Firm Growth through New Establishments

Dan Cao[1] Henry R. Hyatt[2]
Toshihiko Mukoyama[3] Erick Sager[4]

[1]Georgetown University [2]Census Bureau
[3]Georgetown University [4]Federal Reserve Board

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This research was conducted with restricted access to Bureau of Labor Statistics (BLS) data. The views expressed within are those of the authors and do not necessarily reflect those of the Federal Reserve System, the Census Bureau, the Bureau of Labor Statistics, or the U.S. government.

What we do

- We analyze the firm growth in relation to innovation and employment dynamics in the macroeconomic environment.
- Our starting point: a firm is a collection of establishments (locations)
- Divide firm employment growth into
  - Extensive margin: build new establishments
  - Intensive margin: add workers for given establishments.
  - For each firm,

$$\text{Employment} = \underbrace{\frac{\text{Employment}}{\text{Num. Establishments}}}_{\text{intensive margin}} \times \underbrace{\text{Num. Establishments}}_{\text{extensive margin}}$$

- Document the patterns observed in U.S. data.
- Build a innovation-driven growth model with extensive and intensive margins.
- Estimate the model to explain the observed pattern.

Is extensive margin important?

- In the firm dynamics literature, firms and establishments are often considered as identical/interchangeable.
- In the total U.S. economy, 95% of firms have only one establishment. However, they account for only 45% of employment.
- In the manufacturing sector, single-plant firms own 72% of plants but produce only 22% of the value added.
- With the fat tail in the firm size distribution, a large firm has a big impact on macroeconomic outcome ("granular dynamics").
- We find fat tails in three distributions:
  - firm size
  - intensive margin
  - extensive margin

Literature

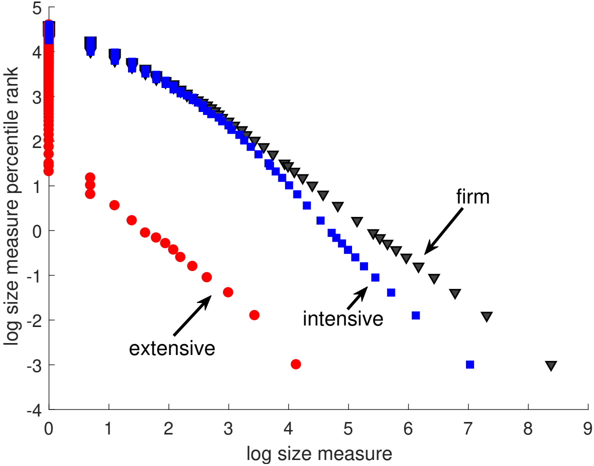
- Extensive and intensive margins: Haltiwanger, Jarmin, and Miranda (2013), Xi (2016)
- Increasing firm sizes: Choi and Spletzer (2012), Hathaway and Litan (2014)
- Increasing concentration: Autor, Dorn, Katz, Patterson, and Van Reenen (2017), De Loecker and Eeckhout (2017)
- Models of innovation and firm dynamics: Klette and Kortum (2004), Luttmer (2011)
- Estimating innovation models: Lentz and Mortensen (2008), Akcigit and Kerr (2016)
- More recent papers: Hsieh and Rossi-Hansberg (2020), Aghion, Bergeaud, Boppart, Klenow, and Li (2020), Benkard, Yorukoglu, and Zhang (2021), Rossi-Hansberg, Sarte, and Trachter (2020), Hershbein, Macaluso, and Yeh (2020).

Data

Dataset: Quarterly Census of Employment and Wages (QCEW)

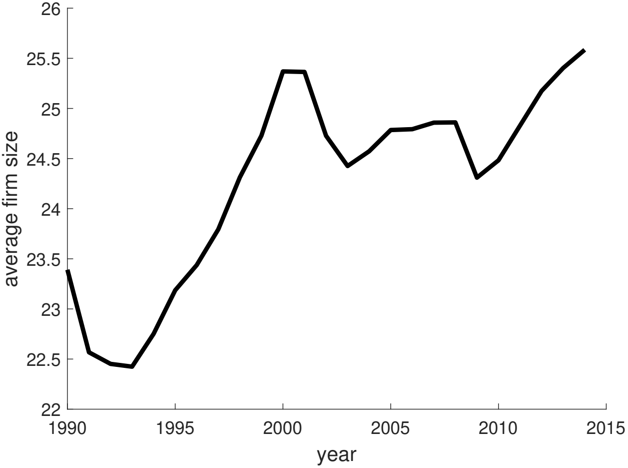
- Administrative data from state unemployment insurance program.
- Contains establishment-level monthly employment and total wage bill
- We use samples from 38 states, 1990-2014. (Some of the numbers below are from 28 states, accessed from the Census Bureau.)
- Our definition of a firm: the employer identification numbers (EINs).
  - Some large firms have multiple EINs (Wal-Mart has five).
  - Multiple EIN occurrences are rare (less than 1.5% of employment within state, for the state EIN).

Cross-sectional distribution



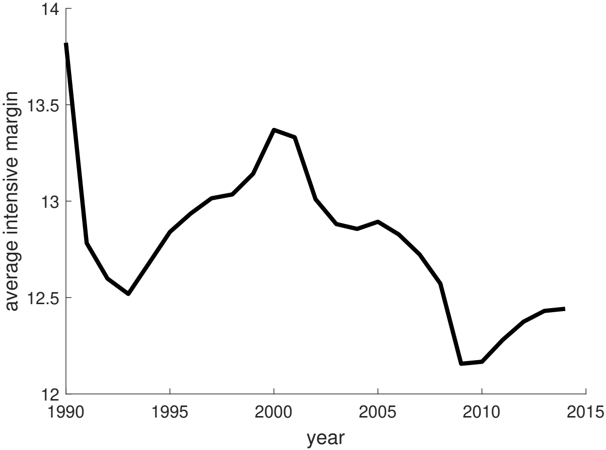
- Firm size, intensive margin, and extensive margin all have Pareto tails.

Time-series pattern: average firm size



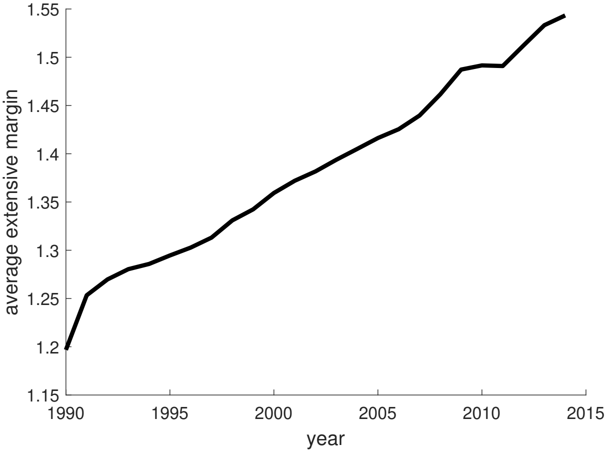
- Average firm size has increased over the last 20 years.

Time-series pattern: average intensive margin



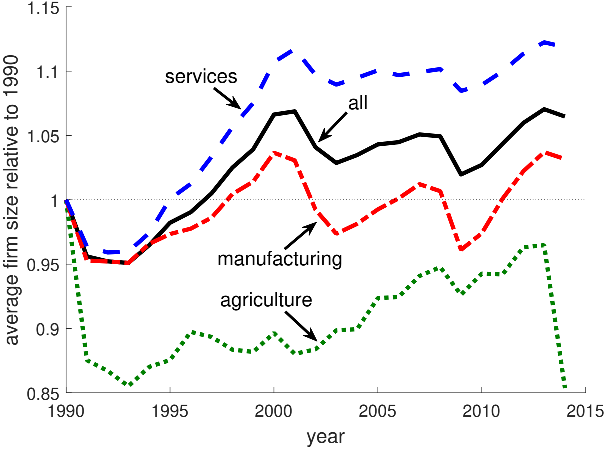
- Average establishment size has no trend (or has slightly decreased).
- The business cycle has some influence on the intensive margin.

Time-series pattern: average extensive margin



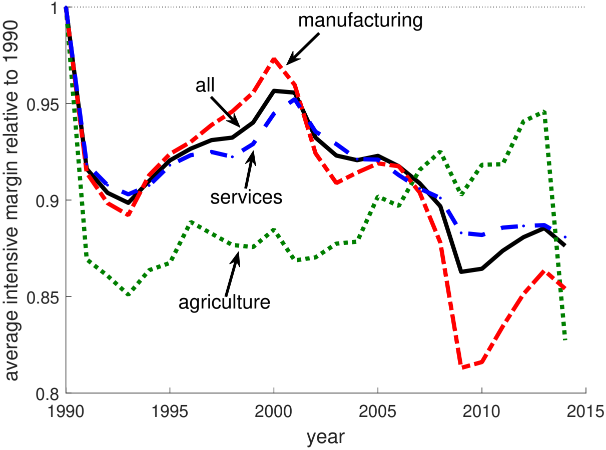
- Average number of establishments per firm has increased.

Average firm size, for different industries



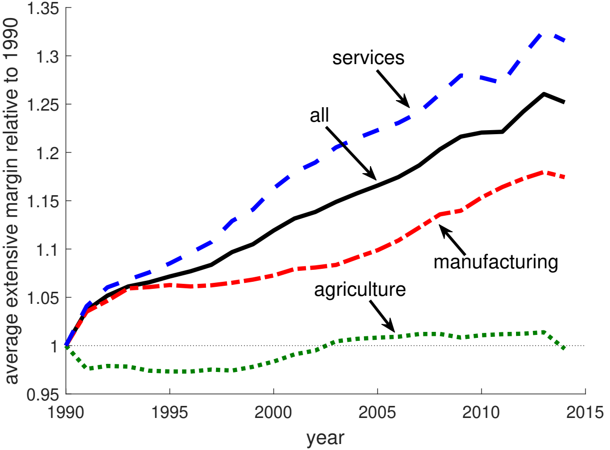
- All industries experienced the size increase during 1991-2013. Service sector has been the most significant.

Average intensive margin, for different industries



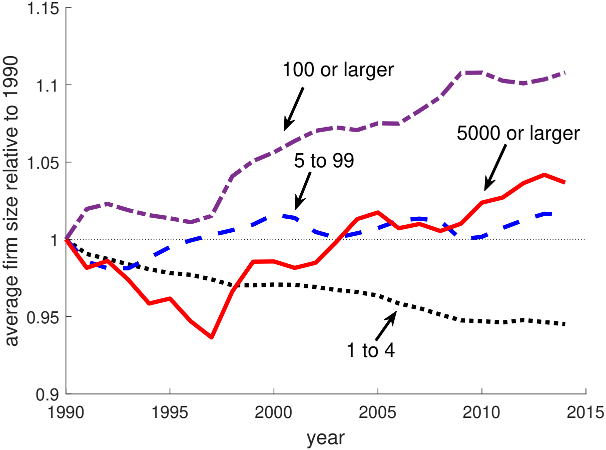
- Average establishment sizes have been either falling or constant. Agriculture shows a somewhat different trend.

Average extensive margin, for different industries



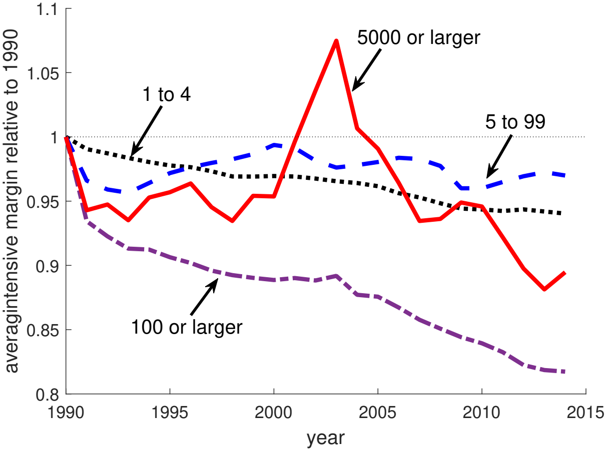
- The number has increased significantly, except for agriculture.

Average firm size, for different size bins



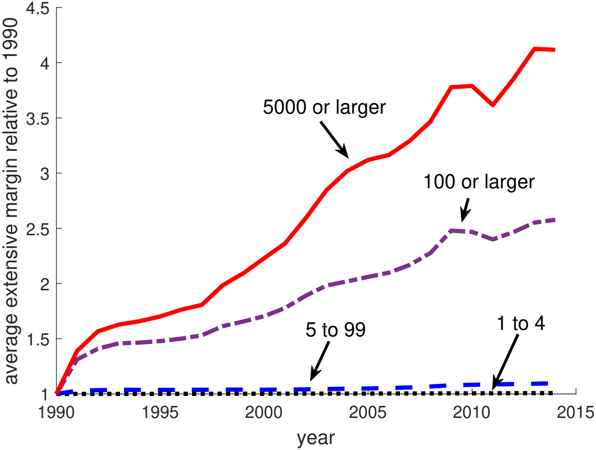
- The size increase has been driven by large firms.

Average intensive margin, for different size bins



- They have been constant (or declining) even for large firms.

Average extensive margin, for different size bins



- The number has increased dramatically for large firms.

Summary and next step

Summary of empirical finding:

- Average firm size has increased during the last 20 years.
- The firm size increase has been driven by the increase in the number of establishments per firm (extensive margin).

Next step:

- Question: What happened during the last 20 years?
  - We build an innovation-based model of firm growth and estimate it with the data moments in 1995 and 2014. We compare the estimated parameters to see what fundamental changes led to the observed changes in the distribution.

Model setup

Overall structure:

- Continuous time, infinite horizon.
- The representative consumer supplies labor inelastically; owns firms; and consumes the final good.
- The final good sector is perfectly competitive and it assembles heterogeneous intermediate goods.
- The intermediate good sector is monopolistically competitive. It uses labor to produce the goods. (Main focus of the paper.)
  - Each good has different quality.
  - Quality can be improved by innovation.
  - Firms can add new goods by innovation.
  - Innovation drives entry, exit, and expansion/contraction in both extensive and intensive margins.

Consumers

Representative consumer:

- Population grows at the rate $\gamma \geq 0$.
- Utility:

$$U = \int_0^\infty e^{-\tilde{\rho} t} L(t) u(C(t)/L(t)) dt,$$

where

$$u(C(t)/L(t)) = (C(t)/L(t))^{1-\sigma}/(1-\sigma) \text{ for } \sigma > 0 \text{ and } \sigma \neq 1$$

or

$$u(C(t)/L(t)) = \log(C(t)/L(t)).$$

Final good producers

Final goods are used for consumption, R&D, fixed operation and entry costs.

- Perfect competition.
- Production function:

$$Y(t) = \left(\int_{\mathcal{N}(t)} q_j(t)^{\beta} x_j(t)^{1-\beta} dj \right)^{\frac{1}{1-\beta}} .$$

- Inverse demand function:

$$p_j(t) = Y(t)^{\beta} \left(\frac{q_j(t)}{x_j(t)} \right)^{\beta} .$$

  Demand is higher for a higher quality good.
- Define the average quality of intermediate goods by

$$Q(t) \equiv \frac{1}{N(t)} \int_{\mathcal{N}(t)} q_j(t) dj.$$

Intermediate good producers

- Monopolistic competition.
- Each good is produced by (up to) one firm. We interpret one product as "one establishment."
- Production function:

$$x_j(t) = Z(t)\ell_j(t),$$

  where

$$Z(t) = (e^{\theta t})^{\alpha} Q(t)^{1-\alpha}.$$

  $\alpha = 1$ case is exogenous growth (the aggregate growth rate is determined only by γ, θ, and β).
- As the result of optimization, the establishment-level employment is proportional to $q_j(t)$.
  $\rightarrow$ The intensive margin directly maps into $q_j(t)$.
- The flow profit $\pi_j(t)$ is also proportional to $q_j(t)$:

$$\pi_j(t) = \bar{\pi}(t) q_j(t),$$

  where $\bar{\pi}(t)$ is constant across establishments.

Innovation for intermediate goods

Two types of innovation: internal and external. There are different types (τ) of firms that differ in innovation costs.

- Internal innovation: improve the quality of existing goods (establishments).
  - The quality of a good improves by

$$\frac{dq_j(t)}{dt} = z_{I,j}(t)q_j(t).$$

  - R&D cost for a type τ firm:

$$R_I^\tau(z_{I,j}(t), q_j(t)) = h_I^\tau(z_{I,j}(t))q_j(t).$$

- External innovation: add a new variety (establishment) with Poisson rate z_X.
  - R&D cost for a type τ firm:

$$R_X^\tau(z_{X,j}(t), q_j(t)) = h_X^\tau(z_{X,j}(t))q_j(t).$$

Innovation for intermediate goods

- Type transition occurs with Poisson rate $\lambda_{\tau\tau'}$.
- Exogenous exit rate of establishment: δ_τ.
- Exogenous exit rate of firm: d_τ.

Entry

- A firm can enter by creating a new product.
- Entry cost is $\phi Q(t)$.
- Initial draw: type probability is m_τ and the relative quality $\hat{q} = q(t)/Q(t)$ distribution is $\Phi_\tau(\hat{q})$.
- The normalized entry value:

$$v^e = \sum_\tau v_\tau m_\tau \int \hat{q} d\Phi_\tau(\hat{q}).$$

  v_τ is a value of an establishment per quality.
- We assume free entry:

$$v^e = \phi.$$

Characterization: growth rate

- Let the growth rate of $Q(t)$ be ζ (intensive margin growth) and the growth rate of $N(t)$ by η (extensive margin growth).
- It turns out that the output growth rate on a balanced growth path can be decomposed into

$$g = \zeta + \eta.$$

- The growth rate follows a simple formula

$$g = \sum_{\tau} s_\tau [z_I^\tau + z_X^\tau - (\delta_\tau + d_\tau)] + \mu_e \int \hat{q} d\Phi(\hat{q}).$$

Here, s_τ is the share of type-τ quality in total quality:

$$s_\tau \equiv \frac{\int_{\mathcal{N}_\tau(t)} q_j(t) dj}{\int_{\mathcal{N}(t)} q_j(t) dj}$$

and μ_e is the entry rate (normalized by $N(t)$).

Characterization: growth rate in one-type economy

Consider a case with one type.

- As in the general case, growth rate can be decomposed into intensive margin growth rate and extensive margin growth rate:
$$g = \zeta + \eta.$$
- Intensive margin growth rate is
$$\zeta = z_I + \mu_e \left(\int \hat{q} d\Phi(\hat{q}) - 1 \right)$$
- Extensive margin growth rate is
$$\eta = z_X - \delta - d + \mu_e.$$

Characterization: one-type economy

Consider a case with one type, and the innovation cost taking the form

$$h_i(z_i) = \chi_i z_i^{\psi},$$

where $i = I, X$.

- **Proposition:** An increase in entry cost ϕ: increases z_I and z_X; reduces μ_e.
- **Proposition:** A decrease in internal innovation cost χ_I: increases z_I and keeps z_X constant; reduces μ_e.
- **Proposition:** A decrease in external innovation cost χ_X: increases z_X and keeps z_I constant; reduces μ_e.

Characterization: distribution

- There are three distributions of interest:
  - Establishment size (intensive margin)
  - Number of establishments per firm (extensive margin)
  - Firm size (combination of both)

Distribution: one-type economy

- **Proposition:** If $\int \hat{q} d\Phi(\hat{q}) < 1$, $z_I > \zeta$ holds. In this case, the establishment size distribution has a Pareto tail with index
$$\frac{\eta + d - (z_X - \delta)}{z_I - \zeta}.$$
A large $z_I - \zeta$ makes the tail thicker.
- **Proposition:** Assuming $z_X > \delta$, the distribution of the number of establishment per firm has a Pareto tail with index
$$\frac{\eta + d}{z_X - \delta}.$$
A large $z_X - \delta$ the tail thicker.
- **Proposition:** Assume that $z_I > \zeta$ and $z_X > \delta$. When the extensive margin tail is thicker, the firm size distribution has a Pareto tail with index
$$\frac{\eta + d}{z_X - \delta + z_I - \zeta}.$$

Estimation in two steps

We assume that the innovation cost functions take the form

$$h_i^\tau(z) = \chi_i^\tau z^\psi.$$

After assigning values to the growth rate and its components $g = \eta + \zeta$ and a subset of parameters $(\beta, \sigma, \rho, \gamma, d_\tau, \delta_\tau, \psi, \alpha)$, we estimate the remaining parameters by the following two steps:

- **Step 1**: Estimate some exogenous parameters $(\lambda_{HL}, m_H, m_L, \varrho, \varsigma)$ from data moments on the distributions and the assigned numbers above. Also estimate some endogenous variables (z_i^τ, μ_e) using the same information.
- **Step 2**: Recover the exogenous parameters (χ_i^τ, ϕ) from the Step 1 information, equilibrium restrictions, and the investment-output ratio.

Estimation: assigned parameter values

Concept	Parameter	Value	Target/Source
Elasticity of demand	β	$1-(1/1.10)$	10% Markup
Intertemporal elasticity	σ	1	Log utility
Discount rate	ρ	0.01	Standard value
Population growth rate	γ	0.011	Census Bureau
Firm exit rates	d_L, d_H	0.4%, 0%	BLS
Establishment exit rates (1995)	δ_L, δ_H	12%, 12%	BLS
Establishment exit rates (2014)	δ_L, δ_H	10%, 10%	BLS
Innovation cost	ψ	2	Quadratic baseline
Partially endogenous growth	α	1	Exogenous baseline
Growth rate (1995)	g	3.1%	
Growth rate (2014)	g	2.3%	
Est/firm growth rate	η	1%	BLS

Estimated results 1

Parameter	Description	Value (1995)	Value (2014)
Innovation Investments			
z_X^H	H-type external innovation	0.3281	0.5120
z_X^L	L-type external innovation	0.0019	0.0002
z_I^H	H-type internal innovation	0.0000	0.0000
z_I^L	L-type internal innovation	0.1040	0.0766
Innovation Costs			
χ_X^H	H-type external innovation cost	0.6998	0.5569
χ_X^L	L-type external innovation cost	84.110	820.33
χ_I^H	H-type internal innovation cost	∞	∞
χ_I^L	L-type internal innovation cost	1.5099	2.3209

Estimated results 2

Parameter	Description	Value (1995)	Value (2014)
Firm Entry			
μ_E	Entry rate	0.0980	0.0740
ϕ	Entry Fixed-Cost	0.1518	0.2294
$\int \hat{q} d\Phi_H(\hat{q})$	Entrant size relative to mean	0.1000	0.1000
ϱ_H	Mean of $\Phi_H(\cdot)$	-2.8185	-3.2303
ς_H	Standard deviation of $\Phi_H(\cdot)$	1.0158	1.3622
$\int \hat{q} d\Phi_L(\hat{q})$	Entrant size relative to mean	0.5021	0.6917
ϱ_L	Mean of $\Phi_L(\cdot)$	-1.6974	-1.4150
ς_L	Standard deviation of $\Phi_L(\cdot)$	1.4201	1.4466
Firm Types			
λ_{HL}	H to L transition rate	0.2523	0.4900
m_H	Fraction of H-type at entry	0.0523	0.0878
m_L	Fraction of L-type at entry	0.9477	0.9122

Estimated results: Decomposition

	Δ in entry rate	$\%\Delta$ in est/firm
Aggregate 1995-2014	-2.47	12.15
Decomposition:		
type fraction and persistence (m_H, λ_{LH})	1.57	-18.82
establishment entry distributions	2.18	-2.40
fixed entry cost (ϕ)	-6.37	3.91
external innovation cost (χ_X^H, χ_X^L)	-0.50	16.75
internal innovation cost (χ_I^H, χ_I^L)	5.11	-1.62
establishment exit rates (δ_H, δ_L)	-3.14	10.37
growth rate g	-1.32	7.45

Conclusion

- We analyzed two margins of firm growth, the extensive margin and the intensive margin.
- Average firm size has increased over the last 20 years.
  - Driven by extensive margin expansions.
  - Observed both in manufacturing and services, but more significant in services.
  - Driven by the behavior of large firms. Right tails are important.
- We built and estimated a model of firm growth by innovations.
- The model estimation tells us (among other things), during past 20 years,
  - External innovation became easier for the firms that are active in such innovations.
  - Internal innovation became harder.
  - Entry cost became higher.

Right tail

	Firm size	Extensive	Intensive
	95th percentile and above		
1995	-1.10	-1.20	-1.35
2014	-0.99	-1.17	-1.32
	99th percentile and above		
1995	-1.17	-1.25	-1.39
2014	-0.99	-1.21	-1.24

- The right tail of the firm size distribution became thicker. Both extensive and intensive margins contribute to this change.

Firm optimization

- Hamilton-Jacobi-Bellman (HJB) equation at establishment level:

$$rV_\tau(q) - \dot{V}_\tau(q) = \max_{z_I, z_X} \left[\begin{array}{l} \pi(q) - R_I^\tau(z_I, q) - R_X^\tau(z_X, q) \\ +z_I \frac{\partial V_\tau(q)}{\partial q} q + z_X V_\tau(q) \\ -(\delta_\tau + d_\tau) V_\tau(q) \\ +\sum_{\tau'} \lambda_{\tau\tau'} (V_{\tau'}(q) - V_\tau(q)) \end{array} \right],$$

- HJB equation in balanced growth:

$$V_\tau(q) = v_\tau q$$

where

$$rv_\tau = \max_{z_I, z_X} \left[\begin{array}{l} \bar{\pi} - h_I^\tau(z_I) - h_X^\tau(z_X) + (z_I + z_X - \delta_\tau - d_\tau) v_\tau \\ +\sum_{\tau'} \lambda_{\tau\tau'} (v_{\tau'} - v_\tau) \end{array} \right].$$

- This means that the choice of innovation intensities depend only on the firm type.

Relating to the existing models

- Klette and Kortum (2004) and Lentz and Mortensen (2008) cannot generate a thick tail of firm size distribution. In their model, the expansion of incumbents is always slower than their contraction forces, because of the "quality ladder" structure.
- Luttmer's (2011) insight: if incumbents can expand without sacrificing other incumbents (due to, for example, population growth), incumbents can expand faster than their contraction forces and thus generate a thick right tail.
- Our paper: for the extensive margin, the mechanism is similar to Luttmer. For the intensive margin, if the entrants are worse than average, they do not take away demand from large incumbents and provides room for incumbents to grow very large.

Estimation

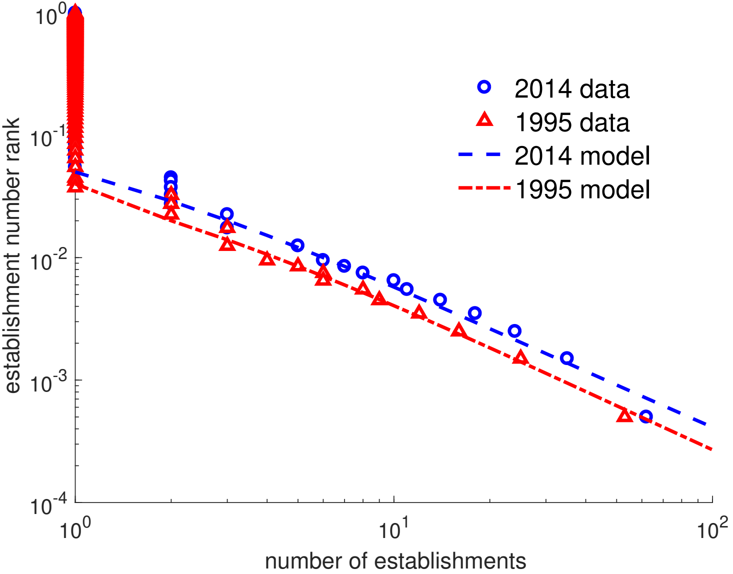
Step 1:

- Endogenous variables (z_X^H, z_X^L, μ_e) and parameters (λ_{HL}, m_H, m_L) are estimated from the distribution of the number of establishment per firm (the fraction of single-establishment firms, the tail parameter, and percentiles), and $\eta = 1\%$.
- Endogenous variables (z_I^H, z_I^L) and parameters (ϱ, ς) are estimated from the establishment size distribution (the tail parameter and percentiles) and $\zeta = g - 1\%$.

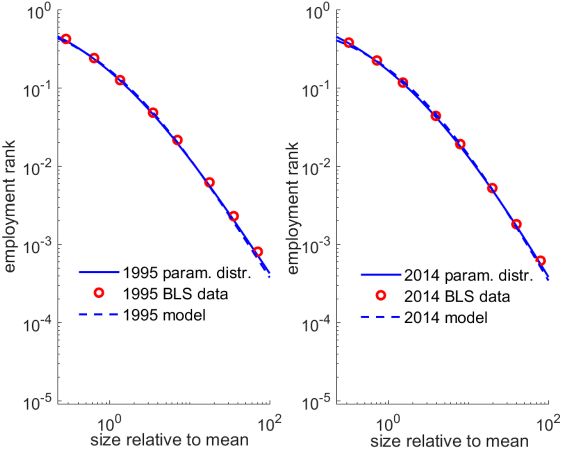
Step 2:

- Recover parameters (χ_i^τ, ϕ) from the Step 1 information, equilibrium restrictions, and the investment-output ratio$= 10\%$.

Estimation: Data fit



Estimation: Data fit



Extra: job creation and destruction

$$
\begin{aligned}
JC &= \frac{\sum_{N_{jt} > N_{j,t-1}} (N_{jt} - N_{j,t-1})}{\sum_j AE_{j,t-1}} \\
&= \frac{1}{\sum_j AE_{j,t-1}} [\underbrace{\sum_{N_{jt} > N_{j,t-1}, cont} \frac{E_{j,t-1} + E_{jt}}{2} (\bar{n}_{jt} - \bar{n}_{j,t-1})}_{\text{intensive (within)}} \\
&\qquad + \underbrace{\sum_{N_{jt} > N_{j,t-1}, cont} \frac{\bar{n}_{j,t-1} + \bar{n}_{jt}}{2} (E_{jt} - E_{j,t-1})}_{\text{extensive (between)}} \\
&\qquad + \underbrace{\sum_{entrant} \bar{n}_{jt} E_{jt}]}_{\text{entry}}
\end{aligned}
$$

Extra: job creation and destruction

$$
\begin{aligned}
JD &= \frac{\sum_{N_{jt}<N_{j,t-1}}(N_{j,t-1}-N_{jt})}{\sum_j AE_{j,t-1}} \\
&= \frac{1}{\sum_j AE_{j,t-1}}[\underbrace{\sum_{N_{jt}<N_{j,t-1},cont} \frac{E_{j,t-1}+E_{jt}}{2}(\bar{n}_{j,t-1}-\bar{n}_{jt})}_{\text{intensive (within)}} \\
&\qquad + \underbrace{\sum_{N_{jt}<N_{j,t-1},cont} \frac{\bar{n}_{j,t-1}+\bar{n}_{jt}}{2}(E_{j,t-1}-E_{jt})}_{\text{extensive (between)}} \\
&\qquad + \underbrace{\sum_{exiter} \bar{n}_{j,t-1}E_{j,t-1}]}_{\text{exit}}
\end{aligned}
$$

Extra: job creation and destruction

- Averages from our dataset:

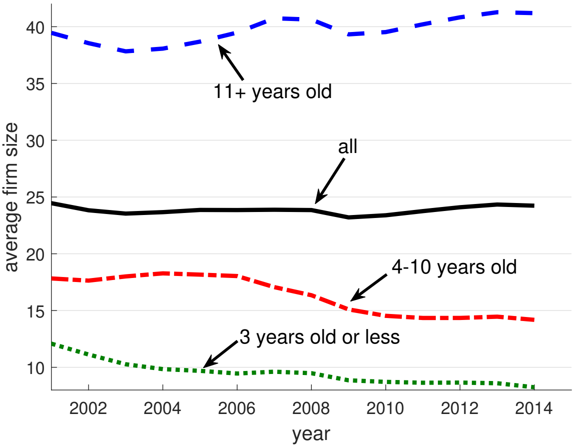
JC				JD			
Total	Intensive	Extensive	Entry	Total	Intensive	Extensive	Exit
6.8%	4.4%	1.2%	1.1%	6.4%	4.9%	0.6%	1.0%

- In terms of share:

JC			JD		
Intensive	Extensive	Entry	Intensive	Extensive	Exit
65.7%	18.2%	16.1%	76.1%	8.7%	15.1%

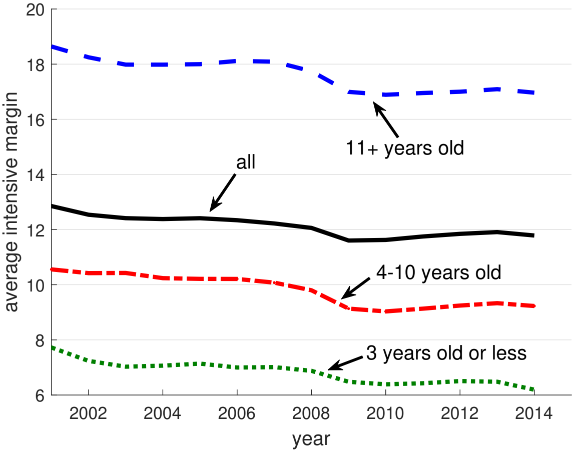
- There are notable asymmetries.

Average firm size, for different age groups



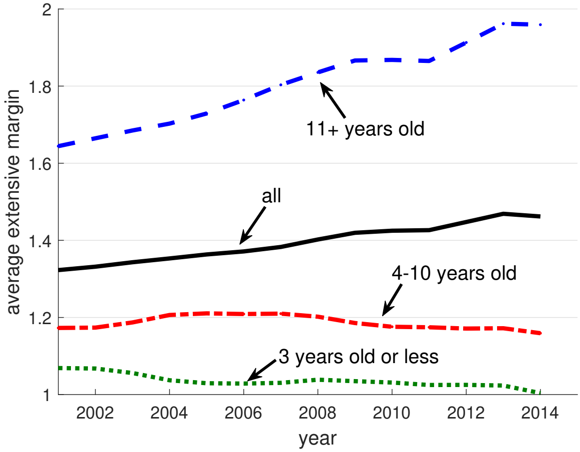
- The size increase has been observed only for old firms.

Average intensive margin, for different age groups



- They have been declining for all age groups.

Average extensive margin, for different age groups

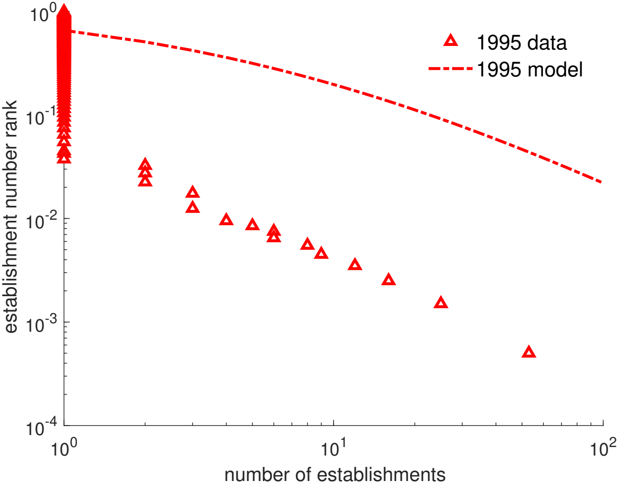


- The number has increased only for old firms.

Estimation: one-type

- $z_X = \frac{\eta+d}{1.25} + \delta = 0.1294$
- $\mu_e = \eta + d + \delta - z_X = 0.0028$
- $z_I = \frac{\eta+d+\delta-z_X}{1.40} + \zeta = 0.0230$
- $\int \hat{q} d\Phi(\hat{q}) = 1 - \frac{z_I - \zeta}{\mu_e} = 0.2857$
- matching establishment size distribution: $\varsigma = 1.1936$ and $\varrho = -1.9652$

Estimation: one-type



Estimation: One-type