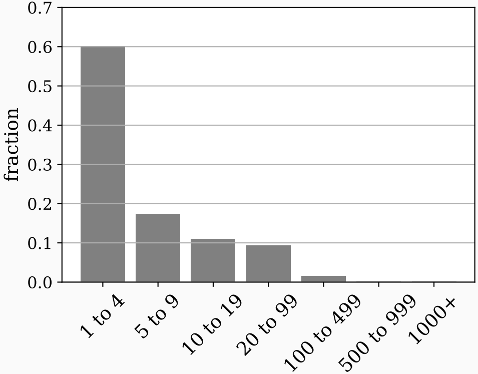
Firm Heterogeneity in the Data

Data Source

- All figures from the U.S. Census Bureau's **Business Dynamics Statistics (BDS)**
- Covers the universe of U.S. employer establishments and firms
- Key distinction:
    - **Establishment**: a fixed physical location of economic activity
    - **Firm**: a collection of establishments under common ownership
- Over 5 million firms and 7 million establishments in the U.S.

Firm Size Distribution

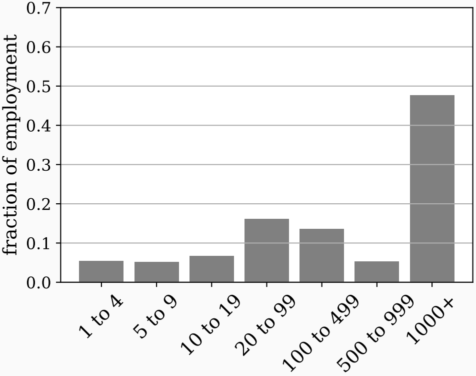
Distribution of firm size in 2022



- Highly dispersed distribution
- Majority are very small firms (1–4 employees)
- Over 10,000 firms with 1,000+ employees
- Over 1,000 firms with 10,000+ employees

Employment Shares

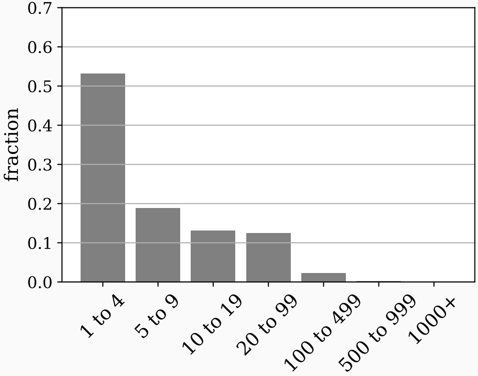
Employment share of each size category in 2022



- Small firms are numerous but large firms employ most workers
- $\approx 50\%$ of employees work at firms with $1{,}000+$ employees
- $\approx 30\%$ work at very large firms ($10{,}000+$ employees)

Establishment Size Distribution

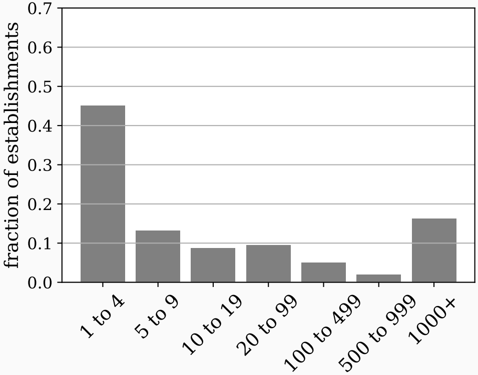
Distribution of establishment size in 2022



- $\approx 50\%$ are very small (1–4 employees)

Firm Size and Establishment Share

Fraction of establishments owned by each firm size category in 2022



- Large firms own many establishments
- Firms with $1{,}000+$ employees own avg ≈ 100 establishments
- Firms with $10{,}000+$ employees own avg ≈ 600 establishments

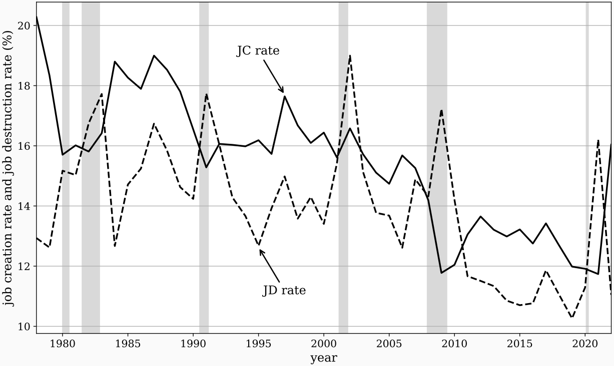
Job Creation and Destruction

Definitions: Let ℓ_{it} be employment of establishment i at year t

$$JC_t \equiv \frac{\sum_{i:\ell_{it}>\ell_{i,t-1}}(\ell_{it}-\ell_{i,t-1})}{\bar{L}_t}, \qquad JD_t \equiv \frac{\sum_{i:\ell_{it}<\ell_{i,t-1}}(\ell_{i,t-1}-\ell_{it})}{\bar{L}_t}$$

where $\bar{L}_t = (L_t + L_{t-1})/2$.

Annual job creation and destruction rates (establishment level)



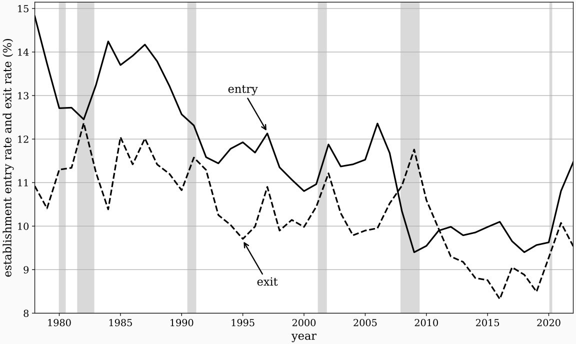
Properties of Job Flows

Three notable properties:

1. **Large magnitudes**: Both JC and JD rates exceed 10% in any given year
2. **Cyclical**: In recessions, JC declines and JD increases
3. **Declining trend**: Both rates have declined over time $\Rightarrow$ **"declining business dynamism"**

Entry and Exit Rates

Annual establishment entry and exit rates

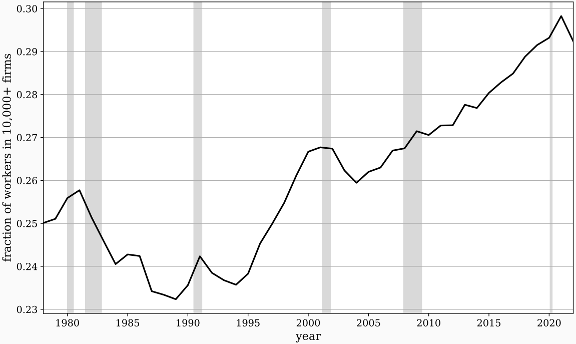


Similar properties as JC/JD:

- Large rates (entry ≈ 10–14%, exit ≈ 8–11%)
- Cyclical patterns
- Declining trend over recent decades

Rise of Large Firms

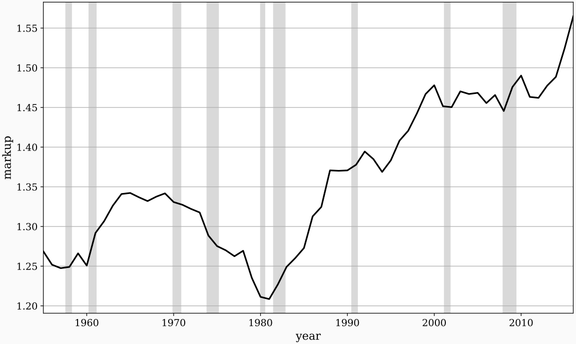
Fraction of employees working in firms with 10,000+ employees



- Steady increase since early 1990s
- Large firms are increasingly dominating the U.S. economy
- Raises concerns about **market power**

Rising Markups

Markup of U.S. public firms



- Average markup (price/marginal cost) of U.S. public firms
- Increasing trend since the 1980s
- Consistent with rising dominance of large firms
- Active research area; see Miller (2025) and Syverson (2025) for surveys

Reallocation and Misallocation

How Much Does Reallocation Matter?

Foster, Haltiwanger, and Krizan (2001) decomposition:

Define output-weighted average productivity: $\bar{A}_t \equiv \sum_i s_{it} a_{it}$

where s_{it} is the output share of establishment i.

$$\Delta \bar{A}_t = \underbrace{\sum_{i \in C} s_{i,t-1} \Delta a_{it}}_{\text{within}} + \underbrace{\sum_{i \in C} (a_{i,t-1} - \bar{A}_{t-1}) \Delta s_{it}}_{\text{between}} + \underbrace{\sum_{i \in C} \Delta a_{it} \Delta s_{it}}_{\text{cross}}$$

$$+ \underbrace{\sum_{i \in N} s_{it} (a_{it} - \bar{A}_{t-1})}_{\text{entry}} - \underbrace{\sum_{i \in X} s_{i,t-1} (a_{i,t-1} - \bar{A}_{t-1})}_{\text{exit}}$$

C: continuing, N: new entrants, X: exiting establishments.

Decomposition Results

Five sources of aggregate productivity growth:

1. **Within**: each establishment raises its own productivity
2. **Between**: high-productivity establishments gain market share
3. **Cross**: interaction of productivity growth and share changes
4. **Entry**: entrants are above average
5. **Exit**: exiting establishments are below average

Key finding (U.S. manufacturing, 1977–1987):

- Only 45% from the within effect
- The remaining 55% from reallocation (between, cross, entry, exit)

$\Rightarrow$ **Reallocation is crucial for aggregate productivity growth**

Misallocation: Setup

Restuccia and Rogerson (2008), Hsieh and Klenow (2009): firm-specific distortions cause misallocation.

Add firm-specific output tax τ_i to the simple model:

$$\max_{m_i} (1-\tau_i) a_i m_i^\gamma - c m_i$$

GDP is still $Y = \int y_i di$. Aggregate production function: $Y = AF(\mathbf{X})^\gamma$

But now:

$$A = \frac{\left[\int a_i^{\frac{1}{1-\gamma}} (1-\tau_i)^{\frac{\gamma}{1-\gamma}}\, di\right]^{1-\gamma}}{\left[\int a_i^{\frac{1}{1-\gamma}} (1-\tau_i)^{\frac{1}{1-\gamma}}\, di\right]^{\gamma}}$$

Reduces to the undistorted A when $\tau_i = 0$ for all i.

Misallocation: Lognormal Case

Assume $(a_i, 1-\tau_i)$ are bivariate lognormal:

$$(\log a_i, \log(1-\tau_i)) \sim N(\mu, \Sigma)$$

with $\mu = (\nu_a - \sigma_a^2/2,\ \nu_\tau - \sigma_\tau^2/2)$ and $\text{Var}(\log(1-\tau_i)) = \sigma_\tau^2$.

Result:

$$A = \exp\left(\nu_a + \frac{\gamma}{1-\gamma}\frac{1}{2}(\sigma_a^2 - \sigma_\tau^2)\right)$$

Key insights:

- Higher dispersion in taxes σ_τ **reduces** aggregate productivity
- Productive firms may not expand if $(1-\tau_i)a_i$ is low
- Unproductive firms may over-expand if $(1-\tau_i)a_i$ is high
- The level of the tax ν_τ does not affect A (no distortion in allocation)

Sources of Misallocation

Specific policies and institutions that cause misallocation:

- Size-specific taxes and regulations
- Entry regulations and barriers
- Regulations governing hiring and firing
- Financial frictions (credit constraints)
- Trade barriers

Quantitative importance: Hsieh and Klenow (2009) estimate that reallocating resources to equalize marginal products would raise manufacturing TFP by:

- 30–50% in China
- 40–60% in India

Firm Heterogeneity in General Equilibrium

The Hopenhayn-Rogerson (1993) Framework

Goal: Analyze reallocation frictions in general equilibrium with forward-looking firms.

Key features:

- Continuum of firms with idiosyncratic productivity shocks
- Endogenous entry and exit
- Labor as the only input
- Free entry condition
- Representative consumer
- Focus on steady state

Experiments: firing taxes and entry barriers (Moscoso Boedo and Mukoyama, 2012).

Production and Profits

- Production function: $y_t = a_t \ell_t^\gamma$
- a_t: idiosyncratic productivity (stochastic)
- Firms pay:
  - Wages: $w_t \ell_t$
  - Fixed operation cost: c_f (units of goods, per period)
  - Firing taxes: $\tau \max(0, \ell_{t-1} - \ell_t)$

Flow profit:

$$\pi(\ell_{t-1}, \ell_t, a_t) = a_t \ell_t^\gamma - w_t \ell_t - c_f - \tau \max(0, \ell_{t-1} - \ell_t)$$

- Output price normalized to 1
- Firing taxes transferred back to consumers

Timing and Productivity Process

Within-period timing:

1. Incumbent decides whether to **exit**
   - If exit: pay firing cost $\tau\ell_{t-1}$
2. If stay: receive current a_t from

$$\log(a_t) = \alpha + \rho \log(a_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2)$$

3. Choose employment ℓ_t, produce

Dynamic element: With $\tau > 0$, hiring is dynamic—firm foresees future firing costs $\Rightarrow$ reluctance to hire even with positive shocks.

Firm's Dynamic Program

$$W(a, \ell_{-1}) = \max_{\ell} \; \pi(\ell_{-1}, \ell, a) + \beta \max \left\{ \mathbb{E}[W(a', \ell) \mid a], \; -\tau\ell \right\}$$

- $\beta \in (0, 1)$: consumer's discount factor
- Inner $\max$: **exit decision**
    - Continue: get $\mathbb{E}[W(a', \ell) \mid a]$
    - Exit: pay firing cost $-\tau\ell$

Entry and Consumer

Free entry:

$$W^e = c_e + \kappa$$

where c_e = technological entry cost, κ = policy-related entry barriers.

$$W^e = \int (W(a,0) + c_f)\, d\nu(a)$$

$\nu(a)$: exogenous distribution for entrant productivity.

Representative consumer:

$$\sum_{t=0}^{\infty} \beta^t [u(C_t) - \chi L_t^s]$$

Steady-state FOC: $wu'(wL^s + \Pi + R) = \chi$

where Π = total firm profits, R = total transfers.

Block-Recursive Structure

Key computational insight (Kaas, 2021):

1. Solve $W(a, \ell_{-1})$ as function of w ($\Rightarrow$ get $W^e(w)$)
2. Free entry pins down w^* ($\Rightarrow W^e(w^*) = c_e + \kappa$)
3. Given w^* and entry mass M: compute stationary distribution
4. Compute $L^d(M)$, $\Pi(M)$, $R(M)$
5. Labor market clearing $L^d(M) = L^s(\Pi(M), R(M))$ pins down M

Advantage: No need to track firm distribution to solve for prices ("block recursive structure")

$\Rightarrow$ Much easier computationally than heterogeneous-consumer models. (Lee and Mukoyama, 2018)

Calibration

- One period = 5 years
- $u(c) = \log(c)$
- $\beta = 0.8$ (4% annual discount rate)
- $\gamma = 0.64$ (labor share)
- Baseline: $(\tau, \kappa) = (0, 0)$ calibrated to the US economy

Targets:

- $\alpha = 0.076$: average firm size ≈ 62 employees
- $\rho = 0.93$: autocorrelation of $\log(\ell)$ is 0.93
- $\sigma = 0.253$: variance of employment growth is 0.53
- $c_f = 18.0$: exit rate $\approx 37\%$ (over 5 years)
- $c_e = 9.04$: free entry holds with $w = 1$

Effects of Firing Taxes

	$\tau = 0$	$\tau = 0.1$	$\tau = 0.2$
Wage	1.000	0.977	0.957
Total output	100	97.7	95.7
Total employment	100	98.3	97.4
Labor productivity	100	99.4	98.3
JC (= JD) rate	0.28	0.25	0.21

Table 1: Model results with firing taxes ($\kappa = 0$).

- $\tau = 0.1 \approx$ 6 months' salary
- $\tau = 0.2 \approx$ 1 year's salary

Interpreting the Firing Tax Results

1. **Employment declines**: Hiring effect dominates firing effect
   - Firms fire less $\Rightarrow$ pushes L up
   - Firms hire less (foresee future firing costs) $\Rightarrow$ pushes L down
   - Net: employment falls
2. **Labor productivity (Y/L) declines**: Misallocation
   - Good-a firms don't expand enough
   - Bad-a firms don't contract enough
   - Marginal products of labor become dispersed
3. **JC rate declines**: Less reallocation
   - Reluctance to hire and fire
   - Closely linked to productivity loss from misallocation

Effects of Entry Barriers

	$\kappa = 0$	$\kappa = 0.5$	$\kappa = 5.0$
Wage	1.000	0.986	0.879
Total output	100	98.6	87.9
Total employment	100	99.5	96.2
Labor productivity	100	99.1	91.4
JC (= JD) rate	0.28	0.28	0.28

Table 2: Model results with entry barriers ($\tau = 0$).

- $\kappa = 5.0$: comparable to low-income country entry costs
- Substantial output and productivity losses

Interpreting the Entry Barrier Results

Mechanism: Higher $\kappa \Rightarrow$ higher firm value in equilibrium $\Rightarrow$ lower wage

Three channels through which low wage affects productivity:

1. Low-productivity firms **less likely to exit** $\Rightarrow$ drags down A $\downarrow$
2. Low exit rate $\Rightarrow$ low entry rate $\Rightarrow$ fewer (less productive) entrants $\Rightarrow$ raises A $\uparrow$
3. Larger incumbent size $\Rightarrow$ DRS $\Rightarrow$ lower productivity $\downarrow$

Net effect: productivity **declines** (channels 1 & 3 dominate).

Political economy: Entry barriers **harm entrants** but **benefit incumbents** $\Rightarrow$ conflict of interest (Mukoyama and Popov, 2014).

Alternative Market Arrangements

Moving Beyond Perfect Competition

Two takeaways:

1. Misallocation insights carry over with minor modifications
2. Market power enables analysis of firm heterogeneity and **markups**

We consider two alternatives:

- **Monopolistic competition** (constant markups)
- **Oligopoly** (endogenous markups) — Atkeson and Burstein (2008)

Monopolistic Competition: Setup

Final good (CES aggregation / Dixit–Stiglitz):

$$Y = \left(\int y_i^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

- Each intermediate good i produced by a monopolist
- Same input structure as the simple model
- Input markets: perfectly competitive

Inverse demand for good i:

$$p_i = y_i^{-1/\sigma} Y^{1/\sigma}$$

Monopolistic Competition: Firm Problem

Producer i solves:

$$\max_{m_i} (a_i m_i^\gamma)^{-1/\sigma} Y^{1/\sigma} \cdot a_i m_i^\gamma - c m_i$$

taking Y as given (firm is small relative to aggregate).

Key result — Constant markup pricing:

$$p_i = \frac{\sigma}{\sigma - 1} \cdot \mathcal{M}$$

where $\mathcal{M}$ is marginal cost. The markup $\sigma/(\sigma-1)$ is **constant**.

Aggregate production function: $Y = AF(\mathbf{X})^\gamma$ with modified A:

$$A \equiv \left(\int a_i^{\frac{\sigma}{\sigma-1} - \gamma} \, di \right)^{\frac{\sigma-1}{\sigma-1-\gamma}}$$

Monopolistic Competition: Key Properties

- Aggregate production function same form as before
- Aggregation uses $\sigma/(\sigma-1)$ instead of $1/(1-\gamma)$
- **Advantage**: Can accommodate **constant** or even some **increasing** returns to scale
    - Monopoly power provides another force limiting firm size
- **Limitation**: Constant markup $\Rightarrow$ cannot analyze endogenous markup changes

Oligopoly: Atkeson-Burstein (2008)

Motivation: Need endogenous markups to study:

- Rising markups since the 1980s
- How changes in competition affect markups

Structure: Two layers of aggregation

1. **Across sectors**: CES with elasticity σ (same as before)

$$Y = \left(\int y_i^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}$$

2. **Within sector** i: J firms ("brands") with elasticity η

$$y_i = \left(\sum_{j=1}^{J} q_{ij}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Assumption: $\eta > \sigma > 1$ (brands more substitutable than sectors).

Endogenous Markup Formula

Pricing rule (Cournot competition):

$$\hat{p}_{ij} = \frac{\varepsilon(s_{ij})}{\varepsilon(s_{ij}) - 1} \cdot \mathcal{M}$$

where s_{ij} is the **sales share** of firm j in sector i:

$$s_{ij} = \frac{\hat{p}_{ij} q_{ij}}{\sum_h \hat{p}_{ih} q_{ih}}$$

and the effective elasticity is:

$$\varepsilon(s_{ij}) = \left(\frac{1 - s_{ij}}{\eta} + \frac{s_{ij}}{\sigma} \right)^{-1}$$

- $\varepsilon = \eta$ when $s_{ij} = 0$ (small firm $\Rightarrow$ low markup)
- $\varepsilon = \sigma$ when $s_{ij} = 1$ (monopoly $\Rightarrow$ high markup)
- Monotonically decreasing in s_{ij}

Endogenous Markups: Properties

Symmetric firms: $s_{ij} = 1/J$

$$\varepsilon = \left(\frac{1}{\eta} \cdot \frac{J-1}{J} + \frac{1}{\sigma} \cdot \frac{1}{J}\right)^{-1}$$

- $J = 1$ (monopoly): $\varepsilon = \sigma \Rightarrow$ same as monopolistic competition
- $J \to \infty$: $\varepsilon \to \eta \Rightarrow$ most competitive case
- **Fewer firms** $\Rightarrow$ **higher markups**

Asymmetric firms: firm heterogeneity in a_{ij} feeds into markup heterogeneity through sales shares.

Bertrand competition: Same formula but with

$$\varepsilon(s_{ij}) = \eta(1 - s_{ij}) + \sigma s_{ij}$$

Business Cycles and Heterogeneous Firms

Do Idiosyncratic Shocks Wash Out?

Law of Large Numbers argument:

Let GDP $Y_t = \sum_{i=1}^{N} y_{it}$, with i.i.d. growth rates: $(y_{i,t+1} - y_{it})/y_{it} = \sigma \varepsilon_{i,t+1}$

Standard deviation of GDP growth:

$$\sigma_Y = \sigma \sqrt{\sum_{i=1}^{N} \left(\frac{y_{it}}{Y_t} \right)^2}$$

Equal-size firms: $y_{it}/Y_t = 1/N$

$$\sigma_Y = \frac{\sigma}{\sqrt{N}}$$

With $N = 5{,}000{,}000$ and $\sigma \approx 15\%$: $\sigma_Y \approx 0.00007 \Rightarrow$ **negligible**!

Aggregate Shocks and Firm Dynamics

Standard approach: Accept LLN, add aggregate shock z_t:

$$y_{it} = z_t \, s_{it} \, \ell_{it}^{\gamma}$$

where $z_t =$ aggregate productivity (as in RBC models).

Results:

- Modified Hopenhayn-Rogerson model replicates well:
  - Cyclical JC and JD rates
  - Cyclical entry and exit rates
- Block-recursive structure $\Rightarrow$ computationally tractable
- Some firm-level statistics harder to match (Lee and Mukoyama, 2018)

Idiosyncratic Productivity and Aggregate: Hulten's Theorem

Setup: N sectors, sector i produces $y_i = a_i F(k_i, \ell_i, x_{i1}, \ldots, x_{iN})$

where x_{ij} = intermediate input from sector j.

Theorem (Hulten, 1978): The first-order output effect of productivity changes:

$$\frac{dY}{Y} = \sum_i D_i \frac{da_i}{a_i}$$

where D_i is the **Domar weight**:

$$D_i = \frac{p_i y_i}{\sum_i p_i c_i} = \frac{\text{Sales of sector } i}{\text{GDP (value added)}}$$

Note: Domar weights use *sales* (not value added) ⇒ can exceed 1!

⇒ Downstream firms have amplified impact through input-output linkages.

Hulten's Theorem: Intuition

Two key intuitions:

1. **Why only a_i matters** (not inputs)?
   - Envelope theorem: in efficient economy, input adjustments have no first-order welfare effect
2. **Why sales (not value added)?**
   - With input-output networks, TFP improvement in a downstream firm also raises the value of intermediate inputs

Limitations (relaxed by Baqaee and Farhi, 2019, 2020):

- Requires efficient economy (no misallocation)
- First-order approximation only

Hulten's Theorem: Simple Example

Two sectors:

- Sector 1 (consumption): $y_1 = a_1 x^{1-\gamma} \ell^\gamma$
- Sector 2 (intermediate): $y_2 = a_2 k$

Fixed K, L. Competitive equilibrium: $x = y_2$

$$Y = a_1 (a_2 K)^{1-\gamma} L^\gamma$$
$$\frac{dY}{Y} = \underbrace{1}_{D_1} \cdot \frac{da_1}{a_1} + \underbrace{(1-\gamma)}_{D_2} \cdot \frac{da_2}{a_2}$$

- $D_1 = 1$ (sales = GDP for downstream sector)
- $D_2 = 1 - \gamma$ (value added share of upstream sector)
- Domar weights sum to $1 + (1-\gamma) > 1$

Large Firms and “Granular” Fluctuations

Gabaix (2011): Fat-tailed firm size distribution changes the LLN result.

If firm sizes follow a **Pareto distribution**: $\Pr[y_i > x] = \chi x^{-\zeta}$ (the US data: $\zeta \approx 1$, called the Zipf's law)

Result: Instead of $\sigma_Y \propto \sigma/\sqrt{N}$:

$$\sigma_Y \sim \frac{v_\zeta \, \sigma}{\log(N)}$$

- $1/\log(1{,}000{,}000) \approx 0.072$ vs. $1/\sqrt{1{,}000{,}000} = 0.001$
- **Two orders of magnitude larger!**
- Idiosyncratic shocks to large firms can generate aggregate-scale fluctuations

Carvalho & Grassi (2019): quantitative business cycle model driven by idiosyncratic shocks to large firms.

Production Networks

Additional amplification: Domar weights $D_i = p_i y_i / \text{GDP}$

- Large downstream firms (Walmart, Amazon, GM): sales $\gg$ value added
- Their Domar weights can be much larger than their GDP share
- $\Rightarrow$ Idiosyncratic shocks amplified through input-output linkages

Beyond Hulten's theorem:

- **Baqaee & Farhi (2019)**: Second-order effects can be quantitatively important
- **Baqaee & Farhi (2020)**: With distortions, network structure matters for first-order effects; decompose Solow residual into misallocation changes + "pure" technology growth

Endogenous Productivity:
Klette-Kortum (2004)

Motivation

- Previous sections: productivity a_i is **exogenous**
- Natural question: What determines a_i?
- **Klette & Kortum (2004)**: Endogenous productivity from **innovation**
- Combines:
    - Quality-ladder growth models (Ch. 13)
    - Firm dynamics (entry, exit, expansion, contraction)
- Major strength: clear definition of a "firm" and its dynamics

Preferences

Representative consumer:

$$\sum_{t=0}^{\infty} \beta^t \log(C_t)$$

Consumption aggregator (unit elasticity of substitution):

$$C_t = \exp\left(\int_0^1 \log\left(\sum_{k=-1}^{J_t(j)} q_t(j,k)\, c_t(j,k)\right) dj\right)$$

- $j \in [0,1]$: product index
- k: generation (quality rung), $J_t(j) =$ cutting-edge generation
- $q_t(j,k)$: quality of product (j,k)
- $c_t(j,k)$: consumption of product (j,k)
- Different generations are **perfect substitutes** (quality-adjusted)

Intratemporal Problem

Equal expenditure shares (unit elasticity):

$$c_t(j,k) = \frac{E_t}{p_t(j,k)}$$

for each j, consumer buys only the generation with lowest quality-adjusted price.

Price index: $P_t = \exp\left(\int_0^1 [\log p_t(j,k) - \log q_t(j,k)]\, dj\right)$

Normalize $P_t = 1$.

Intertemporal Euler equation (balanced growth path):

$$\frac{1}{C_t} = \beta(1+r)\frac{1}{C_{t+1}} \quad \Rightarrow \quad \frac{1}{1+r} = \frac{\beta}{1+\gamma}$$

where γ is the consumption growth rate.

Production and Pricing

Production: 1 unit of labor $\Rightarrow$ 1 unit of any product.

- Unit cost $= w_t$ for all products

Limit pricing: The cutting-edge producer sets:

$$p_t(j, J_t(j)) = \lambda w_t$$

where $\lambda > 1$ is the **quality step** (also the markup).

Period profit per product line:

$$\pi_t = \left(1 - \frac{1}{\lambda}\right) C_t$$

- All product lines earn the same profit.
- Firm size $=$ number of product lines owned

Innovation

Innovation intensity: η per product line

- Cost: $w_t R(\eta)$ units of labor, $R(\cdot)$ increasing and convex
- Success probability: η
- If successful: gain a random product line (quality λ times the current best of that line)
- Fixed total number of products $\rightarrow$ New product always taken from another firm (creative destruction)

Let $\mu =$ probability of **losing** a product line to another innovator.

Bellman equation (per product line):

$$V_t = \max_{\eta} \; \pi_t - w_t R(\eta) + \frac{1}{1+r}(1+\eta-\mu)V_{t+1}$$

Expected continuation: $(1+\eta-\mu)V_{t+1}$ (gain with prob η, lose with prob μ).

Balanced Growth Path

Divide both sides by $(1+\gamma)^t$, use $\beta/(1+\gamma) = 1/(1+r)$ and call $v \equiv V_t/(1+\gamma)^t$:

$$v = \max_{\eta} \left(1 - \frac{1}{\lambda}\right) C_0 - R(\eta) + \beta(1+\eta-\mu)v,$$

FOC: $R'(\eta) = \beta v$

Free entry: An entrant pays c_e units of labor, gets one product line:

$$v = c_e$$

Creative destruction: $\mu = \eta + \nu$ (incumbent innovation + entry rate)

Labor market clearing:

$$\frac{C_0}{\lambda} + R(\eta) + \nu c_e = L$$

Equilibrium

Four unknowns: v, η, C_0, μ

Four equations:

$$v = \left(1 - \frac{1}{\lambda}\right) C_0 - R(\eta) + \beta(1 + \eta - \mu)v$$

$$R'(\eta) = \beta v$$

$$v = c_e$$

$$\frac{C_0}{\lambda} + R(\eta) + \nu c_e = L \qquad (\nu = \mu - \eta)$$

Growth rate: $\gamma = \mu \log(\lambda)$

- Growth driven by creative destruction rate μ and innovation step λ
- Both incumbent innovation η and entry ν contribute to growth

Firm Dynamics and Gibrat's Law

Firm size $=$ number of product lines

Average growth rate of the firm size (number of product lines per firm): $\eta - \mu = -\nu < 0$

- Net growth rate is **negative** (because entry displaces incumbents)
- Same for all firms regardless of size $\Rightarrow$ **Gibrat's Law**

Advantage over exogenous-shock models:

- Policies affect the productivity process itself
- Can analyze how innovation incentives shape firm dynamics
- Clear firm definition through product lines

Generating a Pareto Firm-Size Distribution

Problem: Basic Klette–Kortum doesn't produce a Pareto tail (average growth is negative).

Fix: Suppose large firms have positive constant growth rate $g > 0$, with exit probability δ. When the density at size n is $h(n)$,

Stationarity requires: $(1 + g)h((1 + g)n) = (1 - \delta)h(n)$

Guess Pareto distribution: $h(n) = Fn^{-(\zeta+1)} \Rightarrow$

$$\zeta = -\frac{\log(1 - \delta)}{\log(1 + g)}$$

- ζ small (thick tail) when δ small or g large
- **Modification of the Klette-Kortum model to have** $g > 0$: Allow some innovation to create **new** products.
- If new product creation ξ is large enough:
  $g = \eta - \mu = \xi - \mu > 0$

Counterfactual Predictions and Extensions

Limitations of the basic model:

1. All product lines earn equal profits
   - Unit elasticity $\Rightarrow$ revenue independent of quality
   - Fix: higher substitutability ($\sigma > 1$)
2. No cumulative innovation advantage for incumbents
   - Any firm can innovate at the same cost
   - Fix: allow incumbents to improve own products
3. No Pareto tail in firm-size distribution
   - Fix: allow new product creation alongside creative destruction
   - Luttmer (2011) discusses related insights

Summary

Key Takeaways

1. **Aggregation matters**: Firm heterogeneity affects aggregate productivity through endogenous resource allocation
2. **Data**: U.S. shows dispersed firm sizes, large reallocation, declining dynamism, rising concentration and markups
3. **Misallocation**: Firm-specific distortions reduce aggregate TFP; dispersion in distortions (not level) is what matters
4. **General equilibrium**: Firing taxes and entry barriers have substantial effects on output, productivity, and reallocation
5. **Market power**: Monopolistic competition gives constant markups; oligopoly (Atkeson-Burstein) generates endogenous markups linked to market shares
6. **Business cycles**: Fat tails and production networks can amplify firm-level shocks to aggregate fluctuations
7. **Endogenous productivity**: Klette-Kortum links innovation, creative destruction, and firm dynamics